PERFORMANCE ANALYSIS OF SUPER-RESOLUTION BEAMFORMING IN SMART ANTENNAS

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ABSTRACT

Auto-regressive (AR) extrapolation has been in recent years used to achieve super-resolution capability in spectral estimation and in antenna beamforming. In this paper, the performance of an auto-regressive, super-resolution beamforming technique is analyzed and compared with other high-resolution methods. The AR coefficients, which represent an IIR filter, are determined adaptively using the Least Mean Square (LMS) algorithm. A linear algebra-based analysis is developed to show that the gain in signal to noise ratio is determined by the order of the extrapolation filter. It is also shown that if the filter coefficients are chosen such that there are poles on the unit circle corresponding to each source present, then the interference between the sources can be eliminated. However, if a pole is not placed on the unit circle for any given source, then it may interfere with the other sources. This will yield no improvement in signal to interference plus noise ratio. This observation is of great importance in systems such as Space Division Multiple Access (SDMA), where separating the signals from sources that utilize the same frequency resources is critical.

1. INTRODUCTION

Smart antenna systems have been proposed for increasing capacity and throughput in wireless voice and data networks [1-2]. Smart antenna systems are generally adaptive antenna arrays that can adapt to a changing signal environment. In mobile communication systems, the subscriber is generally in motion, which results in the link between the subscriber and the base station (BS) constantly changing. Smart antenna systems are equipped with fast, adaptive signal processing algorithms, which can adapt to these changes in the communication link.

In smart antennas, adaptive signal processing algorithms accomplish two objectives. First they estimate the direction of arrival (DoA) of the received signals and then use this information to form an antenna beam that places nulls in the directions of signals of no interest and maximizes the gain in the direction of the signal of interest. A multitude of algorithms have been developed for estimating the DoA of the received signals [3-5]. These techniques cover a variety of different signal processing techniques – from the standard delay-and-sum technique [3] to high resolution techniques such as auto-regressive methods [6], the Capon method [4], and signal sub-space methods such as the MUSIC algorithm [5].

High-resolution techniques such as MUSIC provide accurate estimates of DoA. However, they do not preserve the fidelity of the signals. In communications applications, the primary objective is the extraction of information from the received signal with a high degree of fidelity. This requires that the effects of system noise and multiple access interference (MAI) or co-channel interference from other users be kept to a minimum [3]. For high-resolution techniques to be useful in wireless systems, they need to be coupled with high-resolution beamforming algorithms that can preserve the fidelity of the received signals from separate sources.

For an array of size L, the beamwidth is given by λ/L. However, in most wireless applications, arrays are limited to a few elements (*e.g.,* 4). This limits the effective beamwidth of the array. In this study, spatial extrapolation is used for both direction finding and super-resolution beamforming. Unlike standard modern spectral analysis, in this application actual extrapolation is implemented to achieve super-resolution beamforming. The AR coefficients are found adaptively using the LMS algorithm [7]. This makes the algorithm well suited for wireless applications where the users are generally mobile.

2. BEAMFORMING

Figure 1 shows the standard antenna array configuration used in many beamforming problems. The antenna measurements, which are represented by vector $\hat{x}$, are multiplied by the weights of a steering vector and then summed to produce the desired output. The steering vector, $\hat{w}$, will determine the direction of the beam and the associated beam pattern. The output is given by

$$y = \hat{w}^H \hat{x} \quad (1)$$

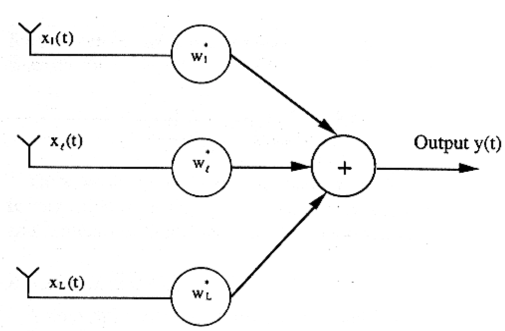

Figure 1 - Antenna array processor

In adaptive beamforming, the beamformer algorithm first estimates the directions of arrival (DoA) of the received signals and then determines the weights of the appropriate steering vector [8]. The next section discusses some of the existing high-resolution DoA algorithms and their performance.

3. HIGH RESOLUTION ALGORITHMS

Two widely used high resolution algorithms are the Minimum Variance Distortionless Response (MVDR) beamformer and the Multiple Signal Classification (MUSIC) algorithm. The spatial power spectrum of the MVDR beamformer (also known as the Capon beamformer) is given by [4]

$$P(\phi) = \frac{1}{\hat{a}^H(\phi) R_{xx}^{-1} \hat{a}(\phi)}, \quad (2)$$

where ϕ is the DoA and $R_{xx} = \left\langle \hat{x}\hat{x}^H \right\rangle$ is the spatial correlation matrix of the input signal. The vector, $\hat{a}(\phi)$, is given by

$$\hat{a}(\phi) = [1, e^{j\Delta}, e^{2j\Delta}, ... e^{j(N_0-1)\Delta}], \quad (3)$$

where $\Delta = k_0 d \sin\phi$ and d is the separation between array elements. Similarly, the spatial power spectrum for MUSIC is given by [5]

$$P(\phi) = \frac{1}{\hat{a}^H(\phi) R_{nn} \hat{a}(\phi)}, \quad (4)$$

where R_{nn} is the spatial correlation matrix of the system noise. The noise correlation matrix is determined by subtracting the signal sub-space from the measurement (signal-plus-noise) space. The signal sub-space is estimated by associating the signal sub-space with the eigenvectors corresponding to the largest eigenvalues of R_{xx}.

Having found the DoA, the next step is to estimate the different signals by designing an antenna beam that maximizes the signal strength from the signal of interest while minimizing (ideally nullifying) the interference from the signals of no interest. If nulls are placed in the directions of the signals of no interest, then the signal of interest will be noise limited since the interference is zero. Figure 2 shows the output signal to interference plus noise ratio ($SINR_o$) for two different input signal to noise ratios (SNR_i), for two transmitters separated by different angles of separation. The DoA's were estimated using the MUSIC algorithm. When the angles of separation are large, $SINR_o$ is approximately 6dB better than SNR_i. This is because the array gain of a 4-element array is 4. However, at smaller angles of separation, the gain in the direction of the signal of interest is reduced. This is because a null cannot be placed in the direction of the signal of no interest, without reducing the gain in the direction of the signal of interest. The reduction in gain will depend on the beamwidth, which is limited by the size of the array aperture.

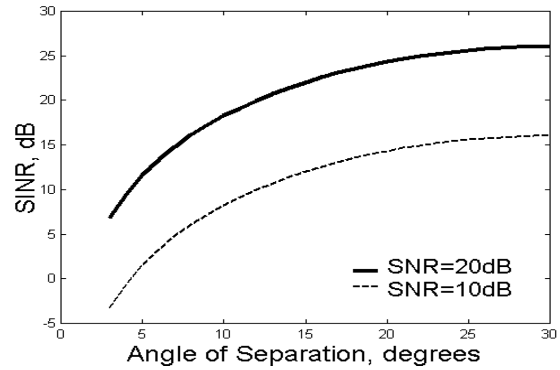

Figure 2 – Output signal to interference plus noise ratio for the noise-limited case.

Another strategy is to point the beam in the direction of the signal of interest while keeping the interference low but not zero. Such systems will be interference limited. MVDR beamformer works this way, when the sources are closely located. In this case, the main beam is pointed in the direction of the signal of interest. However, these systems will also suffer from the same dilemma – how to reduce interference without compromising the array gain in the direction of the signal of interest. Figure 3 shows this result. The $SINR_o$ at the output is plotted as a function of the angle of separation for two transmitters for different input signal to noise ratios. The array is assumed to be a 4-element array. The interference source is assumed to be at the same power level as the signal of interest. For large separations, the signal to noise plus interference ratio is given by the output signal to noise ratio of the system, which in this case is 6dB above SNR_i. However, when the transmitters are closely located, the $SINR_o$ will be interference limited and will quickly approach 0dB, which is the signal to interference ratio.

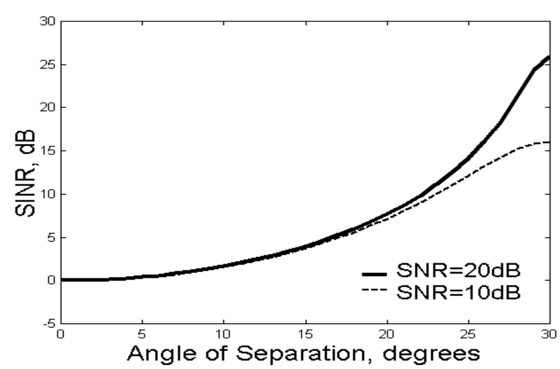

Figure 3 – Output signal to interference plus noise ratio for the interference-limited case.

Figures 2 and 3 suggest that in SDMA-type systems where the separation of signals from closely located transmitters is imperative, standard high-resolution DoA algorithms are of limited use because the high-resolution is only applicable to the DoA. These algorithms do not provide added capability as far as separating signals from closely located transmitters. This requires super-resolution beamforming where the effective beamwidth is narrower than λ/L. In the next section, the performance of AR-based super-resolution beamforming is analyzed with respect to noise and interference.

4. PERFORMANCE OF SUPER-RESOLUTION BEAMFORMING

The AR coefficients can be estimated by minimizing the error between the predicted signal and the measured signal. If $\{x_k\}$ represents the set of signals measured at each antenna element, then the linear predictor may be written as

$$x_n = -\sum_{i=1}^{N} a_i x_{n-i} \,. \qquad (5)$$

In this method, the AR beamforming coefficients are determined adaptively by using the LMS algorithm [7-8]. Applying the LMS algorithm to the antenna array elements yields

$$a_i(n+1) = a_i(n) - \mu x_n \varepsilon_i(n) \,, \qquad (6)$$

where

$$\varepsilon_i(n) = x_n + \sum_{i=1}^{N} a_i(n) x_{n-i} \,. \qquad (7)$$

Equations (6) and (7) are applied to the antenna array measurements to find the linear relationship between the antenna elements. Having found the AR coefficients, which define this linear relationship, the measurements are then extrapolated to a much larger virtual array. The synthesized array size is determined by the desired resolution of the super-resolution processing. The properties of the super-resolution technique are investigated next.

The performance of the super-resolution algorithm is studied by developing an analysis method based on linear algebra. Assume an Mth order extrapolation scheme. If $\hat{x}_{N+1}$ and $\hat{x}_N$ are vectors which represent the sequence of values being extrapolated, such that

$$\hat{x}_N = [x_N \quad x_{N-1} \quad x_{N-2} \quad \ldots \quad \ldots \quad x_{N-M+1}]^T \,, \qquad (8)$$

then

$$\hat{x}_{N+1} = \begin{bmatrix} a_1 & a_2 & \ldots & a_M \\ 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ 0 & \ldots & 1 & 0 \end{bmatrix} \hat{x}_N = H\hat{x}_N \,. \qquad (9)$$

H is an $M\times M$ matrix which can be written in terms of its spectral components as [9]

$$H = \sum_{i=1}^{M} \lambda_i E_i \,, \qquad (10)$$

where the λ_i's are the eigenvalues of H and the E_i's represent the spectral components given by [9]

$$E_i = P D_i P^{-1} \,, \qquad (11)$$

where P represents the matrix consisting of the eigenvectors of H such that

$$P^{-1} H P = D \,, \qquad (12)$$

and D_i is the diagonal matrix, where all the diagonal elements except the ith value of D are set to zero. The eigenvalues of H are given by the roots of its characteristic equation,

$$\lambda^M - a_1 \lambda^{M-1} - a_2 \lambda^{M-2} - \ldots\ldots - a_M = 0 \,. \qquad (13)$$

Equation (13) is the same polynomial equation, which gives the poles of the IIR filter [7]. Hence the eigenvalues of H are the same as the complex poles of the Mth order IIR filter defined by the filter coefficients, $[a_1, a_2, \ldots. a_M]$. Using equation (9), the extrapolation can be generalized for any n as [9]

$$\hat{x}_{N+n} = H^n \hat{x}_N = \left[\sum_{i}^{M} \lambda_i^n E_i\right] \hat{x}_N \,. \qquad (14)$$

This is because $E_i E_j = \delta_{ij}$ [9]. From equation (14), it is clear that only E_i's where $|\lambda_i| \geq 1$ will contribute towards the extrapolation for large values of n. For high signal to noise ratios, the LMS algorithm will produce a single pole on the unit circle for directions corresponding to each of the received signals, while the other poles will fall inside the unit circle [7]. From equation (14), only these poles on the unit circle will contribute towards $\hat{x}_{N+n}$. Equation (14) can then be re-written as

$$\hat{x}_{N+n} \approx \left[\sum_{i}^{M_0} \lambda_i^n E_i\right] \hat{x}_N \,, \qquad (15)$$

where $M_0 < M$ is the number of sources.

4.1 Noise Performance

In standard array processing, the improvement in signal to noise ratio is given by the number of elements in the array. It appears that in the case of extrapolation, the improvement in signal to noise ratio could be quite significant. This would be true if the noise in the extrapolated array was uncorrelated as it is in the case of the real array. However, during extrapolation, noise is extrapolated in such a manner that noise from the portion of the spatial noise spectrum that corresponds to the directions of the received signals will be extrapolated through the elements of the virtual array. Thus the noise in the extrapolated virtual array is not uncorrelated.

The properties of noise can be studied using equation (15). Equation (15) says that for large n, only signal components in a sub-space of dimension M_0, from a measurement space of M, will be extrapolated. If one uses an Mth order extrapolation with only one pole on the unit circle ($M_0 = 1$), then all but one of the spectral components will fall out of equation (15). If the noise is assumed to be white, it will be distributed equally in all M dimensions of the measurement space. However, only noise in a single direction will get extrapolated out for large n. This will yield a signal to noise improvement of M, which is the order of the extrapolation.

4.2 Interference Rejection

The poles of the IIR filter will correspond to the DoA of the different received signals [7]. The LMS algorithm will automatically determine the set of AR coefficients. One of the implicit properties of the AR process is that the IIR filter automatically decouples the contributions from all the poles. However, this will only occur if the poles of the filter are properly associated with DoA of all the sources.

Interference rejection can be studied by using equation (15). If there are two sources and an Mth order extrapolation is used with a single pole corresponding to the direction of one of the sources, on the unit circle ($M_0 = 1$), then the interference between the second source and the first source will depend on the operation $E_1\hat{x}_{02}$, where $\hat{x}_{02}$ is the vector which represents the actual measurements from the second source. For this case, $E_1 = \hat{x}_{01}\hat{x}_{01}^H$, where $\hat{x}_{01}$ is the vector which represents the measurements from the first source. Thus the interference will be given by $\hat{x}_{01}^H\hat{x}_{02}$, which is the projection of the measurement vector representing the second source onto the measurement vector representing the first source. This will be exactly the same as what is shown in Figure 3, where the main beam is pointed at the first source and the interference will depend on the projection of the second source onto the direction of the first source.

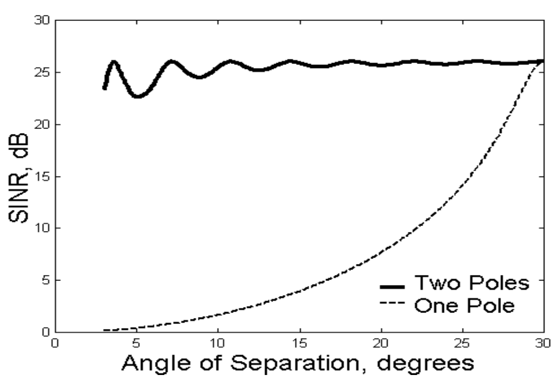

Figure 4 - Output signal to interference plus noise ratio for the extrapolated array where n=32.

If instead of one pole, two poles corresponding to the directions of each of the two sources are used, then since $E_iE_j = \delta_{ij}$, $E_1\hat{x}_{02} = 0$ and $E_2\hat{x}_{01} = 0$. Thus there will be no interference between the two sources. Figure 4 shows the $SINR_o$ for two separate fourth order extrapolation cases with two sources of equal power separated by different angles of separation. SNR_i is assumed to be 20dB for both cases. The extrapolation is carried out for n=32. The first extrapolation is done using a single pole corresponding to the signal of interest on the unit circle. The second extrapolation is done using two poles corresponding to the two directions of the two sources, on the unit circle. In the first case, $SINR_o$ follows the same general trend as the corresponding curve in Figure 3. In the second case, $SINR_o$ remains flat around 26dB (a gain of 4) down to very small angles of separation, when it appears to decrease because the extrapolation distance was not sufficient to separate the two sources. Increasing the extrapolation distance can improve the performance for these angles of separation.

5. CONCLUSIONS AND FUTURE WORK

A linear algebra-based formulation is used to analyze the performance of super-resolution beamforming techniques, which utilize AR models. It is shown that the improvement in signal to noise ratio is given by the order of the AR process. It is also shown that if the AR coefficients produce poles on the unit circle for the directions corresponding to each of the received sources, then it is possible to suppress the interference between the sources. This observation is of great importance in systems such as Space Division Multiple Access (SDMA), where separating the signals from sources that utilize the same frequency resources is an absolute imperative.

The analysis presented in this paper deals with direct path propagation only. In the presence of multipath propagation, reducing interference will be significantly more complicated. In the future, attempts will be made to minimize multipath interference using super-resolution beamforming. Algorithms, which utilize the correlation between the multipath components, will be investigated.

6. REFERENCES

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