MINIMUM DISTORTION TEQ EQUALIZER DESIGN FOR DMT SYSTEM

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ABSTRACT

In this paper the problem of time domain equalizer (TEQ) design for discrete multitone DMT applications is dealt with based on matrix time domain derivation of total distortion power of intersymbol interference ISI and noise in multicarrier modulation schemes with cyclic prefix. We argue for modification of popular form of weighting shortened impulse response tail and head samples contributing to ISI power. We stress the total power (not per-channel) nature of the solution, what means it is not directly bit rate oriented. However, the extension to frequency domain bit rate maximizing approach has been sought as well and described in cited reference. Despite all the numerical results are presented for modification of one exemplary design algorithm presented recently, i.e. eigenfilter of Vaidyanathan and Tkacenko, the main observation is valid for wide range of design rules originating from Maximum Shortening SNR approach.[1]

1. INTRODUCTION

The multicarrier transmission is commonly used nowadays in digital subscriber loop (DSL) cable modems and wireless communication [1, 2]. The research effort is focused on providing higher bit rates, lower power expense and reliable data transfer. The general structure of such multicarrier modem in specific setup called DMT is shown in figure 1. The orthonormal modulator $\mathbf{F}^{\dagger}$ (implemented by IFFT) acts as a trasmultiplexer of parallel complex data $\mathbf{u}$ (codes of QAM constellations) to real values $\mathbf{x}$ grouped into frames. Cyclic prefix (CP) addition and serialization is performed by $\mathbf{T}$. Fading transmission line $\mathbf{c}$ is equalized by the time domain equalizer $\mathbf{w}$ (TEQ) having FIR structure of length L_w, affecting also line noise $\mathbf{n}$. Channel $\mathbf{c}$ and TEQ $\mathbf{w}$ form together time equalized (shortened) line $\mathbf{h}$ described by FIR model. Serial to parallel converting and guard interval discarding $\mathbf{R}$ precedes demodulator $\mathbf{F}$ (FFT) which together with frequency domain equalizer $\mathbf{E}$ (FEQ) reconstructs parallel data $\hat{\mathbf{u}}$ from received distorted frame $\mathbf{y}$.

Signal interferences result from the dispersive nature of transmission line. A shortened (time equalized) impulse response (SIR) of the line can be expressed as follows (see figure 2):

$$\mathbf{h} = [\ \underbrace{h_{-D},\ldots,h_{-1}}_{precursor},\underbrace{h_0,\ldots,h_M}_{cursor},\underbrace{h_{M+1},\ldots,h_L}_{postcursor}\ ] \tag{1}$$

with specific partition and indexing used further.

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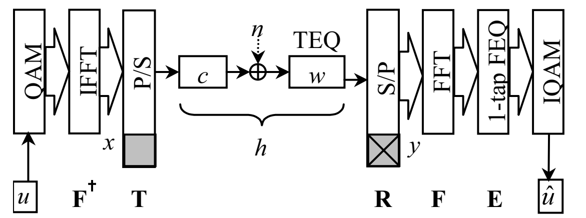

Fig. 1. Simplified block structure of DMT modem.

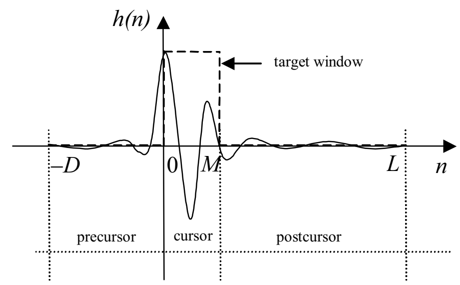

Fig. 2. Partition of shortened (time equalized) line impulse response $\mathbf{h}$ from figure 1 having length K=D+L+1. M is the length of cyclic prefix. Negative indexing of precursor (delay) term is used for convenience.

The idea of separate signal and interference paths is commonly used [3, 4], where the power on the output of interference path is a measure of intersymbol (also called interblock) interference. Referring to figure 2, these separate signal and interference paths can be written as:

$$\mathbf{h}_{approx}^{signal} = [0,\ldots,0,h_0,\ldots,h_M,0,\ldots,0] \tag{2}$$

$$\mathbf{h}_{approx}^{ISI} = [h_{-D},\ldots,h_{-1},0,\ldots,0,h_{M+1},\ldots,h_L] \tag{3}$$

This separation, originating in equalizer design algorithms for single carrier modulation (SCM) systems, is, as shown in further analysis, inaccurate as DMT operates on blocks of samples (fact commented in [2]). However, it is commonly used due to its simplicity. As is shown in this paper, accurate model of interferences is easy to incorporate in description of DMT operation and can be exploited for the design of better equalizers. Derived matrix description can also serve for frequency domain analysis of interferences [5], what we also used for bit rate oriented equalization [6].

2. DESCRIPTION OF INTERSYMBOL AND INTRASYMBOL INTERFERENCES

Let us introduce the following denotations (see figure 1):

1) transmitter (receiver) modification matrix, adding (canceling) M redundant samples into (from) each block of N data, respectively:

$$\mathbf{T} = \left[\begin{array}{c:c} \mathbf{0}_{Mx(N-M)} & \mathbf{I}_M \\ \hdashline \multicolumn{2}{c}{\mathbf{I}_N} \end{array}\right], \mathbf{R} = \left[\begin{array}{c:c} \mathbf{0}_{NxM} & \mathbf{I}_N \end{array}\right] \tag{4}$$

2) equalized line matrix of linear convolution containing samples of non-ideally shortened channel impulse response (precursor and postcursor parts, negative indexing for delay part) [7]:

$$\mathbf{C}^{lin} = \Big[\underbrace{\begin{matrix} h_L & \cdots \\ & \ddots \\ \mathbf{0} & \end{matrix}}_{\mathbf{C}^{lin}_{-1}} \;\Big|\; \underbrace{\begin{matrix} h_0 & \cdots & h_{-D} & \\ & \ddots & & \ddots \\ \ddots & & \ddots & \\ & h_L & \cdots & h_0 \end{matrix}}_{\mathbf{C}^{lin}_{0}} \;\Big|\; \underbrace{\begin{matrix} & \mathbf{0} \\ \ddots & \\ \cdots & h_{-D} \end{matrix}}_{\mathbf{C}^{lin}_{1}}\Big] \tag{5}$$

To avoid writing down huge matrices, $\mathbf{C}^{lin}$ can be schematically represented in a graphical form as in figure 3.

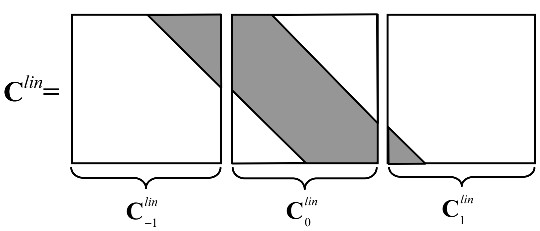

Fig. 3. Graphical representation of the equalized line matrix.

Final input/output relations describing data transmission over the line for any output and three consecutive input frames can be expressed as follows:

$$\mathbf{y} = \Big[\underbrace{\mathbf{R}\mathbf{C}^{lin}_{-1}\mathbf{T}}_{\mathbf{C}_{-1}} \;\Big|\; \underbrace{\mathbf{R}\mathbf{C}^{lin}_{0}\mathbf{T}}_{\mathbf{C}_{0}} \;\Big|\; \underbrace{\mathbf{R}\mathbf{C}^{lin}_{1}\mathbf{T}}_{\mathbf{C}_{1}}\Big] \begin{bmatrix} \mathbf{x}_{-1} \\ \mathbf{x}_0 \\ \mathbf{x}_1 \end{bmatrix} \tag{6}$$

The content of these matrices is shown in figure 4:

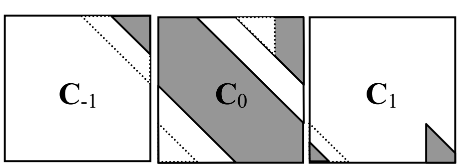

Fig. 4. Graphical representation of *signal* ($\mathbf{C}_0$) and *previous*/*next* ($\mathbf{C}_{-1}$/ $\mathbf{C}_1$) interference matrices in DMT with CP.

From (6) intersymbol interferences do not occur when

$$\left[\mathbf{C}_{-1} \;\vdots\; \mathbf{C}_0 \;\vdots\; \mathbf{C}_1\right] = \left[\mathbf{0}_N \;\vdots\; \mathbf{A}_N \;\vdots\; \mathbf{0}_N\right] \equiv \left[\mathbf{C}^{ref}_{-1} \;\vdots\; \mathbf{C}^{ref}_0 \;\vdots\; \mathbf{C}^{ref}_1\right] \tag{7}$$

Thus, the desired (reference) $\mathbf{C}_{-1}$ and $\mathbf{C}_1$ in DMT case should be matrices with zero elements, and $\mathbf{C}_0$ should be the circulant matrix $\mathbf{A}_N$ with the first row given by $[h_0, h_{-1},\ldots, h_{-D}, 0,\ldots, 0, h_L,\ldots,h_1]$ as shown in figure 5. Then:

$$\mathbf{F}\mathbf{C}^{ref}_0\mathbf{F}^{\dagger} = \mathbf{D} = diag\left(H(0), H\left(\tfrac{2\pi}{N}\right), \ldots, H\left(\tfrac{2\pi(N-1)}{N}\right)\right)$$

and FEQ equalizer is $\mathbf{E} = \mathbf{D}^{-1}$, i.e. with equalized line transform inverse on the diagonal.

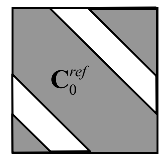

Fig. 5. Graphical representation of reference signal matrix in DMT.

Having established reference matrices $\mathbf{C}^{ref}_{-1}$, $\mathbf{C}^{ref}_0$ and $\mathbf{C}^{ref}_1$ for DMT modulation-demodulation setup, we are able to perform analysis of interferences at the input of the demodulator **F.** The interference signal is expressed as:

$$\tilde{\mathbf{y}} = \mathbf{y} - \mathbf{y}^{ref} = \Big[\underbrace{\mathbf{C}_{-1} - \mathbf{C}^{ref}_{-1}}_{\tilde{\mathbf{C}}_{-1}} \Big| \underbrace{\mathbf{C}_0 - \mathbf{C}^{ref}_0}_{\tilde{\mathbf{C}}_0} \Big| \underbrace{\mathbf{C}_1 - \mathbf{C}^{ref}_1}_{\tilde{\mathbf{C}}_1}\Big] \begin{bmatrix} \mathbf{x}_{-1} \\ \mathbf{x}_0 \\ \mathbf{x}_1 \end{bmatrix} \tag{8}$$

where individual matrices $\tilde{\mathbf{C}}_{-1}$ and $\tilde{\mathbf{C}}_1$ correspond to *previous* and *next* part of *intersymbol* interference while $\tilde{\mathbf{C}}_0$ represents *intrasymbol* interference.

Thanks to the cyclic prefix, matrix $\mathbf{C}_0$ becomes closer to the desired circulant matrix $\mathbf{C}^{ref}_0$ (compare figures 4 and 5). Their difference, corresponding to intrasymbol interference, contains two nonzero triangular parts, which are circularly shifted copies of the matrices $\mathbf{C}_{-1}$, $\mathbf{C}_1$ corresponding to *previous* and *next* components of intersymbol interference. That observation allows us to consider total interference power as twice that of intersymbol interference.

In the following analysis we make two undemanding and realistic assumptions:

- length of the impulse response of equalized transmission line $K=D+L+1$ is shorter than the frame length N,
- signal samples have white spectrum with power σ_x^2.

The mean power of intersymbol interference can be calculated as:

$$P_{if} = \frac{1}{N} E\left[\tilde{\mathbf{y}}^T \tilde{\mathbf{y}}\right] \tag{9}$$

Samples in consecutive frames are assumed independent, hence from (8):

$$P_{if} = \frac{1}{N} E\left[\mathbf{x}^T_{-1}\tilde{\mathbf{C}}_{-1}{}^T\tilde{\mathbf{C}}_{-1}\mathbf{x}_{-1} + \mathbf{x}^T_0\tilde{\mathbf{C}}_0{}^T\tilde{\mathbf{C}}_0\mathbf{x}_0 + \mathbf{x}^T_1\tilde{\mathbf{C}}_1{}^T\tilde{\mathbf{C}}_1\mathbf{x}_1\right] \tag{10}$$

As samples inside every frame are assumed independent the above can be expressed in compact form as a sum of column products, where $\tilde{\mathbf{C}}^{(n)}_{-1,0,1}$ is the n-th column of the corresponding interference matrix.

$$P_{if} = \frac{\sigma_x^2}{N} \sum_{k=-1,0,1} \sum_{n=1}^{N} \tilde{\mathbf{C}}_k^{(n)T} \tilde{\mathbf{C}}_k^{(n)} \tag{11}$$

In other words P_{if} is proportional to the sum of squares of all elements appearing in intersymbol and intrasymbol interference matrices.

Specific content of interference matrices results in the following formula:

$$\begin{aligned} P_{if} &= \frac{2\sigma_x^2}{N}\left(\sum_{m=M+1}^{L} \sum_{l=m}^{L} h_l^2 + \sum_{m=1}^{D} \sum_{l=m}^{D} h_{-l}^2 \right) \\ &= \frac{2\sigma_x^2}{N}\left(\sum_{m=M+1}^{L} (m-M) h_m^2 + \sum_{m=1}^{D} m h_{-m}^2 \right) \\ &= \frac{2\sigma_x^2}{N} \sum_{m=-D}^{L} q_m h_m^2 = \frac{2\sigma_x^2}{N} \mathbf{hQh}^T \end{aligned} \tag{12}$$

$$q_m = \begin{cases} -m, & m = -D, \dots, -1 \\ 0, & m = 0, \dots, M \\ m - M, & m = M+1, \dots, L \end{cases}$$

where: $\mathbf{Q}$ is the $K \times K$ ramp weighting matrix (q_m on its diagonal).

As we see, the impact of SIR samples on interference changes with sample index. That specific ramp weighting is compared with proposed or used by others in figure 6.

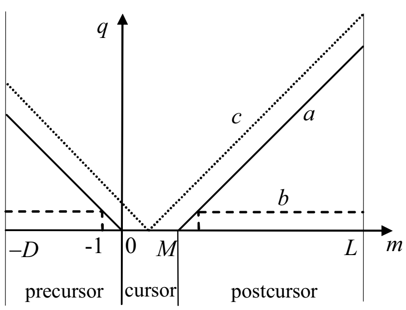

Fig. 6. Weighting sequences (see figure 2): a) ramp (proposed by the authors), b) wall (proposed in [4]), c) linear absolute (best solution in [4]).

The proposed weighting, although considered, has not been used in practice due to supposed increased computational complexity [2]. The next section shows that complexity is not the obstacle.

3. MINIMUM DISTORTION EQUALIZER DESIGN

The assumed objective of equalizer design is minimization of power of signal distortions, meant as a joint power of interferences P_{if} and power of noise after equalization P_n, relative to signal power after equalization P_x. Thus the optimization criterion takes the form:

$$J = \frac{P_{if} + P_n}{P_x} \tag{13}$$

The power of interferences was derived in preceding section as (12), what can be noted using matrix operations as

$$P_{if} = \frac{2\sigma_x^2}{N} \mathbf{wCQC}^T \mathbf{w}^T \tag{14}$$

where $\mathbf{C}$ is the Toeplitz line convolution matrix of linear of dimensions $L_w \times (K+L_w-1)$ ($\mathbf{w}$, $\mathbf{h}$ assumed real). The power of noise after equalization, depending on the form of noise description, is expressed either as

$$P_n = w(k) * r_n(k) * w(k)\big|_{k=0} \tag{15}$$

when the noise is described by autocorrelation sequence $r_n(k)$ or

$$P_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_n\left(e^{j\Omega}\right) \left|W\left(e^{j\Omega}\right)\right|^2 d\Omega \tag{16}$$

when noise is described by its power spectral density $S_n\left(e^{j\Omega}\right)$. Taking into account FIR structure of $\mathbf{w}$, (15) can be written in short matrix form as:

$$P_n = \mathbf{wR}_n \mathbf{w}^T \tag{17}$$

where $\mathbf{R}_n$ is the noise autocorrelation matrix of dimensions ($L_w \times L_w$). Signal power after equalization is, due to white input spectrum, simply

$$P_x = \sigma_x^2 \mathbf{hh}^T = \sigma_x^2 \mathbf{wCC}^T \mathbf{w}^T \tag{18}$$

Putting (14), (17) and (18) into (13) we obtain the matrix formula:

$$J = \frac{\mathbf{w}\left(\frac{2}{N} \mathbf{CQC}^T + \frac{1}{\sigma_x^2} \mathbf{R}_n \right) \mathbf{w}^T}{\mathbf{w}\left(\mathbf{CC}^T\right)\mathbf{w}^T} \tag{19}$$

Following the eigenfilter approach applied in [4] $\mathbf{CC}^T$ is Cholesky-decomposed to $\mathbf{U}^T\mathbf{U}$, where $\mathbf{U}$ is upper triangular of dimensions ($L_w \times L_w$). By transforming above to the Rayleigh quotient using $\mathbf{v}=\mathbf{U}\mathbf{w}^T$, we obtain:

$$J = \frac{\mathbf{v}^T \mathbf{Bv}}{\mathbf{v}^T \mathbf{v}}, \quad \mathbf{B} = \left(\mathbf{U}^{-1}\right)^T \left(\frac{2}{N} \mathbf{CQC}^T + \frac{1}{\sigma_x^2} \mathbf{R}_n \right) \left(\mathbf{U}^{-1}\right) \tag{20}$$

Finally we arrive to the optimum equalizer - unique solution of:

$$\mathbf{w}_{opt} = \min_{\mathbf{w}} \{J\} = \mathbf{v}_{\min}^T \left(\mathbf{U}^{-1}\right)^T, \quad J_{opt} = \lambda_{\min} \tag{21}$$

where $\lambda_{\min}, \mathbf{v}_{\min}$ is the minimum eigenvalue and the corresponding eigenvector of matrix $\mathbf{B}$.

4. COMPARISON OF NUMERICAL RESULTS

Simulation of DMT system has been performed in order to verify effectiveness of the proposed modifications in TEQ filter design methodology. The new MD-TEQ (Minimum Distortion Time EQualizer) has been created on the basis of EF-TEQ (Eigen Filter Time Equalizer) taken from [4] and has been compared with it. Intersymbol and intrasymbol interferences are treated in different manner in MD-TEQ and EF-TEQ: we use weighting directly connected with SIR samples impact on the overall interference power (see figure 6). The relative weighting of noise and interference components is also power based. Minimization of the cost function (19) leads to MD-TEQ and DMT system with minimum joint power of interferences and noise, what is confirmed by simulation results (see figure 7).

However, minimization of the joint distortion power does not necessarily lead to bit rate maximization [5, 6] as it is noticeable in figure 8. Interferences have different share in overall distortion power for different level of AWGN noise power (see figure 9), as the design is always compromise between these two distortion components. For AWGN power higher than

-140 dBm/Hz noise power is dominating for CSA loop 2. Frequency response of the original EF-TEQ and the modified MD-TEQ equalizer designed for the DMT system with narrow-band noise localized at frequency $0.2f_s$ are presented in figure 10. We can observe that both of them highly attenuate this frequency, but differ in other channels.

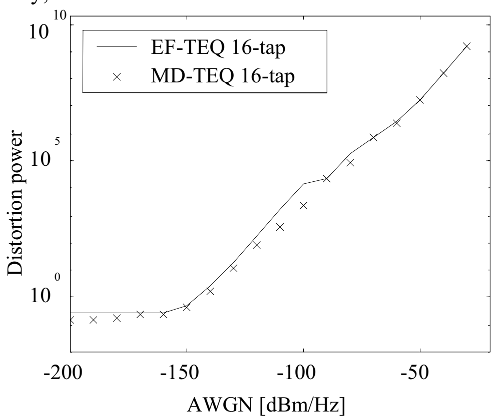

Fig. 7. Distortion power P_n+P_{if} for 16-tap equalizers.

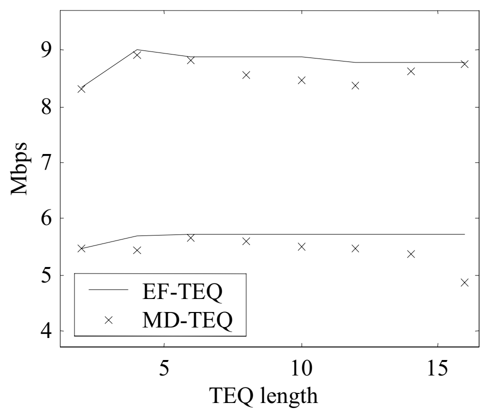

Fig. 8. Bit rate performance for different filter length in presence of AWGN of level –140 dBm/Hz (upper two graphs) and –120 dBm/Hz (lower two graphs).

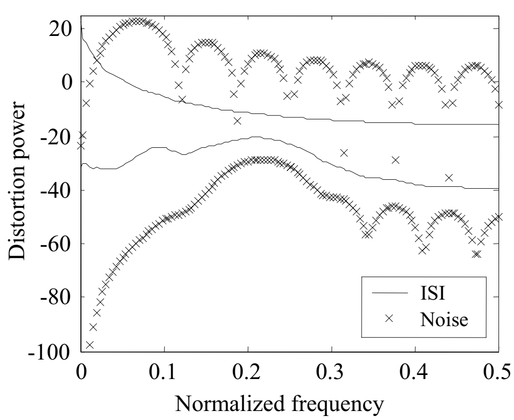

Fig. 9. Distortion power of ISI (P_{if}) and Noise (P_n) for MD-TEQ in presence of AWGN –100 dBm/Hz (upper two graphs) and –160 dBm/Hz (lower two graphs).

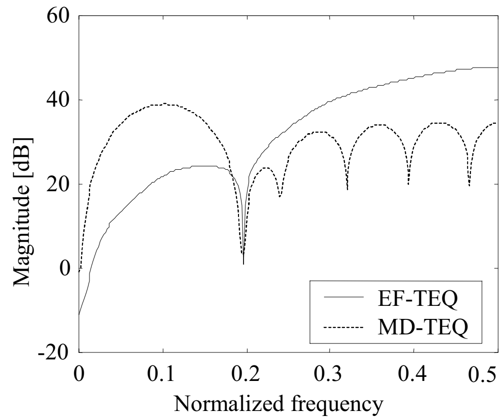

Fig. 10. Frequency response of two 16-tap TEQs designed for DMT system with narrowband interference of level –30dBm located in $0.2\,f_s$.

CONCLUSIONS

New time domain equalizer minimizing joint power of additive noise and interferences is presented in this paper. The proposed method is different from approaches that are used at present in choosing the ramp weighting function and in scaling the interferences power in respect to the noise power. Both weighting and scaling factors are not matched by heuristics but derived from power analysis. The presented methodology can be applied to improve different TEQs designed according to SIR weighting as MSSNR and MBR. Modification of one of them, the eigenfilter TEQ, has been presented in the paper. We would like to stress that bit rate optimization of DMT system has not been the goal of this research.

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