FAST ESTIMATION OF POWER SPECTRAL DENSITY OF ISI/ICI INTERFERENCES FOR ADSL MODEM

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ABSTRACT

Simple time domain matrix description of intersymbol and interchannel interferences (ISI/ICI) existing in multicarrier modulation schemes with guard interval are presented in the paper and exploited for design of two fast algorithms for computation of the ISI/ICI power spectral density. The discussion is carried out for ADSL modem with general guard interval including cyclic prefix and trailing zeros as special cases**.** The proposed algorithms can be utilized for estimation of the modem bitrate, for example during optimization of the time equalization (TEQ) filter[1].

1. INTRODUCTION

The multicarrier transmission is commonly used nowadays in asymmetric digital subscriber line (ADSL) cable modems and wireless communication [1]. The research effort is focused on providing higher bit rates, lower power expense and reliable data transfer. The general structure of such multicarrier modems (MCMs) is shown in figure 1. The orthonormal modulator **G** acts as a trasmultiplexer of parallel complex data **u** (codes of QAM constellations) to real values **x** grouped into frames. Guard interval addition (typically cyclic prefix or trailing zeros) and serialization is performed by **T**. Fading transmission line **c** is equalized by the time domain equalizer **w** (TEQ) having FIR structure and affecting also line noise **n**. Channel **c** and TEQ **w** form together time equalized (shortened) line **h** described by FIR model. Serial to parallel converting and guard interval discarding **R** precedes demodulator **F**, which together with frequency domain equalizer **E** (FEQ) reconstructs parallel data $\hat{\mathbf{u}}$ from received distorted frame **y**.

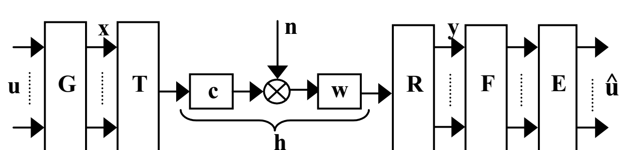

Fig. 1. Simplified block structure of MCM modem

Knowledge of noise and interferences power spectral density is crucial for bit rate evaluation of ADSL modem [1]. Signal interferences are result of dispersive nature of the transmission line. A shortened (time equalized) impulse response (SIR) of the line can be expressed as follows:

$$\mathbf{h} = [\ \underbrace{h_{-D},\ldots,h_{-1}}_{precursor},\underbrace{h_0,\ldots,h_M}_{cursor},\underbrace{h_{M+1},\ldots,h_L}_{postcursor}\ ] \quad (1)$$

with specific partitioning and indexing used further. Popular interference definition relies on describing the amount of overlapping between samples from consecutive transmit frames dispersed by the line.

In this paper an intuitive matrix time-domain ISI/ICI interference description will be presented and exploited for design of two fast algorithms for computation of its power spectral density. The discussion will be carried out for ADSL modem with general guard interval including cyclic prefix and trailing zeros as special cases**.** The proposed algorithms can be utilized for estimation of the modem bitrate, for example during optimization of the time equalization (TEQ) filter [2, 3, 4, 5].

2. DESCRIPTION OF INTERFERENCES

Let us introduce the following denotations (see figure 1):

1) input $\mathbf{x}_m$ and output $\mathbf{y}_m$ m^{th} data frame (N samples long) transmitted over the line, respectively (**G** - modulator, **F** - demodulator, **E** - frequency equalizer):

$$\mathbf{x}_m = \begin{bmatrix} x_{mN} \\ x_{mN+1} \\ \vdots \\ x_{mN+N-1} \end{bmatrix} = \mathbf{G}\mathbf{u}_m\,, \quad \hat{\mathbf{u}}_m = \mathbf{E}\mathbf{F}\mathbf{y}_m = \mathbf{E}\mathbf{F}\begin{bmatrix} y_{mN} \\ y_{mN+1} \\ \vdots \\ y_{mN+N-1} \end{bmatrix} \quad (2)$$

2) transmitter (receiver) modification matrix, adding (canceling) M redundant samples into (from) each block of N data samples, respectively [6, 7, 8] (CP - cyclic prefix, TZ – trailing zeros):

$$\bullet\ \text{CP:} \quad \mathbf{T} = \left[\begin{array}{c:c} \mathbf{0}_{Mx(N-M)} & \mathbf{I}_M \\ \hdashline \multicolumn{2}{c}{\mathbf{I}_N} \end{array}\right], \quad \mathbf{R} = \left[\begin{array}{c:c} \mathbf{0}_{NxM} & \mathbf{I}_N \end{array}\right] \quad (3)$$

$$\bullet\ \text{TZ:} \quad \mathbf{T} = \left[\begin{array}{c} \mathbf{I}_N \\ \hdashline \mathbf{0}_{MxN} \end{array}\right], \quad \mathbf{R} = \left[\begin{array}{c:c} \mathbf{I}_N & \begin{array}{c} \mathbf{I}_M \\ \hdashline \mathbf{0}_{(N-M)xM} \end{array} \end{array}\right] \quad (4)$$

3) equalized line matrix of linear convolution including samples of non-ideally shortened channel impulse response (pre and postcursor parts, negative indexing for delay part):

$$\mathbf{C}^{lin} = \left[\begin{array}{cc:ccc:cc} & h_L \ \cdots & h_0 & \cdots & h_{-D} & & \\ & \ddots & & \ddots & & \ddots & \mathbf{0} \\ \mathbf{0} & & \ddots & & \ddots & \ddots & \\ & & & h_L & \cdots \ h_0 & \cdots & h_{-D} \end{array}\right] \quad (5)$$

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$$\mathbf{C}^{lin} = \left[\mathbf{C}_{-1}^{lin} \mid \mathbf{C}_0^{lin} \mid \mathbf{C}_1^{lin}\right]_{(N+M)x3(N+M)} \qquad (6)$$

Such a decomposition of the matrix $\mathbf{C}^{lin}$ as above is used in [8], in the context of perfect equalization of the ADSL modem without guard interval.

Final input/output relations describing data transmission over the line for any output and three consecutive input frames can be expressed as follows:

$$\mathbf{y} = \mathbf{R} \cdot \left[\mathbf{C}_{-1}^{lin} \mid \mathbf{C}_0^{lin} \mid \mathbf{C}_1^{lin}\right] \cdot \begin{bmatrix} \mathbf{T}\mathbf{x}_{-1} \\ \mathbf{T}\mathbf{x}_0 \\ \mathbf{T}\mathbf{x}_{+1} \end{bmatrix} \qquad (7)(8)$$

$$\mathbf{y} = \left[\mathbf{R}\mathbf{C}_{-1}^{lin}\mathbf{T} \mid \mathbf{R}\mathbf{C}_0^{lin}\mathbf{T} \mid \mathbf{R}\mathbf{C}_1^{lin}\mathbf{T}\right] \begin{bmatrix} \mathbf{x}_{-1} \\ \mathbf{x}_0 \\ \mathbf{x}_{+1} \end{bmatrix} \mathbf{y} = \left[\mathbf{C}_{-1} \mid \mathbf{C}_0 \mid \mathbf{C}_1\right] \begin{bmatrix} \mathbf{x}_{-1} \\ \mathbf{x}_0 \\ \mathbf{x}_{+1} \end{bmatrix}$$

Input/output relation for the whole modem including remaining elements has form (see figure 1):

$$\hat{\mathbf{u}} = \underbrace{\left[\mathbf{EFC}_{-1}\mathbf{G} \mid \mathbf{EFC}_0\mathbf{G} \mid \mathbf{EFC}_1\mathbf{G}\right]}_{\mathbf{S}} \begin{bmatrix} \mathbf{u}_{-1} \\ \mathbf{u}_0 \\ \mathbf{u}_{+1} \end{bmatrix} \qquad (9)$$

Transmitted data is perfectly reconstructed when $\mathbf{S}=[\mathbf{0}_N \mid \mathbf{I}_N \mid \mathbf{0}_N]$, what can be achieved approximately by proper choice of component matrices. The desired (reference) values of $\mathbf{C}_{-1}$, $\mathbf{C}_0$ and $\mathbf{C}_1$ in (8) are modulator/demodulator dependent. In ADSL case $\mathbf{C}_{-1}^{ref}$ and $\mathbf{C}_1^{ref}$ should be matrices with zero elements, and $\mathbf{C}_0^{ref}$ - a circulant matrix with the first row given by $\left[h_0, h_{-1}, \ldots, h_{-D}, 0, \ldots, 0, h_L, \ldots, h_1\right]$ as in (10), diagonalized by the discrete Fourier transform matrix of modulator/demodulator.

Having established reference matrices $\mathbf{C}_{-1}^{ref}$, $\mathbf{C}_0^{ref}$ and $\mathbf{C}_1^{ref}$ for specific modulation-demodulation setup, we are able to perform analysis of interferences at the input of the demodulator **F**. Description of the interference signal expressed as:

$$\tilde{\mathbf{y}} = \mathbf{y} - \mathbf{y}^{ref} = \left[\tilde{\mathbf{C}}_{-1} \mid \tilde{\mathbf{C}}_0 \mid \tilde{\mathbf{C}}_1\right] \begin{bmatrix} \mathbf{x}_{-1} \\ \mathbf{x}_0 \\ \mathbf{x}_{+1} \end{bmatrix} \qquad (11)$$
$$= \tilde{\mathbf{C}}_{-1}\mathbf{x}_{-1} + \tilde{\mathbf{C}}_0\mathbf{x}_0 + \tilde{\mathbf{C}}_1\mathbf{x}_{+1} = \tilde{\mathbf{y}}_{-1} + \tilde{\mathbf{y}}_0 + \tilde{\mathbf{y}}_{+1}$$

where individual matrices correspond to:

$\tilde{\mathbf{C}}_{-1} = \mathbf{C}_{-1} - \mathbf{C}_{-1}^{ref}$ - *previous* part of *intersymbol* interference,

$\tilde{\mathbf{C}}_1 = \mathbf{C}_1 - \mathbf{C}_1^{ref}$ - *next* part of *intersymbol* interference,

$\tilde{\mathbf{C}}_0 = \mathbf{C}_0 - \mathbf{C}_0^{ref}$ - *intrasymbol* interference,

allows us to calculate its mean power spectral density (PSD).

In the following analysis intersymbol and intrasymbol interferences in ADSL modems with guard interval will serve as an example. Cyclic prefix (CP) or trailing zeros (TZ) of length M separate frames and decrease *previous* term of intersymbol interference together with the corresponding part of intrasymbol interference. As can be checked, in both cases multiplication of linear convolution matrices $\mathbf{C}_{-1,0,1}^{lin}$ by matrix of guard interval addition $\mathbf{T}$ and canceling $\mathbf{R}$ leads to the following effective matrices $\mathbf{C}_{-1,\mathrm{CP}}$, $\mathbf{C}_{0,\mathrm{CP}}$, $\mathbf{C}_{1,\mathrm{CP}}$ and $\mathbf{C}_{-1,\mathrm{TZ}}$, $\mathbf{C}_{0,\mathrm{TZ}}$, $\mathbf{C}_{1,\mathrm{TZ}}$ given by equations (12)-(16) ($L>D>M$, $K=L-M$, CP – cyclic prefix, TZ – trailing zeros):

$$\mathbf{C}_{-1,\mathrm{CP}} = \mathbf{C}_{-1,\mathrm{TZ}} = \begin{bmatrix} & h_L & \cdots & h_K \\ \mathbf{0} & & \ddots & \vdots \\ & \mathbf{0} & & h_L \\ & & \mathbf{0} & \end{bmatrix} \qquad (12)$$

$$\mathbf{C}_{1,\mathrm{CP}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & & \\ & & h_{-D} & & \\ \mathbf{0} & \mathbf{0} & \vdots & \ddots & h_{-D} \\ h_{-D} & & h_{-2} & & \vdots \\ h_{-K} \cdots h_{-D} & \mathbf{0} & h_{-1} & \cdots & h_{-M} \end{bmatrix} \quad \mathbf{C}_{1,\mathrm{TZ}} = \begin{bmatrix} h_{-M} \cdots h_{-D} & & \mathbf{0} \\ \vdots & \ddots & \\ h_{-1} \cdots & \cdots h_{-D} & \\ \mathbf{0} & & \mathbf{0} \\ h_{-D} & \mathbf{0} & \\ h_{-K} \cdots h_{-D} & & \mathbf{0} \end{bmatrix} \qquad (13)(14)$$

Thanks to the guard interval, matrices $\mathbf{C}_{0,\mathrm{CP}}$ (15) and $\mathbf{C}_{0,\mathrm{TZ}}$ (16) become closer to the desired circulant matrix $\mathbf{C}_0^{ref}$ (10). The missing components, corresponding to intrasymbol interference, are two nonzero triangular parts, which are circularly shifted copies of the matrices {$\mathbf{C}_{-1,\mathrm{CP}}$, $\mathbf{C}_{1,\mathrm{CP}}$} (12)(13), and respectively {$\mathbf{C}_{-1,\mathrm{TZ}}$, $\mathbf{C}_{1,\mathrm{TZ}}$} (12)(14), corresponding to *previous* and *next* components of intersymbol interference. We are making use of this fact in further analysis of all components of interference in ADSL case.

$$\mathbf{C}_0^{ref} = \begin{bmatrix} h_0 & \cdots & h_{-D} & & h_L & \cdots & h_1 \\ \vdots & & & \ddots \;\mathbf{0} & & \ddots & \vdots \\ h_L & & \cdots & h_{-D} & \mathbf{0} & & h_L \\ \mathbf{0} & \ddots & & & \ddots & & \\ h_{-D} & \mathbf{0} & h_L & & \cdots & & h_{-D} \\ \vdots & \ddots & & \ddots & & & \vdots \\ h_{-1} & \cdots & h_{-D} & \mathbf{0} & h_L & \cdots & h_0 \end{bmatrix} \qquad \mathbf{C}_{0,\mathrm{CP}} = \begin{bmatrix} h_0 & \cdots & h_{-D} & \mathbf{0} & h_M & \cdots & h_1 \\ \vdots & & & \ddots & h_L & \ddots & \vdots \\ h_L & & \cdots & h_{-D} & & & h_L \\ \mathbf{0} & \ddots & & & \mathbf{0} & \ddots & \\ & \mathbf{0} & h_L & & \cdots & & h_{-D} \\ \mathbf{0} & & \mathbf{0} & \ddots & h_L & \cdots & h_0 \end{bmatrix} \qquad \mathbf{C}_{0,\mathrm{TZ}} = \begin{bmatrix} h_0 & \cdots & h_{-D} & & h_L & \cdots & h_1 \\ \vdots & & & \mathbf{0} & \ddots & h_L & \cdots h_M \\ h_L & & \cdots & h_{-D} & \mathbf{0} & & \\ \mathbf{0} & \ddots & & & \ddots & & \\ & \mathbf{0} & h_L & & \cdots & & h_{-D} \\ \mathbf{0} & & \mathbf{0} & \ddots & h_L & \cdots & h_0 \end{bmatrix}$$

(10) (15) (16)

3. PSD OF ISI/ICI INTERFERENCES

Noise and interference power spectra are fundamental quantities in bit rate evaluation of the ADSL modem. According to (11) every component of interference is expressed as:

$$\tilde{\mathbf{y}}_k = \tilde{\mathbf{C}}_k \mathbf{x}_k = \sum_{n=1}^{N} \tilde{\mathbf{C}}_k^{(n)} \mathbf{x}_k(n), \quad k = -1, 0, 1 \tag{17}$$

where $\tilde{\mathbf{C}}_k^{(n)}$ denotes the n^{th} column of $\tilde{\mathbf{C}}_k$ and $\mathbf{x}_k(n)$ designates the n^{th} element of vector $\mathbf{x}_k$. Let us take the Fourier transform (FT) of this interference and calculate the mean power spectral density (PSD) of the result:

$$\tilde{Y}_k\left(e^{j\Omega}\right) = \mathrm{FT}\left[\sum_{n=1}^{N} \tilde{\mathbf{C}}_k^{(n)} \mathbf{x}_k(n)\right] = \sum_{n=1}^{N} \mathrm{FT}\left[\tilde{\mathbf{C}}_k^{(n)}\right] \mathbf{x}_k(n) \tag{18}$$

$$\begin{aligned} P_k\left(e^{j\Omega}\right) &= \tfrac{1}{N} E\left[\tilde{Y}_k\left(e^{j\Omega}\right) \tilde{Y}_k\left(e^{j\Omega}\right)^*\right] \\ &= \tfrac{1}{N} E\left[\sum_{n=1}^{N} \mathbf{x}_k^2(n) \mathrm{FT}\left[\tilde{\mathbf{C}}_k^{(n)}\right] \mathrm{FT}\left[\tilde{\mathbf{C}}_k^{(n)}\right]^*\right] = \tfrac{\sigma_x^2}{N} \sum_{n=1}^{N} \left|\mathrm{FT}\left[\tilde{\mathbf{C}}_k^{(n)}\right]\right|^2 \end{aligned} \tag{19}$$

The PSD of all interferences is expressed as a sum of PSDs of independent inter and intrasymbol components and thus expre-ssion for mean power spectrum of interferences has the form:

$$P_{IF}\left(e^{j\Omega}\right) = \tfrac{\sigma_x^2}{N} \sum_{k=-1,0,1} \sum_{n=1}^{N} \left|\mathrm{FT}\left[\tilde{\mathbf{C}}_k^{(n)}\right]\right|^2 \tag{20}$$

In ADSL case only *nonzero* submatrices of intersymbol interference matrices $\tilde{\mathbf{C}}_{-1}$, $\tilde{\mathbf{C}}_1$ have to be taken into account:

$$\tilde{\mathbf{C}}_{-1}^{nz} = \begin{bmatrix} h_L & \cdots & h_{M+1} \\ & \ddots & \vdots \\ \mathbf{0} & & h_L \end{bmatrix}, \quad \tilde{\mathbf{C}}_1^{nz} = \begin{bmatrix} h_{-D} & & \mathbf{0} \\ \vdots & \ddots & \\ h_{-1} & \cdots & h_{-D} \end{bmatrix} \tag{21}$$

They are the same for both cyclic prefix and trailing zeros guard interval (after column/row reordering) as depicted in (12)(13)(14). The general expression (20) can be reduced for ADSL (after exploiting equality of intrasymbol and intersymbol interference PSD, see end of section 2) to:

$$\begin{aligned} P_{IF}\left(e^{j\Omega}\right) &= 2\left(P_{-1}\left(e^{j\Omega}\right) + P_1\left(e^{j\Omega}\right)\right) \\ &= \frac{2\sigma_x^2}{N}\left(\sum_{n=1}^{L-M} \left|\mathrm{FT}\left[\tilde{\mathbf{C}}_{-1}^{nz,(n)}\right]\right|^2 + \sum_{n=1}^{D} \left|\mathrm{FT}\left[\tilde{\mathbf{C}}_1^{nz,(n)}\right]\right|^2\right) \end{aligned} \tag{22}$$

where upper matrix indexing (n) means (as before) n^{th} column of the matrix. These columns are obviously parts of SIR *postcursor* and *precursor* respectively. The above results are generalization of those obtained in [3, 9]. The problem there was the complex algorithm resulting from direct application of derived formulas. This algorithm is approximately characterized with the following number of real multiplications and additions (for ADSL modem, $L_p=D+L-M=D+K$ times computation of $N/2=256$-point FFTs is required after exploiting the fact that from $N=512$ real-valued transformed data $N/2$ samples long complex vector is formed):

real mults = $4*(N/2*\log_2(N/2))*L_p + 2*N/2*L_p$,
real adds = $2*(N/2*\log_2(N/2))*L_p + 2*N/2*L_p$.

Our next goal is to simplify calculations making use of existing regularities.

4. FAST ALGORITHMS

Recursive discrete Fourier transform method. Let us assume that $X_0(k)$ denotes the discrete Fourier transform (DFT) of the N samples long signal $x(n)$, starting from $n=0$:

$$X_0(k) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j(2\pi/N)kn}, \quad k = 0, 1, 2, ..., N-1 \tag{23}$$

and $X_1(k)$ is the DFT of the same signal but starting from the $n=1$ sample (the sample $x(0)$ is removed and the sample $x(N)$ is added to the input vector):

$$X_1(k) = \frac{1}{N} \sum_{n=1}^{N} x(n) e^{-j(2\pi/N)k(n-1)}, \quad k = 0, 1, 2, ..., N-1 \tag{24}$$

The last equation can be rewritten in the following form:

$$X_1(k) = e^{j(2\pi/N)k}\left[X_0(k) + \frac{1}{N}\left(x(N) - x(0)\right)\right] \tag{25}$$

that shows how to modify the spectrum $X_0(k)$, $k = 0, 1, 2, ..., N-1$, in order to obtain $X_1(k)$.

Since consecutive columns of intersymbol interference matrices $\tilde{\mathbf{C}}_{-1}$, $\tilde{\mathbf{C}}_1$ in (21) differ only in one entry we can utilize the formula (25) for fast computation of equation (22). Exemp-lary Matlab program for fast recursive computation of the *previous* intersymbol interference PSD:

$$\sum_{n=1}^{K=L-M} \left|\mathrm{FT}\left[\tilde{\mathbf{C}}_{-1}^{nz,(n)}\right]\right|^2$$

is given in table 1. As we can see, in practice no FFT is performed in the beginning since we start from null spectrum. Input samples are shifted into the end of the data buffer.

The recursive method for computing the interference PSD presented above requires ($L_p=D+L-M=D+K$):

real mults = $6*(N/2+1)*L_p$, real adds = $5*(N/2+1)*L_p$

and is significantly faster than the direct approach.

Table 1. Matlab program for calculation of the *previous* intersymbol interference PSD via recursive DFT method.

```
hp = h(L:-1:M+1); ph=exp(j*2*pi*(0:N/2)/N);
Hp = zeros(1,N/2+1); Pp = Hp;
for m = 1:L-M
  Hp = ph.*(Hp+(hp(m)-0));  Pp = Pp+abs(Hp).^2;
end
Pp = Pp/N;
```

Autocorrelation method. Let us recall basic power spectrum - autocorrelation matrix relation:

$$\sum_n \left|\mathrm{FT}\left[\mathbf{C}^{(n)}\right]\right|^2 = \sum_n \mathrm{FT}\left[\mathbf{r}^{(n)}\right] = \mathrm{FT}\left[\sum_n \mathbf{r}^{(n)}\right] = \mathrm{FT}[\mathbf{r}] \tag{26}$$

where $\mathbf{C}^{(n)}$ denotes the n^{th} column of matrix $\mathbf{C}$, $\mathbf{r}^{(n)}$ is its autocorrelation sequence and $\mathbf{r}$ represents the sum of $\mathbf{r}^{(n)}$. Now equation (22) can be rewritten as

$$P_{IF}\left(e^{j\Omega}\right) = \frac{2\sigma_x^2}{N} FT\left(\sum_{n=1}^{L-M} r_{-1}^{(n)} + \sum_{n=1}^{D} r_1^{(n)}\right) = \frac{2\sigma_x^2}{N} FT\left(r_{-1} + r_1\right) \tag{27}$$

We note that all components of autocorrelation sequences of columns of interest are found in the matrix products $\tilde{\mathbf{C}}_{-1}^{nz\,\mathrm{T}}\tilde{\mathbf{C}}_{-1}^{nz}$ and $\tilde{\mathbf{C}}_{1}^{nz}\tilde{\mathbf{C}}_{1}^{nz\,\mathrm{T}}$ respectively, because of shifting characteristic of autocorrelation definition apparent in consecutive columns of interference matrices. The wanted sum of column autocorrelations is a sum of elements lying on diagonals of the given matrix product. Despite the fact that the speed-up obtained this way is significant (matrix product and one FT instead of *L-M* or *D* FTs) we can further simplify calculation of the one-sided autocorrelation making use of the following relations:

$$r_{-1} = \tilde{\mathbf{C}}_{-1}^{nz}\tilde{\mathbf{c}}_{-1}^{nz\,\mathrm{T}} = \begin{bmatrix} h_L & \cdots & h_{M+1} \\ & \ddots & \vdots \\ \mathbf{0} & & h_L \end{bmatrix} \begin{bmatrix} (L-M)h_L \\ \vdots \\ 2h_{M+2} \\ h_{M+1} \end{bmatrix} \tag{28}$$

$$r_1 = \tilde{\mathbf{C}}_{1}^{nz\,\mathrm{T}}\tilde{\mathbf{c}}_{1}^{nz} = \begin{bmatrix} h_{-D} & \cdots & h_{-1} \\ & \ddots & \vdots \\ \mathbf{0} & & h_{-D} \end{bmatrix} \begin{bmatrix} Dh_{-D} \\ \vdots \\ 2h_{-2} \\ h_{-1} \end{bmatrix} \tag{29}$$

where $\tilde{\mathbf{c}}_{-1}^{nz}$ is the first (longest nonzero) row of $\tilde{\mathbf{C}}_{-1}^{nz}$, $\tilde{\mathbf{c}}_{1}^{nz}$ is the first column of $\tilde{\mathbf{C}}_{1}^{nz}$, both with elements weighted decreasingly with position index.

Final computational expense for the autocorrelation method is lower than for previous approaches and equal:

real mults = $4*(N/2*\log_2(N/2))*2 + 0.5D^2 + 0.5K^2$,
real adds = $2*(N/2*\log_2(N/2))*2 + 0.5D(D-1) + 0.5K(K-1)$.

The exemplary Matlab program for calculation of the *previous* intersymbol interference PSD is presented in table 2. In the proposed implementation one matrix by vector multiplication and one FFT should be performed instead of $L-M$ twice-oversampled FFTs as in [3].

Table 2. Matlab program for calculation of the *previous* intersymbol interference PSD via correlation method. We assume here *L-M*<*N*/2.

```
hp=h(L:-1:M+1);
Cp=toeplitz([hp(1) zeros(1,Lhp-1)], hp(1:Lhp));
rp=Cp*(Cp(1,:).*[Lhp:-1:1])';
Pp=abs(fft([flipud(rp(2:Lhp));rp], N))/N;
```

5. CONCLUSIONS

The bit rate model of ADSL modem heavily relies on strict description of interferences being part of overall disturbance. The presented matrix description of interference gave the same results as in [3] but its interpretation is much easier. Proposed implementations of the PSD interference estimation algorithm are faster than already reported in literature.

For example, for the following values of parameters: N = 512, M = 32, L_e = 16 (time equalizer length), D = 28, L = $N+(L_e-1)-(D+1)$ = 498, K = L-M = 466, the proposed recursive DFT method requires 5.6 times less multiplications and 3.6 times less additions than the direct method while the autocorrelation method has 34 times less multiplications and 19 times less additions. When interpolated spectra are computed, the obtained computational speed-up is significantly better. For 8 order interpolation realized via zero padding of DFT/FFT input, 7.7 and 4.8 times less multiplications and additions, respectively, are required for the recursive DFT method and 161/122 times less operations for the autocorrelation approach (see figure 2).

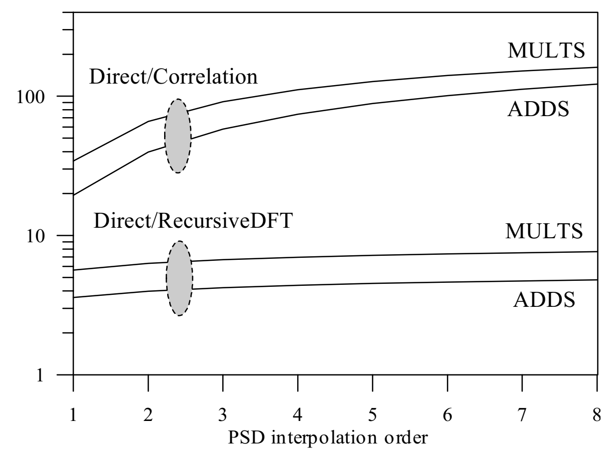

Fig. 2. Computational speed-up of fast vs. direct implementations

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