PAPR REDUCTION VIA A FIXED FREQUENCY-DOMAIN WEIGHTING ACROSS MULTIPLE OFDM BAUDS

Timothy A. Thomas
Motorola Labs - Communication Systems Research Laboratory
1301 E. Algonquin Road, Schaumburg, IL 60196
T.Thomas@motorola.com

ABSTRACT

This paper introduces a PAPR reduction technique for OFDM that uses the same PAPR reduction weights over multiple OFDM bauds. The PAPR reduction weights are applied on each subcarrier in the frequency domain and are designed to either be fixed over a group of subcarriers (in addition to multiple bauds) or the weights are parameterized by time taps. The advantage of these methods is to simplify channel estimation by not destroying the channel frequency correlation (for weights fixed over groups of subcarriers) or by increasing the effective length of the channel to be estimated (for the weights parameterized by time-taps). Simulation results show up to a 6.0 dB decrease in PAPR where PAPR is measured over an OFDM baud.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) [1] is a frequency-domain communication method that not only is being considered for future communication systems but is in current standards (e.g., 802.11a). It is well know that OFDM has a very poor Peak-to-Average Power Ratio (PAPR) and various solutions have been proposed to improve the PAPR [2]-[29]. These existing PAPR reduction techniques can be broken up into seven categories: 1) Clip and filter [2]-[5]. These techniques apply a type of clipping of the time-domain signal and then apply a filter to improve the frequency-response of the signal. 2) Mapping [6]-[12]. These techniques map the original symbols to an alternative symbol set that results in lower PAPR. These techniques also include using coding techniques designed to lower the PAPR. Typically, side information must be sent with these techniques to identify which alternative symbol set (or code type) was used. 3) Partial transmit sequences [13]-[19]. These techniques operate on a single OFDM baud and apply a different weight on non-overlapping groups of subcarriers in the frequency domain. The weights are determined to lower the PAPR on the single baud. These techniques can rely on side information to determine the weight or can embed pilots in each subcarrier grouping. 4) Additional redundancy [20]-[22]. These techniques typically reserve subcarriers for the sole purpose of PAPR reduction and then fill these subcarriers with symbols designed to lower the PAPR of the baud. Because these subcarriers do not carry data, these techniques lower the system throughput. 5) Signal addition [23]-[25]. These techniques add a signal either in the time domain or the frequency domain that lowers the PAPR on a single baud. Side information is typically used to determine which sequence is used. 6) IFFT spreading [26][27]. These techniques transmit spread OFDM where the spreading sequence is the IFFT or a similar transform that gives low PAPR in the time domain. The PAPR of these techniques is similar to single carrier and could benefit from further reduction especially for higher order modulations. 7) Constellation alteration [28][29] techniques modify the transmitted signal constellation to improve the PAPR on a single baud. The modifications will slightly effect BER performance at the receiver since the constellation shape is changed from what the receiver is expecting.

One common thread between all of these PAPR reduction techniques is that they all operate on a single OFDM baud at a time (where a baud is defined as the group of K symbols that are IFFT'd at the transmitter). In this paper, weights are designed that are fixed over B OFDM bauds and reduce the PAPR of all B bauds. By fixing the weights in time, channel estimation can be simplified since the weights can be determined by the receiver through the normal channel estimation process. Also, the proposed methods do not send any side information to identify the frequency-domain weighting and therefore the throughput is unchanged. The first method finds one weight on each non-overlapping group of subcarriers and B OFDM bauds, and the second method parameterizes the frequency-domain weights (that are fixed over B bauds) by a number of time taps.

2. OFDM SYSTEM DESCRIPTION

An OFDM system is assumed with an FFT size of N with K usable subcarriers. The frequency-domain data or pilot symbols on subcarrier k and baud b, $X(k,b)$, are multiplied by a frequency-dependent weight, $V(k)$, that is designed to lower the PAPR on each of the B bauds in a transmission block ($0 \le k \le K-1$ and $0 \le b \le B-1$). The transmitted symbols, $S(k,b)$, are given by:

$$S(k,b) = V(k)X(k,b) \quad (1)$$

The over-sampled (by Q) time-domain symbols on block b, $s(n,b)$ are given as ($0 \le n \le NQ-1$):

$$s(n,b) = \sum_{k=0}^{K-1} S(k,b) e^{j2\pi kn/NQ} \quad (2)$$

For best PAPR performance, the algorithms presented below should sample $s(n,b)$ by at least Q=4 to capture time-domain peaks that happen between sample-spaced samples.

3. PAPR WEIGHTS FIXED OVER TIME AND BLOCKS OF SUBCARRIERS

To be able to use channel estimators that work on groups of subcarriers, $V(k)$ must be constant over the group. In equation form, the transmitted frequency-domain symbols are given as:

$$S(k,b) = V_B\left(\left\lfloor \tfrac{N_B k}{K} \right\rfloor\right) X(k,b) \quad (3)$$

where N_B is the number of subcarrier groups, $V_B(\ell)$ is the weight on group ℓ, and $\lfloor x \rfloor$ means the largest integer less than or equal to x. $\mathbf{V}_B=[V_B(0),\ldots,V_B(N_B\text{-}1)]^T$ can be found using the following gradient approach. The algorithm steps are:

1. Set $\mathbf{V}_B$ equal to a $N_B\times 1$ vector of all ones, choose a PAPR target threshold, P_t, choose the maximum number of iterations, N_I, and choose a step size, α.
2. Set the counter, i=0, and calculate $s(n,b)$ for $n=0,\ldots,NQ\text{-}1$ and $b=0,\ldots,B\text{-}1$.
3. Determine the time and baud number pair (k_ℓ, b_ℓ) for the $\ell=1\ldots N_p$ peaks in $s(n,b)$ that are above the threshold P_t up to a maximum number of peaks, $N_{p,max}$.
4. Determine the $N_B\times 1$ gradient vector, $\mathbf{g}_B$, according to:

$$\mathbf{g}_B=\sum_{\ell=1}^{N_p}\mathbf{U}^T\mathbf{X}^*(b_\ell)\mathbf{f}(n_\ell)\mathbf{f}^H(n_\ell)\mathbf{X}(b_\ell)\mathbf{U}\mathbf{V}_B \tag{4}$$

where $K\times K$ $\mathbf{X}(b)=\mathrm{diag}(X(0,b),\ldots,X(K\text{-}1,b))$ and $K\times 1$ $\mathbf{f}(n)$ and $K\times N_B$ $\mathbf{U}$ are:

$$\mathbf{f}(n)=\begin{bmatrix}1\\ e^{j2\pi n/NQ}\\ \vdots\\ e^{j2\pi(K-1)n/NQ}\end{bmatrix} \tag{5}$$

$$\mathbf{U}=\begin{bmatrix}\mathbf{1} & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & \mathbf{1} & \ddots & \cdots\\ \vdots & \ddots & & \mathbf{0}\\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{1}\end{bmatrix} \tag{6}$$

where $\mathbf{1}$ is an $\frac{K}{N_B}\times 1$ vector of all ones and $\mathbf{0}$ is an $\frac{K}{N_B}\times 1$ vector of all zeros.

5. Update $\mathbf{V}_B=\mathbf{V}_B-\alpha\mathbf{g}_B$.
6. Normalize $\mathbf{V}_B$ according to one of the following:

   a. $\mathbf{V}_B=\dfrac{\sqrt{N_B}\mathbf{V}_B}{\sqrt{\mathbf{V}_B^H\mathbf{V}_B}}$

   b. $V_B(\ell)=e^{j\angle V_B(\ell)}$ where $\angle V_B(\ell)$ means the phase of $V_B(\ell)$.

   c. $V_B(\ell)=\begin{cases}V(\ell)\text{ if }\beta_1\le|V(\ell)|\le\beta_2\\ \beta_1 e^{j\angle V(\ell)}\text{ if }V(\ell)<\beta_1\\ \beta_2 e^{j\angle V(\ell)}\text{ if }V(\ell)>\beta_2\end{cases}$ plus $\mathbf{V}_B=\dfrac{\sqrt{N_B}\mathbf{V}_B}{\sqrt{\mathbf{V}_B^H\mathbf{V}_B}}$

7. Update the counter, $i=i+1$, and calculate $s(n,b)$ for $n=0,\ldots,NQ\text{-}1$ and $b=0,\ldots,B\text{-}1$.
8. Determine the time and baud number pair (k_ℓ, b_ℓ) for the $\ell=1\ldots N_p$ peaks in $s(n,b)$ that are above the threshold P_t up to a maximum number of peaks, $N_{p,max}$.
9. If N_p=0 or if $i=N_I$ then stop, otherwise go to Step 4.

4. PAPR REDUCTION WEIGHTS THAT ARE PARAMETERIZED BY TIME TAPS

The frequency-domain weights, $V(k)$, for this method are given as the N_t-point DFT of L time-domain weights, $v(n)$, as follows:

$$V(k)=\sum_{k=0}^{L-1}v(n)e^{-j2\pi kn/N_t} \tag{7}$$

The advantage of parameterizing the frequency-domain weights by time-domain weights is when channel estimation is performed. The reason for the advantage is that the channel estimator sees a composite channel that is the original channel multiplied by the frequency-domain weights:

$$\widetilde{H}(k)=H(k)V(k) \tag{8}$$

Therefore if the channel has a finite time duration (i.e., $H(k)$ can be expressed as the N_t-point DFT of L_H time-taps, $h(n)$), then the composite channel can be expressed as the N_t-point DFT of L_H+L time-taps (i.e., $h(n)$ convolved with $v(n)$).

In matrix form, the relationship between the frequency-domain weights and the time-domain weights is given as:

$$\mathbf{V}=\mathbf{F}\mathbf{v} \tag{9}$$

where $L\times 1$ $\mathbf{v}=[v(0),\ldots,v(L)]^T$ and $K\times L$ $\mathbf{F}$ is:

$$\mathbf{F}=\begin{bmatrix}1 & 1 & \cdots & 1\\ 1 & e^{-j2\pi/N_t} & \cdots & e^{-j2\pi(L-1)/N_t}\\ \vdots & & & \vdots\\ 1 & e^{-j2\pi(K-1)/N_t} & \cdots & e^{-j2\pi(L-1)(K-1)/N_t}\end{bmatrix} \tag{10}$$

The algorithm steps are:

1. Set $\mathbf{v}=[v(0),\ldots,v(L\text{-}1)]^T$ equal to a $L\times 1$ vector of all zeros except $v(L/2)=1$, choose a PAPR target threshold, P_t, choose the maximum number of iterations, N_I, and choose a step size, α.
2. Set the counter, i=0, and calculate $s(n,b)$ for $n=0,\ldots,NQ\text{-}1$ and $b=0,\ldots,B\text{-}1$.
3. Determine the time and baud number pair (k_ℓ, b_ℓ) for the $\ell=1\ldots N_p$ peaks in $s(n,b)$ that are above the threshold P_t up to a maximum number of peaks, $N_{p,max}$.
4. Determine the $L\times 1$ gradient vector, $\mathbf{g}_t$, according to:

$$\mathbf{g}_t=\sum_{\ell=1}^{N_p}\mathbf{F}^H\mathbf{X}^*(b_\ell)\mathbf{f}(n_\ell)\mathbf{f}^H(n_\ell)\mathbf{X}(b_\ell)\mathbf{F}\mathbf{v} \tag{11}$$

where $K\times K$ $\mathbf{X}(b)=\mathrm{diag}(X(0,b),\ldots,X(K\text{-}1,b))$ and $K\times 1$ $\mathbf{f}(n)$ is given in (5).

5. Update $\mathbf{v}=\mathbf{v}-\alpha\mathbf{g}_t$.
6. Normalize $\mathbf{v}$ according to one of the following:

   a. $\mathbf{V}=\mathbf{F}\mathbf{v}$, $\quad\mathbf{v}=\dfrac{\sqrt{K}\mathbf{v}}{\sqrt{\mathbf{V}^H\mathbf{V}}}$

   b. $V(k)=e^{j\angle V(k)}$ where $\angle V(k)$ means the phase of $V(k)$. Then $v(\ell)$ is the first L samples of the N_t–point IFFT of $V(0)$ through $V(K\text{-}1)$.

   c. $V(k)=\begin{cases}V(k)\text{ if }\beta_1\le|V(k)|\le\beta_2\\ \beta_1 e^{j\angle V(k)}\text{ if }V(k)<\beta_1\\ \beta_2 e^{j\angle V(k)}\text{ if }V(k)>\beta_2\end{cases}$ plus $\mathbf{V}=\dfrac{\sqrt{K}\mathbf{V}}{\sqrt{\mathbf{V}^H\mathbf{V}}}$. Then $v(\ell)$ is the first L samples of the N_t–point IFFT of $V(0)$ through $V(K\text{-}1)$.

7. Update the counter, $i=i+1$, and calculate $s(n,b)$ for $n=0,\ldots,NQ\text{-}1$ and $b=0,\ldots,B\text{-}1$.
8. Determine the time and baud number pair (k_ℓ, b_ℓ) for the $\ell=1\ldots N_p$ peaks in $s(n,b)$ that are above the threshold P_t up to a maximum number of peaks, $N_{p,max}$.
9. If N_p=0 or if $i=N_I$ then stop, otherwise go to Step 4.

5. SIMULATION RESULTS

The simulated OFDM system has an FFT size of N=1024 with K=750 subcarriers used for data. The baud duration is 50 μsec with a cyclic prefix length of 256 samples (10 μsec). All algorithms use N_I=50 iterations and an over-sampling factor, Q=4. The PAPR is measured on the eight times over-sampled OFDM time-domain waveform and is calculated on a per-baud basis (i.e., the peak is found over all 8192 samples in an OFDM over-sampled baud and the PAPR statistics are calculated over many OFDM baud realizations).

Figure 1 shows a PAPR reduction example for weights that are different on each subcarrier (i.e., N_B=K=750) but fixed over 26 bauds. The left side of the plot shows the original oversampled OFDM waveform (without cyclic prefixes) and the right side of the plot shows the PAPR reduced waveform. As can be seen, there is significant PAPR reduction. To quantify the statistical performance of the algorithm, 11,000 Monte Carlo simulations were run and the results (in dB) are presented in Table 1 for different criteria on the weights. As the weight criteria gets more stringent (i.e., the weight amplitudes go from having any value to being constant modulus), the PAPR reduction is reduced. Table 2 shows similar results for weights that are fixed over 10 subcarriers (i.e., N_B=75) as well as being fixed over B bauds. Finally, Table 3 shows the results for weights that are parameterized by L=256 time taps with a DFT size of N_t =1024. Only the full frequency search was performed for these weights since the BER performance was satisfactory as is shown next.

Figure 2 shows BER results (without channel estimation) for an AWGN channel to show the effects of the weights (that are different on each subcarrier but fixed over 26 bauds) on BER performance. Very little impact on BER performance is seen when the weight amplitudes are restricted (in fact in a Rayleigh-faded frequency-selective channel, the difference between all of the amplitude limiting schemes is less than 0.1 dB). The BER performance for the weights fixed over 10 subcarriers is identical to these results. Finally, Figure 3 shows the BER performance of the weights parameterized by 256 time taps. The performance without restricting the weight amplitudes ("Full search") is not too degraded because the initial weights start out as a delta function in the time domain and the final weights do not deviate too far from a delta function.

6. CONCLUSION

This paper introduced PAPR reduction weights for OFDM where the weights are applied in the frequency domain to the transmitted symbols but are fixed in time over multiple bauds. By fixing the weight in time, channel estimation can be simplified. Since the weights can be determined by the receiver through the normal channel estimation process, no side information is required. In addition, the weights can be designed to retain the frequency correlation in the channel thus further assisting channel estimation. Simulation results show up to a 6.0 dB decrease in PAPR.

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Table 1. Results (in dB) for different weights on each subcarrier

Search type	B	90% CDF point	99% CDP point	99.9% CDF point
No reduction	-	10.0062	10.9301	11.6175
Full search	6	4.0564	4.121	4.2262
Full search	16	5.114	5.2581	5.3777
Full search	26	5.6515	5.7933	5.9085
$.8<\|V(k)\|<1.2$	6	4.3738	4.5126	4.6418
$.8<\|V(k)\|<1.2$	16	5.4658	5.5879	5.6796
$.8<\|V(k)\|<1.2$	26	5.9605	6.0634	6.1296
$.9<\|V(k)\|<1.1$	6	4.5769	4.7208	4.87
$.9<\|V(k)\|<1.1$	16	5.6244	5.7454	5.8465
$.9<\|V(k)\|<1.1$	26	6.1079	6.219	6.3219
$\|V(k)\|=1$	6	4.8822	5.187	5.9247
$\|V(k)\|=1$	16	5.8941	6.1433	6.6823
$\|V(k)\|=1$	26	6.3573	6.5511	6.9681

Table 2. Results for fixed weights over groups of 10 subcarriers

Search type	B	90% CDF point	99% CDP point	99.9% CDF point
No reduction	-	10.0062	10.9301	11.6175
Full search	6	6.5095	6.8389	7.7294
Full search	16	7.3575	7.604	8.1405
Full search	26	7.729	7.9294	8.3917
$.8<\|V(k)\|<1.2$	6	6.8687	7.2579	7.964
$.8<\|V(k)\|<1.2$	16	7.6258	7.9007	8.512
$.8<\|V(k)\|<1.2$	26	7.9546	8.1681	8.5261
$.9<\|V(k)\|<1.1$	6	7.0437	7.5456	8.3215
$.9<\|V(k)\|<1.1$	16	7.7603	8.1249	8.7837
$.9<\|V(k)\|<1.1$	26	8.0837	8.3733	8.9078
$\|V(k)\|=1$	6	7.4997	8.3043	9.0917
$\|V(k)\|=1$	16	8.0879	8.6965	9.3225
$\|V(k)\|=1$	26	8.3588	8.9143	9.6173

Table 3. Results for weights parameterized by 256 time taps

Search type	B	90% CDF point	99% CDP point	99.9% CDF point
No reduction	-	10.0062	10.9301	11.6175
Full search	6	6.5647	6.7635	6.922
Full search	16	7.3653	7.5469	7.6794
Full search	26	7.7209	7.885	8.0015

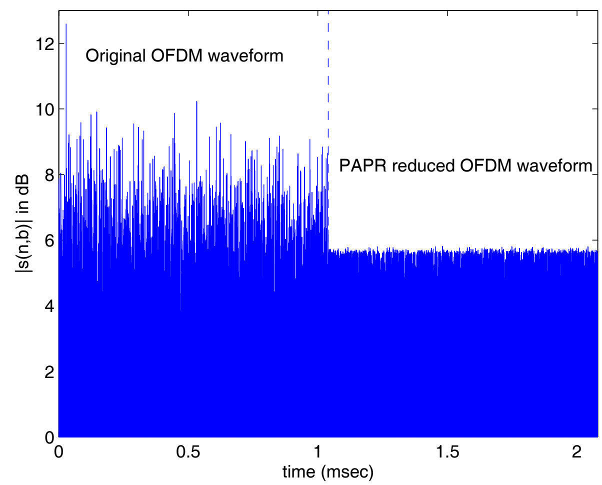

Figure 1. PAPR reduction for different weights on each subcarrier that are fixed over 26 bauds.

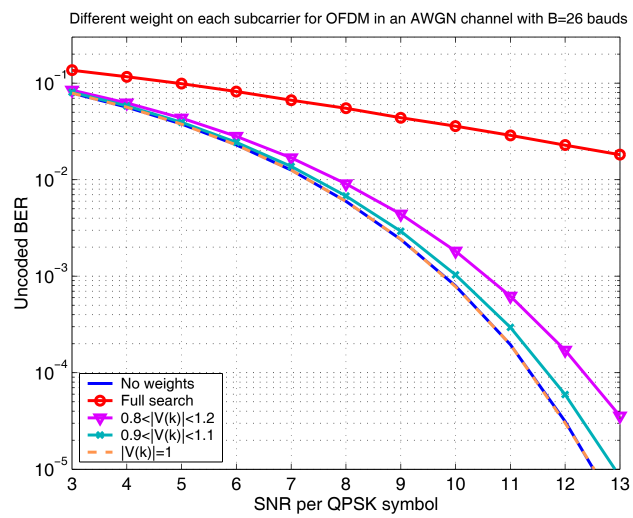

Figure 2. BER results for different weights on each subcarrier (fixed over B=26 bauds) on an AWGN channel.

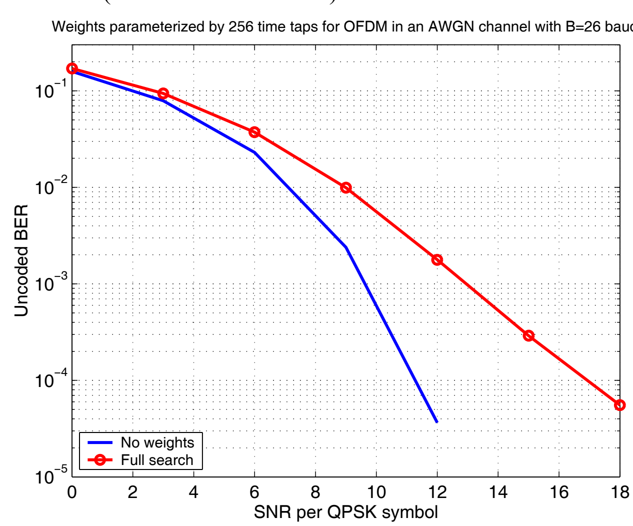

Figure 3. BER results for weights parameterized by 256 time taps (fixed over B=26 bauds) on an AWGN channel.