ADAPTIVE COMPENSATION OF GAIN/PHASE IMBALANCES AND DC-OFFSETS USING CONSTANT MODULUS ALGORITHM

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ABSTRACT

A novel technique based on the constant modulus algorithm (CMA) is proposed to compensate the gain/phase imbalances and DC-offsets in a transceiver. The algorithm is first described when implemented in the transmitter to compensate its circuit distortions. It is shown that the gain/phase imbalances and DC-offsets of the modulator can be corrected even in presence of high noise level. The symbol error rate (SER) performance approaches that of a system without imperfection. In the second approach, the CMA algorithm is implemented at the receiver to compensate the receiver distortions and also the transmitter ones. It is shown that the algorithm compensates for most of the receiver distortions and partly the transmitter ones.

1. INTRODUCTION

There has been considerable effort to optimize transceiver architecture to be efficient in terms of cost, physical size, and electrical power consumption. Direct conversion is a step toward such an efficient transceiver architecture where the down-conversion of the RF signal to baseband and the up-conversion of the baseband signal to RF are carried out without any intermediate frequencies (IF) [1]. The performance of quadrature modulators used in direct transmitters is sensitive to gain and phase imbalances and DC voltage offsets presented in its inputs. Similarly quadrature demodulators in direct receivers introduce gain and phase distortion as well as DC offsets in the recovered complex signal. It is well known that these imperfections degrade the overall communication link performance [2][3].

To mitigate the performance loss associated to the imperfect quadrature modulators and demodulators, a digital compensation technique is proposed along with a novel adaptive technique to estimate the compensation coefficients. The novel technique is based on the constant modulus algorithm (CMA), a blind technique that has been widely applied to equalization in communication systems [4]. The CMA seeks to minimize a cost function defined by a constant modulus (CM) criterion which penalizes deviations in the modulus (i.e., magnitude) of the signal away from a fixed value. In certain ideal conditions, minimizing the CM cost can be shown to result in perfect (zero-forcing) recovery of the received signal [5]. The CM criterion can successfully recover not only signals possessing a constant modulus, such as M-PSK, but also those characterized by source alphabets not possessing a constant modulus, such as 16-QAM signals.

In this paper, the CM criteria are defined for the transceiver gain/phase imbalance and DC-offset compensation problem and their corresponding adaptive algorithms are developed. We first study the algorithm implementation in the transmitter to estimate the pre-compensation coefficients required to mitigate the quadrature modulator imperfections. Then the algorithm is developed for implementation in the receiver to compensate both the quadrature demodulator and the quadrature modulator imperfections. Computer simulations are used to validate and assess the performance of the novel technique.

2. COMPENSATION MODELS

By definition, the in-phase (I) and quadrature (Q) channels of a quadrature communication system are orthogonal to each other. Quadrature transceivers with good balance in the I and Q channels rely on stringent specifications on components and thus are costly. In practice, there are often some gain/phase imbalances between the I and Q channels and DC-offsets in the I and Q channels. A baseband processing based on the CMA is proposed to compensate these imperfections.

2.1. *Pre-Compensation of Transmitter*

To model the quadrature modulation transformation with imperfect implementation, a complex envelope as a length-2 column vector containing the real and imaginary components is used. The quadrature modulator can be expressed as

$$\mathbf{v}_s(n) = \mathbf{M}\mathbf{v}_m(n) + \mathbf{a} \tag{1}$$

where $\mathbf{v}_s(n) = [I_s(n)\ \ Q_s(n)]^{\mathrm{T}}$, $\mathbf{v}_m(n) = [I_m(n)\ \ Q_m(n)]^{\mathrm{T}}$, $\mathbf{a} = [a_I\ \ a_Q]^{\mathrm{T}}$, $\mathbf{M}$ is defined as

$$\mathbf{M} = \begin{bmatrix} 1 & -\alpha_m \sin(\phi_m) \\ 0 & \alpha_m \cos(\phi_m) \end{bmatrix}$$

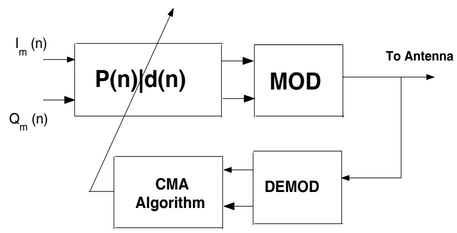

Figure 1. Imbalance and offset compensation at transmitter.

$I_s(n)$ and $Q_s(n)$ are the I and Q components of the actual modulator output at time n, $I_m(n)$ and $Q_m(n)$ are the I and Q components of the ideal output, α_m and ϕ_m represent the gain and phase imbalances between the I and Q channels, a_I and a_Q are DC-offsets of the I and Q channels, respectively.

The pre-compensation procedure of the transmitter's imperfections is shown in Figure 1. The I and Q components of the transmitted signal are

$$\tilde{\mathbf{v}}_m(n) = \mathbf{M}(\mathbf{P}(n)\mathbf{v}_m(n) + \mathbf{d}(n)) + \mathbf{a} \qquad (2)$$

where $\mathbf{P}(n)$ is a compensation matrix for gain/phase imbalances. It is defined as

$$\mathbf{P}(n) = \begin{bmatrix} p_{11}(n) & p_{12}(n) \\ p_{21}(n) & p_{22}(n) \end{bmatrix}$$

and $\mathbf{d}(n) = [d_I(n) \;\; d_Q(n)]^{\mathrm{T}}$ is a compensation vector for the DC-offsets.

It can be shown that $\tilde{\mathbf{v}}_m(n)$ becomes $\mathbf{v}_m(n)$ when $\mathbf{P}(n) = \mathbf{M}^{-1}$ and $\mathbf{d}(n) = -\mathbf{M}^{-1}\mathbf{a}$. That is, the phase/gain imbalances and DC-offsets are corrected and the actual output of the quadrature modulator is ideal.

It is noted that when $\mathbf{P}(n)\mathbf{M} = \mathbf{I}$ and $\mathbf{d}(n) + \mathbf{M}^{-1}\mathbf{a} = 0$, Eq. (2) can be rewritten as

$$\tilde{\mathbf{v}}_m(n) = \mathbf{P}(n)(\mathbf{M}\mathbf{v}_m(n) + \mathbf{a}) + \mathbf{d}(n). \qquad (3)$$

Then when $\mathbf{P}(n)$ and $\mathbf{d}(n)$ are close to the optimal solution, it is reasonable to approximate Eq. (2) as

$$\tilde{\mathbf{v}}_m(n) \approx \mathbf{P}(n)(\mathbf{M}\mathbf{v}_m(n) + \mathbf{a}) + \mathbf{d}(n). \qquad (4)$$

Eq. (4) will be used to develop a blind estimation method based on the CM criterion instead of Eq. (3).

2.2. *Compensation of Receiver*

Assume that the I and Q components of the received signal are

$$\mathbf{v}_r(n) = \mathbf{R}(n)\mathbf{v}_s(n) + \mathbf{w}(n) \qquad (5)$$

where $\mathbf{v}_r(n) = [I_r(n) \; Q_r(n)]^{\mathrm{T}}$, $\mathbf{w}(n) = [I_w(n) \; Q_w(n)]^{\mathrm{T}}$, $I_w(n)$ and $Q_w(n)$ are two independent white Gaussian channel processes with zero mean and variance σ^2, and $\mathbf{R}(n)$

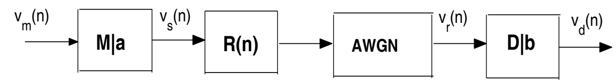

Figure 2. Matrix model of the whole system with imperfection implementation.

represents the channel phase shift which can be represented by a rotation matrix

$$\mathbf{R}(n) = \begin{bmatrix} \cos(\delta(n)) & -\sin(\delta(n)) \\ \sin(\delta(n)) & \cos(\delta(n)) \end{bmatrix}.$$

The carrier phase tracking can be implemented in the receivers by direct adjustment of the local oscillator (LO) phase. In this case, $\mathbf{R}(n)$ is constant, and equal to the identity matrix $\mathbf{I}$ if the phase error is zero.

The received signal including the gain/phase imbalances and DC-offsets in the quadrature receiver can be represented by

$$\mathbf{v}_d(n) = \mathbf{D}\mathbf{v}_r(n) + \mathbf{b} \qquad (6)$$

where $\mathbf{v}_d(n) = [I_d(n) \; Q_d(n)]^{\mathrm{T}}$, $\mathbf{b} = [b_I \;\; b_Q]^{\mathrm{T}}$, $\mathbf{D}$ is defined as

$$\mathbf{D} = \begin{bmatrix} 1 & -\alpha_d \sin(\phi_d) \\ 0 & \alpha_d \cos(\phi_d) \end{bmatrix}$$

α_d and ϕ_d represent the gain and phase imbalances in the demodulator, b_I and b_Q are DC-offsets.

As depicted in Figure 2, the overall effect of the system imperfection and channel noise can be modelled by the following

$$\mathbf{v}_d(n) = \mathbf{H}(n)\mathbf{v}_m(n) + \mathbf{c}(n) + \mathbf{m}(n) \qquad (7)$$

where $\mathbf{H}(n) = \mathbf{D}\mathbf{R}(n)\mathbf{M}$, $\mathbf{c}(n) = \mathbf{H}(n)\mathbf{a}+\mathbf{b}$, and $\mathbf{m}(n) = \mathbf{D}\mathbf{w}(n)$.

The compensation transform illustrated in Figure 3 is used to reduce or eliminate the distortion

$$\tilde{\mathbf{v}}_m(n) = \mathbf{P}(n)\mathbf{v}_d(n) + \mathbf{d}(n) \qquad (8)$$

where $\tilde{\mathbf{v}}_m(n) = [\tilde{I}_m(n) \; \tilde{Q}_m(n)]^{\mathrm{T}}$ is the estimate of $\mathbf{v}_m(n)$. Substituting Eq. (8) into Eq. (7) results to

$$\tilde{\mathbf{v}}_m(n) = \mathbf{P}(n)\mathbf{H}(n)\mathbf{v}_m(n)+\mathbf{P}(n)\mathbf{c}(n)+\mathbf{d}(n)+\mathbf{P}(n)\mathbf{m}(n) \qquad (9)$$

The gain/phase imbalances and DC-offsets in the demodulator and modulator can be compensated all at once as $\mathbf{P}(n) = \mathbf{H}^{-1}(n)$ and $\mathbf{d}(n) = -\mathbf{a} - \mathbf{H}^{-1}(n)\mathbf{b}$. The compensation estimation procedure becomes then a problem to track $\mathbf{P}(n)$ and $\mathbf{d}(n)$.

3. CONSTANT MODULUS ESTIMATION ALGORITHM

The objective of the CMA in the transmitter is to restore the output $\tilde{\mathbf{v}}_m(n)$ to a constant envelope signal on average by

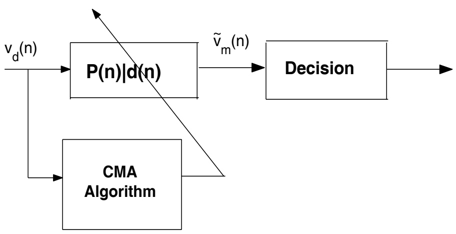

Figure 3. Imbalance and offset compensation at receiver.

adjusting the $\mathbf{P}(n)$ and $\mathbf{d}(n)$ to minimize the cost function $J(n)$, which provides a measure of the amplitude fluctuation. $J(n)$ is defined as

$$J(n) = E\left[\left(\|\tilde{\mathbf{v}}_m(n)\|^2 - R_2\right)^2\right] \tag{10}$$

where $E[\cdot]$ is the mathematical expectation operator and R_2 is a constant depending only on the input data symbol $\mathbf{v}_m(n)$. R_2 is defined as

$$R_2 = \frac{E\left[\|\mathbf{v}_m(n)\|^4\right]}{E\left[\|\mathbf{v}_m(n)\|^2\right]}. \tag{11}$$

The compensation coefficients $\mathbf{P}(n)$ and $\mathbf{d}(n)$ can be adapted by using a stochastic gradient algorithm. To simplify the presentation, let $\mathbf{p}(n) = [p_{11}(n) \;\; p_{12}(n) \;\; p_{21}(n) \;\; p_{22}(n) \;\; d_I(n) \;\; d_Q(n)]^{\mathrm{T}}$.

For the transmitter, the gradient with respect to $\mathbf{p}(n)$ is given from (4) as

$$\frac{\partial J(n)}{\partial \mathbf{p}(n)} = e(n)\mathbf{x}_t(n), \tag{12}$$

where $e(n) = \|\tilde{\mathbf{v}}_m(n)\|^2 - R_2$ and $\mathbf{x}_t(n) = [\tilde{I}_m^2(n) \;\; \tilde{I}_m(n)\tilde{Q}_m(n) \;\; \tilde{Q}_m(n)\tilde{I}_m(n) \;\; \tilde{Q}_m^2(n) \;\; \tilde{I}_m(n) \;\; \tilde{Q}_m(n)]^{\mathrm{T}}$.

The adaptation consists of adjusting $\mathbf{p}(n)$ with a step size μ in the opposite direction of the estimated gradient and is given by

$$\mathbf{p}(n+1) = \mathbf{p}(n) - \mu e(n)\mathbf{x}_t(n). \tag{13}$$

The value of the step size μ is a tradeoff between the speed of convergence and the estimate jitter in the steady state.

When the adaptive algorithm and compensation is only implemented in the receiver, the gradient with respect to $\mathbf{p}(n)$ is given by

$$\frac{\partial J(n)}{\partial \mathbf{p}(n)} = e(n)\mathbf{x}_r(n), \tag{14}$$

where $\mathbf{x}_r(n) = [\tilde{I}_m(n)I_d(n) \;\; \tilde{I}_m(n)Q_d(n) \;\; \tilde{Q}_m(n)I_d(n) \;\; \tilde{Q}_m(n)Q_d(n) \;\; \tilde{I}_m(n) \;\; \tilde{Q}_m(n)]^{\mathrm{T}}$. The adaptation rule in the receiver is then

$$\mathbf{p}(n+1) = \mathbf{p}(n) - \mu e(n)\mathbf{x}_r(n). \tag{15}$$

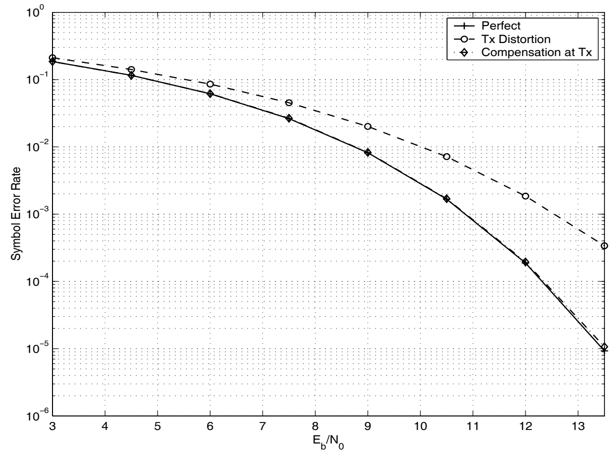

Figure 4. Performance of CMA based pre-compensation in the transmitter for an 8-PSK system.

4. SIMULATION RESULTS

In this section the performance of the novel technique based on the CMA is studied using computer simulations of an 8-PSK communication system. Assume that the carrier and symbol timings are perfectly synchronized, i.e., $\delta(n) = 0$ in (5).

Figure 4 shows the SER performance of the CMA-based pre-compensation system at the transmitter having the following phase/gain imbalances and DC-offsets

$$\alpha_m = 0.95,\; \phi_m = \pi/36,\; a_I = 0.05,\; a_Q = 0.05. \tag{16}$$

The imperfections in the modulator largely degrade the SER performance of the overall transceiver. From the figure, we note about 2dB E_b/N_0 loss at SER= 10^{-3}. The pre-compensation at the transmitter brings back the performance to the theoretical one.

Figure 5 plots the performance when the compensation is implemented in the receiver to correct gain/phase imbalances and DC-offsets of the modulator. We note that the CMA technique can improve significantly the SER performance. However, the effect of the modulator imperfection can not be fully eliminated. This was expected as reported in [6] that the gain/phase imbalances of the modulator results in signal-to-noise ratio (SNR) loss in the received signal which cannot be recovered fully at the receiver.

To investigate the capability of the technique to mitigate receiver gain/phase imbalances and DC-offsets, we first consider an ideal transmitter and a receiver with the following imperfections

$$\alpha_d = 0.95,\; \phi_d = \pi/36,\; b_I = -0.05,\; b_Q = -0.05. \tag{17}$$

The SER performance of the CMA compensation technique is shown in Figure 6. The gain/phase imbalances and DC-offsets in the demodulator degrade the performance significantly. Adding the compensation based on the CMA recovers almost all the performance loss.

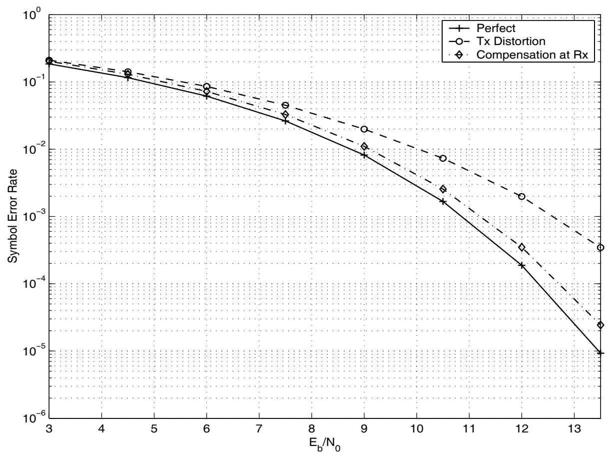

Figure 5. Performance of CMA based compensation of modulator distortions in the receiver for an 8-PSK system.

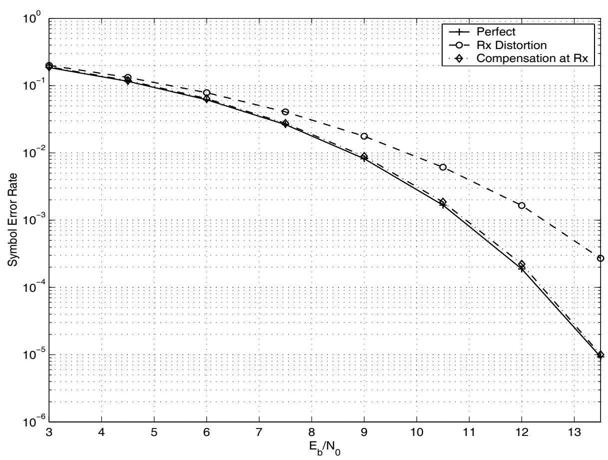

Figure 6. Performance of CMA based compensation of demodulator distortions in the receiver for an 8-PSK system.

Figure 7 gives the SER performance when both the transmitter and the receiver have the imperfections noted above ((16) and (17)) and the compensation is only performed at the receiver. It can be seen from Figure 7 that the CMA can largely improve the performance of the overall communication system. The remaining performance loss is due to the SNR loss in the received signal caused by the modulator imperfections.

5. CONCLUSIONS

A novel compensation coefficient estimation technique based on the constant modulus algorithm (CMA) is proposed to mitigate the effects of the gain/phase imbalances and DC-offsets in transceivers. Application of this technique at the transmitter to pre-compensate for its imperfections is presented and it is shown that the algorithm can effectively

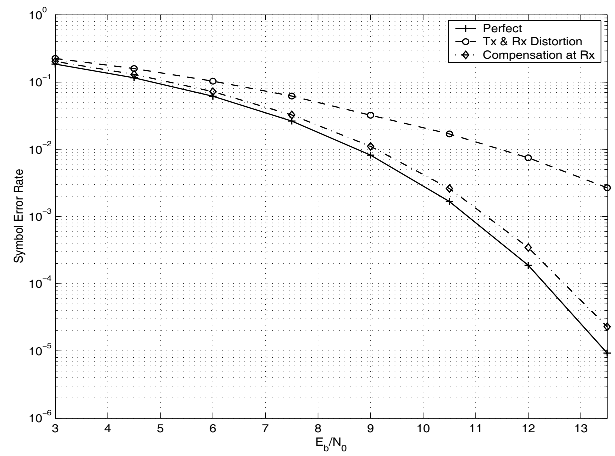

Figure 7. Performance of CMA based compensation of the modulator and demodulator distortions in the receiver for an 8-PSK system.

fully mitigate the performance loss. Use of the technique in the receiver to compensate receiver imperfections is also presented. It is shown that the technique can almost fully recover from the imperfections. When the receiver algorithm is also use to compensate for the transmitter imperfections, the simulation results show that the CMA can effectively estimate the compensation coefficients to minimize the performance loss.

6. REFERENCES

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