SUBBAND DECOMPOSITION INDEPENDENT COMPONENT ANALYSIS AND NEW PERFORMANCE CRITERIA

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ABSTRACT

We introduce a new extended model for independent component analysis (ICA) and/or blind source separation (BSS), in which the assumption of the standard ICA model that the source signals are mutually independent (or spatio-temporally uncorrelated) is relaxed. In the new model, the source is presumed to be the sum of some independent and/or dependent subcomponents. We show a practical solution for this class of blind separation problems by using the subband decomposition (SD) and the independence test by analyzing global mixing-demixing matrices obtained for various subbands or multi-bands. This is very simple but efficient technique, and users just apply the proposed method to conventional ICA/BSS algorithms as pre- and post-processing. The method proposed in the paper has been tested for blind separation problems with partially dependent sources. The results indicate that the method is promising for the signal separation problem of speech, image, EEG data and so on.

1. INTRODUCTION

Independent component analysis (ICA) has extensively been studied for solving blind signal separation problems and several efficient algorithms have been proposed (see for overview, e.g., [1,2]). To apply ICA method for separation, generally source signals to be estimated must satisfy the strong assumption that source signals are mutually independent and non-Gaussian except one source. However, the assumption of statistical independence is very strong restriction. In fact, there are a lot of real world problems in which the independence of sources cannot be assumed. For example, the same tones generated by different instruments may have correlation, that is, they are generally dependent. Consider the estimation of brain source signals from observed electroencephalograph (EEG) data. In general, brain source signals are not completely independent. It is difficult to estimate them as long as we apply conventional ICA algorithms. How we can solve this problem and how to find whether we have extracted true sources or not (e.g. brain signals, voices, instrumental sounds and so on) are main objective of this paper.

We firstly propose a relaxed linear mixing model in which it is assumed that each source is the sum of several narrow-band sub-components, and at least two of them are mutually independent. Then, we show a solution for this problem using a surprisingly simple method consisting of linear signal decomposition for the mixtures and applying blind performance criterion for the estimated separation matrices. In this method, we apply the so-called subband decomposition (SD) for extracting independent sub-components or subbands to the observed mixtures. Then, we apply existing ICA algorithms [3–5] to several sub-components, which yields a set of the corresponding estimated separation matrices. In order to find independent subbands only from the separation matrices, we propose new performance criteria to blindly identify independent subbands. Finally, we show experimental results and compare the proposed method with the conventional ICA/BSS to claim our proposed methods to be effective in blind source separation.

2. PROPOSED MIXING MODEL AND SUBBAND DECOMPOSITION ICA

In this section, we propose the model of mixture in which the assumption of mutual independence of original signals is considerably relaxed. We propose the use of subband decomposition (SD) as a pre-processing in order to decompose the observed data into several subbands.

In the ICA problem, it is assumed that the sources $s_i(t)$ are mutually independent and non-Gaussian. We will relax this assumption of independence as follows. We assume that all sources $s_i(t)$ are not necessarily independent, but can be represented as the sum of several sub-components as

$$s_i(t) = s_{i,1}(t) + s_{i,2}(t) + \cdots + s_{i,L}(t), \tag{1}$$

where $s_{i,k}$, $k = 1, \ldots, L$, are narrow-band sub-components. We further assume that at least two of such sub-components are statistically independent. In more general scenario, we must find at least two groups of sub-component which are mutually independent.

We now consider the observed signals $x_i(t)$ which are linearly mixed by an unknown matrix $\boldsymbol{A} \in \mathbb{R}^{n \times n}$ as

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t), \tag{2}$$

where $\boldsymbol{s}(t) = [s_1(t), \ldots, s_n(t)]^T$ and $\boldsymbol{x}(t) = [x_1(t), \ldots, x_n(t)]^T$. The ICA problem is to find a separation (demixing) matrix $\boldsymbol{W}$ which gives the independent components (ICs) $\boldsymbol{y}(t) = \boldsymbol{W}\boldsymbol{x}(t)$.

Most ICA/BSS algorithms require the assumption of the independence/uncorrelatedness of the source signals. Therefore, under our relaxed assumption, those algorithms cannot be applied directly to the separation problem. Some pre-processing of the observed data is necessary. However, if we have the linear operator (or linear time-invariant filter) T_k which exactly extract at least one sub-component in (1) given as

$$\boldsymbol{s}_k(t) = T_k[\boldsymbol{s}(t)], \tag{3}$$

where $\boldsymbol{s}_k(t) = [s_{1,k}(t), \ldots, s_{n,k}(t)]^T$, then by applying the operator T_k to the mixing model (2), we have

$$\boldsymbol{x}_k(t) = T_k[\boldsymbol{x}(t)] = T_k[\boldsymbol{A}\boldsymbol{s}(t)] = \boldsymbol{A}T_k[\boldsymbol{s}(t)] = \boldsymbol{A}\boldsymbol{s}_k(t). \tag{4}$$

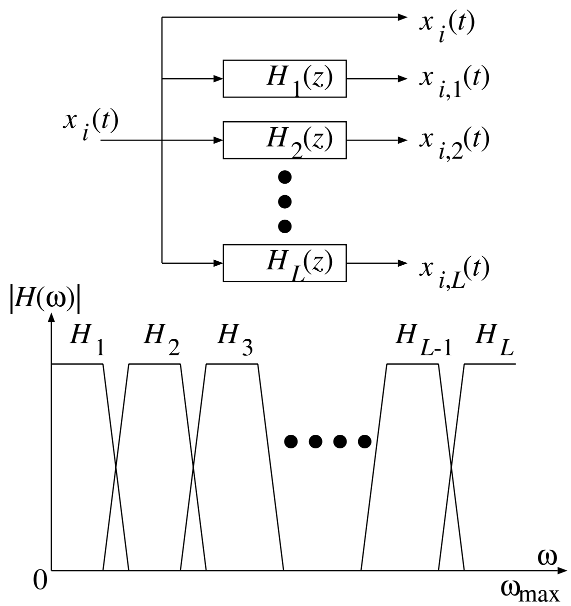


Fig. 1. Subband decomposition: A filter bank structure (top), and the associated subbands (bottom). The subbands can be overlapped or not and have more complex multi-subband forms.

This means that if we apply an ICA/BSS algorithm to $\boldsymbol{x}_k(t)$, we can obtain the separation matrix $\boldsymbol{W}_k$ with respect to $\boldsymbol{A}$. Now, the question is how we find proper operators or filters T. Of course, such filters depends on input data.

Actually, it is generally difficult to find the most appropriate filter T without information about sources. We instead introduce here a method which utilizes the SD. Figure 1 illustrates the basic structure of the SD. The transform consists of a set of bandpass filters whose transfer functions are $H_1(z), \ldots, H_L(z)$ with the associated impulse responses $h_1, \ldots, h_L$, respectively. Note that we only focus here on real-valued signals; therefore, the maximum radian frequency $\omega_{\max}$ shown in Fig. 1 is less than or equal to the Nyquist π. It depends on the energy distribution in the frequency domain of signals $\boldsymbol{x}(t)$ to be analyzed. After subband filtering, we have the L subband components given as

$$x_{i,l}(t) = (x_i * h_l)(t), \quad l = 1, \ldots, L; i = 1, \ldots, n, \tag{5}$$

where $*$ denotes the convolution. If we introduce a linear operator T_l with respect to h_l, the above equation can be written as

$$\boldsymbol{x}_l(t) = T_l[\boldsymbol{x}(t)]. \tag{6}$$

In our method, we estimate the true mixing and separating matrices using the decomposed subband observed signals instead of the original raw observed (mixed) data. After successful estimation of the true mixing matrix, we project the global raw data via the estimated separating matrix to reconstruct the original (dependent) sources. We now have another question: How can we find independent (or nearly independent) subbands from the subband signal set $\{\boldsymbol{x}_l(t)\}$. The difficulty lies in the fact we have to find them in a blind manner. An approach to this problem will be given in the following section.

3. NEW PERFORMANCE CRITERIA

3.1. Cross-Global Matrix

The key to solve this problem is a *cross-global* matrix. In the context of ICA/BSS, the global matrix $\boldsymbol{G}$ is defined as the product of the mixing and the separating matrices, i.e.,

$$\boldsymbol{G} = \boldsymbol{W}\boldsymbol{A}. \tag{7}$$

By applying an ICA/BSS algorithm several times to the observed signal $\boldsymbol{x}(t)$ and its subband signals $\boldsymbol{x}_l(t)$, we obtain the sequence of separating matrices: $\boldsymbol{W}_0, \boldsymbol{W}_1, \ldots, \boldsymbol{W}_L$, where $\boldsymbol{W}_0$ is the separation matrix estimated from the raw signal $\boldsymbol{x}(t)$ and $\boldsymbol{W}_l$, $l = 1, \ldots, L$ is that estimated from the subband signal $\boldsymbol{x}_l(t)$ by applying some ICA/BSS algorithm. Using them, we can define the set of cross-global matrices as

$$\boldsymbol{G}_{i,m} = \boldsymbol{W}_l \boldsymbol{W}_m^{-1}, \; l \neq m, \tag{8}$$

for $l, m = 0, \ldots, L$, where again $\boldsymbol{W}_l$ is the estimating separating matrix for the l-th frequency subband and $\boldsymbol{W}_l^{-1}$ means the inverse matrix which will be equal to the mixing matrix if the estimation by ICA is correctly performed, i.e., if the transformed or filtered source signals are independent.

If the specific sub-components of interest are mutually independent, at least for two subbands, say, for the subband 'l' and subband 'm', then the cross-global matrix $\boldsymbol{G}_{l,m}$ is a sparse generalized permutation matrix $\boldsymbol{P}$ with a special structure with only one non-zero (or in non-perfect case, strongly dominated) element in each row and each column. It follows from the simple mathematical observation that in such case the both matrices $\boldsymbol{W}_l$ and $\boldsymbol{W}_m$ represent the inverses of the same *true* mixing matrix $\boldsymbol{A}$ (ignoring nonessential and unavoidable arbitrary scaling and permutation of the columns). In this manner, we can blindly identify essential and very important information for which sub-components are independent.

This concept can be generalized for any linearly transformed or pre-processed signals, for example, the time-frequency or other transformed data. For each transformed data, we can easily estimate the mixing and/or separating matrices.

3.2. Blind Performance Index

We describe here a method to find the most independent subband by evaluating cross-global matrices introduced above. A well-known criterion for measuring the separation performance is *performance index* (PI) [1] which is defined as

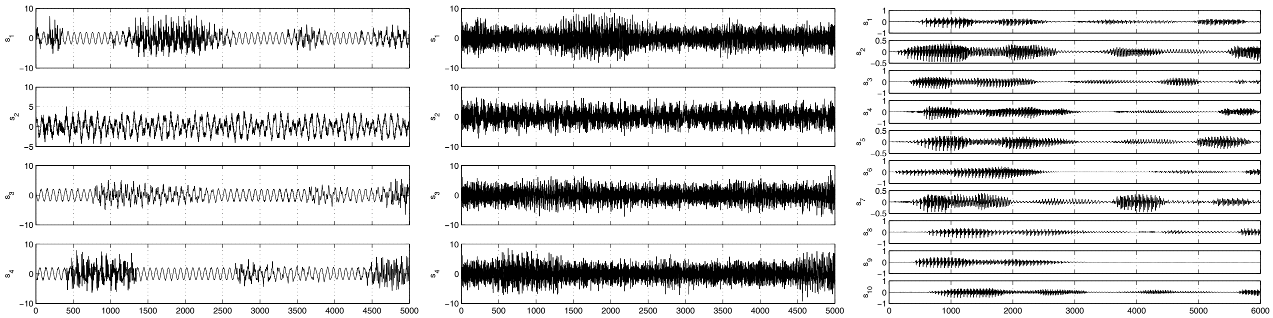
$$PI(\boldsymbol{G}) = \frac{1}{m(n-1)} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\frac{[\boldsymbol{G}]_{i,j}}{\max_j [\boldsymbol{G}]_{i,j}} - 1 \right), \tag{9}$$

where $[\boldsymbol{G}]_{i,j}$ is the (i, j)-element of the matrix $\boldsymbol{G}$. The smaller PI implies usually better performance in separation. It should be noted that the PI can be used for performance evaluation in experimental settings only, because the knowledge of the mixing matrix $\boldsymbol{A}$ is required.

In a similar way, we can define the performance index

$$BPI(\boldsymbol{G}_{l,m}) = PI(\boldsymbol{G}_{l,m}), \tag{10}$$

for the cross-global matrix $\boldsymbol{G}_{l,m}$. Since this $\boldsymbol{G}_{l,m}$ does not exploit knowledge of $\boldsymbol{A}$, we call it *blind performance index* (BPI). If the



(a) `Speech4+sin` (b) `Speech4+high` (c) `10halo`

Fig. 2. Source signals used in numerical examples.

Table 1. Performance index comparison of Experiment I: The frequency range $[0, \omega_{\max}]$ is uniformly divided into L subbands without overlapping by eighth-order Butterworth filters.

Algorithm	l_1	l_2	$BPI(\boldsymbol{G}_{l_1,l_2})$	$BPI(\boldsymbol{G}_{l_2,l_1})$	$PI(\boldsymbol{G}_{l_1})$	$PI(\boldsymbol{G}_{l_2})$	$PI(\boldsymbol{G})$
SOBI	3	5	0.140	0.111	0.0838	0.111	0.181
SANG	7	9	0.0350	0.0359	0.0240	0.0405	0.0927
Pearson	7	6	0.0317	0.0319	0.0237	0.0322	0.0949

(a) `Speech4+sin`: $L = 10$ and $\omega_{\max} = 0.982\pi$.

Algorithm	l_1	l_2	$BPI(\boldsymbol{G}_{l_1,l_2})$	$BPI(\boldsymbol{G}_{l_2,l_1})$	$PI(\boldsymbol{G}_{l_1})$	$PI(\boldsymbol{G}_{l_2})$	$PI(\boldsymbol{G})$
SOBI	2	4	0.146	0.185	0.0347	0.159	0.327
SANG	2	5	0.0527	0.0533	0.0185	0.0494	0.380
Pearson	2	5	0.0483	0.0482	0.0184	0.0468	0.403

(b) `Speech4+high`: $L = 8$ and $\omega_{\max} = \pi$.

Algorithm	l_1	l_2	$BPI(\boldsymbol{G}_{l_1,l_2})$	$BPI(\boldsymbol{G}_{l_2,l_1})$	$PI(\boldsymbol{G}_{l_1})$	$PI(\boldsymbol{G}_{l_2})$	$PI(\boldsymbol{G})$
SOBI	1	7	0.221	0.230	0.141	0.201	0.231
SANG	7	1	0.0697	0.0675	0.0313	0.0458	0.0819
Pearson	7	1	0.0635	0.0617	0.0315	0.0411	0.0762

(c) `10halo`: $L = 8$ and $\omega_{\max} = 0.652\pi$.

Table 2. Performance index comparison of Experiment II: We compare two BPIs and check consistency and validity of two different multi-band preprocessed data.

Algorithm	$BPI(\boldsymbol{G}_{1,2})$	$BPI(\boldsymbol{G}_{2,1})$	$PI(\boldsymbol{G}_1)$	$PI(\boldsymbol{G}_2)$
SOBI	0.0912	0.0860	0.0619	0.0124
SANG	0.0800	0.0639	0.0275	0.0190
Pearson	0.0662	0.0560	0.0236	0.0167

(a) `Speech4+sin`: $L = 10$, $N_1 = 3$, and $N_2 = 6$.

Algorithm	$BPI(\boldsymbol{G}_{1,2})$	$BPI(\boldsymbol{G}_{2,1})$	$PI(\boldsymbol{G}_1)$	$PI(\boldsymbol{G}_2)$
SOBI	0.139	0.0925	0.0488	0.0326
SANG	0.0274	0.0276	0.0235	0.0182
Pearson	0.0150	0.0153	0.0244	0.0248

(b) `Speech4+high`: $L = 10$, $N_1 = 3$, and $N_2 = 4$

Algorithm	$BPI(\boldsymbol{G}_{1,2})$	$BPI(\boldsymbol{G}_{2,1})$	$PI(\boldsymbol{G}_1)$	$PI(\boldsymbol{G}_2)$
SOBI	0.104	0.105	0.106	0.0412
SANG	0.0159	0.0158	0.0211	0.0152
Pearson	0.0164	0.0163	0.0215	0.0151

(c) `10halo`: Two sub-components are generated by the 1st- and the 2nd-order differentiator.

cross-global matrices $\boldsymbol{G}_{l,m}$ and $\boldsymbol{G}_{m,l}$ are the generalized permutation matrices, then the subband sources $\boldsymbol{s}_l(t)$ and $\boldsymbol{s}_m(t)$ are mutually independent. This implies that if both $BPI(\boldsymbol{G}_{l,m})$ and $BPI(\boldsymbol{G}_{m,l})$ are small enough, the subband sources $\boldsymbol{s}_l(t)$ and $\boldsymbol{s}_m(t)$ are likely to be independent. In other words, BPI helps us to judge whether the separation has succeeded or not.

4. EXPERIMENTAL RESULTS

To show the advantage, the proposed method is compared with a standard ICA. All experiments are done by using the modified version of the ICALAB [6]. The ICA/BSS algorithms applied here are SOBI (second order blind identification) [3], SANG (self adaptive natural gradient algorithm with nonholonomic constraints) [4], and Pearson-opt. (Pearson system optimized) [5].

Due to lack of space, we illustrate the performance of separation for only three data sets called

1. `Speech4+sin`: Four speech signals corrupted by sinusoids of 50 Hz with different phase delays. Specifically, we consider the case where the speech signals are corrupted as follows:

$$s_i(t) = \hat{s}_i(t) + \sin\left[2\pi\left(\frac{f}{f_m}t + \alpha_i\right)\right], \quad i = 1, \ldots, 4, \tag{11}$$

where $\hat{s}_i(t)$ is a speech signal [1], $f = 50$ Hz, f_m is a sampling frequency, and α_i is a phase delay such that $0 \leq \alpha_i < 1$ (we set in this test that $\alpha_i = (i-1)/5$). This example is somewhat artificial; however, many real world signals can

[1]Readers can obtain the speech signals $\hat{s}_i(t)$ called the benchmark `Speech4` in the ICALAB [6].

be affected by the power supply, which yields a sinusoidal interference.

2. `Speech4+high`: Four speech signals corrupted by the same additive high frequency noise. This source signal can be written as $s_i(t) = \hat{s}_i(t) + \xi(t)$, where $\xi(n)$ is a high frequency signal which is generated as a Gaussian noise. In other words, $\xi(n)$ is a time series of autoregressive model with a highpass filter.

3. `10halo`: Speech signals in which ten different people say the same sentence simultaneously. All signals may not be mutually independent.

The signals of those sets are depicted in Fig. 2. The mixing process is done by randomly generated matrices.

4.1. Experiment I

We examine the following test:

- Apply ICA/BSS for the raw mixture and all single subband signals, and obtain a set of separation matrices $\{\boldsymbol{W}_l\}_{l=0}^{L}$.
- Compute BPIs for all cross-global matrices $BPI(\boldsymbol{G}_{l,m})$, for $l, m = 0, \ldots, L$.
- Find two subbands l_1 and l_2 that correspond to the minimal BPI, i.e., $BPI(\boldsymbol{G}_{l_1,l_2}) = \min_{l,m=0,\ldots,L} BPI(\boldsymbol{G}_{l,m})$.

The performance indexes are listed in Table 1. The frequency range $[0, \omega_{\max}]$ is uniformly divided into L subbands without overlapping. It can be observed in comparison that the proposed method results in consistently superior performance to the standard ICA/BSS algorithm. It should be noted that in particular, the improvement in `Speech4+high` is significantly large.

4.2. Experiment II

The concept of the SD can be extended to the multi-band decomposition or any frequency transform. We examine here the following test:

- For `Speech4+sin` and `Speech4+high`:
  - Apply the same SD as in Experiment I, check the ℓ_p-norm (we have chosen here $p = 1/2$) of each component $\boldsymbol{x}_l(t)$, $l = 0, \ldots, L$, and choose the N_1 subband signals $\boldsymbol{x}_{l(j)}(t)$, $j = 1, \ldots, N_1$ that give the N_1 smallest norms.
  - Set $\boldsymbol{x}^{(1)}(t) = [\boldsymbol{x}_{l(1)}^T(t), \ldots, \boldsymbol{x}_{l(N_1)}^T(t)]^T$.
  - In the same way, compose $\boldsymbol{x}^{(2)}(t)$ of $N_2 \neq N_1$ subband signals that have N_2 smallest norms.
- For `10halo`: Define $\boldsymbol{x}^{(1)}(t)$ and $\boldsymbol{x}^{(2)}(t)$ as filtered signals $\boldsymbol{x}(t)$ with the first-order and second-order differentiators, respectively.
- Obtain $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ for $\boldsymbol{x}^{(1)}(t)$ and $\boldsymbol{x}^{(2)}(t)$, respectively.
- Compute BPIs for the cross-global matrices $BPI(\boldsymbol{G}_{1,2})$ and $BPI(\boldsymbol{G}_{2,1})$.

The performance indexes are listed in Table 2 for the same mixing matrix. If both $BPI(\boldsymbol{G}_{1,2})$ and $BPI(\boldsymbol{G}_{2,1})$ produce very small values, then with high probability, the estimated sources are true sources. First of all, it should be noted that the use of multiple subband signals improves separation performance significantly in many cases. For example, by using the multi-band signals for `Speech4+sin`, PI of Pearson is improved from 0.403 as shown in Table 2(b) to 0.0182 as shown in Table 3(b). It follows from Table 2 that in general, when BPIs are small, true PIs are also small. In contrast, as we see for SOBI algorithm Table 3(b), PI of the sub-component 1 is relatively large, i.e., $PI(\boldsymbol{G}_1) = 0.106$, and the corresponding two BPIs are also large. This fact shows the consistency of our results. This implies that we can blindly know that the sub-components 1 and/or 2 are not independent.

5. CONCLUSIONS AND FUTURE WORK

We have introduced a novel model for ICA/BSS, in which the source is presumed to be the superposition of independent and dependent sub-components. We have shown a method for the separating problem by using the SD and the independence test with the cross-global matrix. Our extensive experiments have confirmed that the use of the proposed procedure can often produce better performance in separation than the conventional single use of ICA/BSS algorithm. Moreover, the blind test based on BPIs tells us which sub-components are likely to be independent. Therefore, the method proposed in this paper is promising to the more general signal separation problems, especially for EEG data processing.

In similar way we can check consistency of the various ICA algorithms Let assume that two different algorithms, say, algorithm "p" and algorithm "q" generate two different separating matrix $\boldsymbol{W}_p$ and $\boldsymbol{W}_q$ If the results of multiplication $\boldsymbol{W}_p\boldsymbol{W}_q^{-1}$ is the permutation matrix or close to permutation matrix this means the both algorithms give the same consistent results. This can be useful if the number of components is large and checking the consistency by comparing and visualize them manually is very time consuming and not exact. There are several applications based on this strategy in biomedical signal processing. These problems will be addressed in the near future.

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