PERFORMANCE OF A NOVEL PHASE NOISE COMPENSATION SCHEME FOR COMMUNICATION RECEIVERS IN THE PRESENCE OF WIENER PHASE NOISE

Liang Zhao and Won Namgoong

Department of Electrical Engineering
University of Southern California
Los Angeles, CA 90089
{liangzha,namgoong}@usc.edu

ABSTRACT

The phase noise (PHN) is one of the primary factors that limits the achievable performance in many communication systems. Recently, the authors have proposed an effective "2-paths" PHN compensation scheme [1]. The approach uses the information provided by an additional signal path added to the receiver front-end to better estimate the PHN. There are generally two types of PHN found in the literature: stationary PHN and Wiener PHN. Previous results have shown the effectiveness of the "2-paths" scheme for stationary PHN. This paper addresses the applicability of the "2-paths" scheme for combating Wiener PHN. It shows that simple modifications of the “2-paths” approach work effectively for Wiener PHN as well. PHN estimation error and signal-to-noise ratio (SNR) after PHN compensation are analyzed. Simulation results for a 64-QAM system demonstrate the significant improvement over conventional approaches in the presence of Wiener PHN.

1. INTRODUCTION

The oscillator instability due to noise, which manifests itself as phase noise (PHN), is one of the primary factors that limits the achievable performance in many communication systems [2][3]. This is especially true when integrated oscillators are employed. Although many high quality off-chip oscillators are available, it is often preferable from both a cost and power perspective to employ noisier on-chip oscillators. Consequently, considerable effort has been expended in minimizing the performance degradation caused by PHN.

The existing work in this area has focused on developing signal processing techniques to best compensate for the effects of the phase noise of a given oscillator [4][5][6]. A high performance adaptive PHN compensation scheme that uses signal processing techniques together with circuit techniques has been proposed recently [1]. In [1], the receiver front-end is modified by adding an additional signal path that helps to compensate the PHN digitally in the back-end. This approach will be referred to as the “2-paths” approach. There are two commonly used mathematical models of PHN: stationary PHN and Wiener PHN. [1] has shown that the “2-paths” approach can combat stationary PHN very effectively.

This paper extends the “2-paths” scheme to receivers that suffer from Wiener PHN. By a piece-wise approach, a joint prediction and smoothing Wiener filter is formed to optimally estimate Wiener PHN in the minimum mean-squared error (MMSE) sense. Although the analytical methods and results for Wiener PHN are significantly different from that for stationary PHN, only minor changes need to be made to the receiver structure in [1] to accommodate Wiener PHN. In this paper, quantization effects due to analogue to digital converter (ADC) is also discussed.

2. SYTEM MODEL

Fig. 1 shows a generic digital communication receiver equipped with the “2-paths” PHN compensation scheme [1]. Two signal flow paths can be recognized, i.e., the received signal path (Path I) in the upper half, and the additional path (Path II, in the dotted block) added to enhance the LO PHN estimation.

The complex received passband signal is $r(t)e^{j\omega_o t}$, where ω_o is the carrier frequency, and

$$r(t) = \sum_k a_k g(t-kT) + w_1(t) \qquad (1)$$

is the baseband signal. In (1), a_k is the complex data symbol, $g(t)$ is the convolution of the transmitter pulse with the channel response, and $w_1(t)$ is the baseband complex white Gaussian noise. The channel is assumed here to be an additive white Gaussian noise (AWGN) channel.

Let $e^{-j[\omega_o t - \theta(t)]}$ be the noisy local oscillator (LO) output in baseband complex form, where $\theta(t)$ is a time-varying phase. When the LO output is phase-locked, $\theta(t)$ is modeled as a stationary process and called stationary PHN. When the LO is only frequency-locked, sometimes also called free-running oscillator, the time-varying phase $\theta(t)$ is modeled as a Wiener process [2][7], which is the integration of a white Gaussian random process. $\theta(t)$ is defined as the PHN in

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Wiener PHN model. Although PHN $\theta(t)$ is nonstationary, the LO output $e^{j\theta(t)}$ is asymptotically stationary with a Lorentzian power spectrum [7]. For Wiener PHN $\theta(t)$,

$$E[\theta(t)]=0 \qquad E[\theta(t_1)\theta(t_2)]=4\pi\beta\min(t_1,t_2), \tag{2}$$

where β is the one-sided 3 dB bandwidth (unit Hz) of the Lorentzian power spectrum. Discrete-time Wiener PHN can be expressed as $\theta[n+1]=\theta[n]+w_\theta[n]$ where $w_\theta[n]$ is zero-mean white Gaussian noise with variance σ_θ^2. Letting T be the sampling period, σ_θ^2 and β can be related by $\sigma_\theta^2=4\pi\beta T$.

In Path I, the mixture of the received passband signal with the LO output is followed by an ideal low pass filter F(ω) to produce the baseband signal modulated by the PHN. The delay block $\delta(t-t_1)$ models the delay associated with the analog part of Path I. After sampling at rate $1/T_s$, the baseband signal in Path I is:

$$r_1[m]=r(mT_s-t_1)e^{j\theta(mT_s-t_1)}. \tag{3}$$

$r_1[m]$ is then fed into a digital pulse matched filter (MF), whose impulse response is $g_{MF}(t)=g(-t)$. Assuming that symbol timing is achieved [8], the MF output is decimated to the symbol rate T. The resulting signal is

$$z[n]=\sum_m r_1[m]g_{MF}(nT-mT_s+t_1). \tag{4}$$

Index n is subsequently used to represent the symbol-rate discrete-time index at the MF output, and index m is used to denote the sampling-rate discrete-time index before the MF. The delay block after decimation in Path I is added because the PHN estimation filter in Path II runs as a smoothing filter.

In Path II, The oscillator output is downconverted to the baseband by mixing itself with a delayed and conjugated replica. The mixer output is

$$r_2(t)=e^{j[\theta(t)-\theta(t-t_2)+\omega_o t_2]}, \tag{5}$$

where t_2 is the delay added by the delay element as shown in Fig. 1, and the constant $\omega_o t_2$ is denoted by γ. $r_2(t)$ is sampled at the symbol rate $1/T$, although lower sampling rates can be employed:

$$r_2[n]=r_2(nT)=e^{j[\theta(nT)-\theta(nT-t_2)+\gamma]}+w_2[n], \tag{6}$$

where $w_2[n]$ is the additive white quantization noise with uniform distribution, zero-mean and variance σ_{w2}^2. $\varphi[n]$, the phase of $r_2[n]$, is applied to a smoothing filter, the output of which is combined with the PHN estimate from Path I to improve the phase estimate.

PHN is compensated by multiplying $z[n]$ with $e^{-j\hat{\theta}_{n|n-1}}$, where $\hat{\theta}_{n|n-1}$ is the PHN estimate to be elaborated in Section 3. The compensated signal

$$y[n]=z[n]e^{-j\hat{\theta}_{n|n-1}} \tag{7}$$

is then used for phase error estimation and data detection.

3. WIENER PHASE NOISE ESTIMATION

Due to the narrow bandwidth of the LO output $e^{j\theta(t)}$, the PHN changes slowly compared to the data signal and noise, allowing the PHN term to be moved outside the convolution operation with little loss in accuracy. For normalized root raised cosine pulse shaping, the MF output $z[n]$ can be shown to be approximately

$$z[n]=a_n e^{j\theta[n]}+w_g[n], \tag{8}$$

where $w_g[n]\equiv\sum_m e^{j\theta(mT_s-t_1)}w_1(mT_s-t_1)g_{MF}(nT-mT_s+t_1)$ is the output of the MF, approximately zero-mean white Gaussian with variance σ_{w1}^2, and

$$\begin{aligned}\theta[n]&\triangleq\theta(\lfloor (nT+t_1)/T_s\rfloor T_s-t_1)\\&\triangleq\theta(m_nT_s-t_1)=\theta(m_oT_s+nT-t_1)\end{aligned} \tag{9}$$

is the PHN of the n[th] symbol to be estimated. In (9), $\lfloor x\rfloor$ denotes the largest integer less than or equal to x.

Fig. 2 illustrates the PHN estimation block in Fig. 1. In Path I, decision-directed one-step PHN prediction is employed, where past data decisions are available and correct to estimate PHN $\theta[k]$: $\tilde{a}_n=a_n$ for $n<k$ [8]. $z[n]/a_n$ gives the maximum-likelihood estimate of the phasor $e^{j\theta[n]}$, and its phase can be approximated to

$$\hat{\theta}_{n|n}=\arg(z[n]/a_n)\approx\theta[n]+\mathrm{Im}[\xi_n](1-\mathrm{Re}[\xi_n]), \tag{10}$$

where $\xi_n=w_g[n]e^{-j\theta[n]}/a_n$ and $|\xi_n|\ll 1$. Herein, the subscript "n|n" is used for an estimate of time n using decisions up to symbol n, and similarly, "n|n-1" for a prediction of time n using decisions up to symbol n-1. Equivalently, $\hat{\theta}_{n|n-1}$ can be obtained by $\hat{\theta}_{n|n}=\hat{\theta}_e[n]+\hat{\theta}_{n|n-1}$, as shown in Fig. 2, where

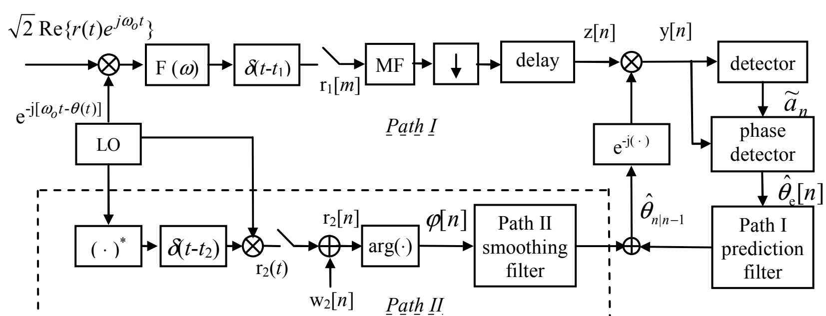

Fig. 1. Receiver model.

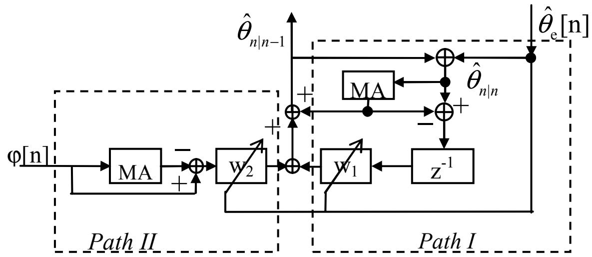

Figure 2. PHN estimation block.

$\hat{\theta}_e[n] = \arg(y[n]/\tilde{a}_n)$ is the output of the phase detector that estimates the estimation error $\theta_e[n] = \theta[n] - \hat{\theta}_{n|n-1}$.

To predict the desired phase $\theta[k]$, N_1 prior values of $\hat{\theta}_{k-i|k-i}$ ($1 \le i \le N_1$) are used. Although $\hat{\theta}_{k-i|k-i}$ is nonstationary and its variance increases as k increases, its centralized value $\hat{\theta}_{k-i|k-i} - \frac{1}{N}\sum_{l=1}^{N}\hat{\theta}_{k-l|k-l}$ has the same statistical properties for any k. The centralized values represented in vector form is

$$\mathbf{\theta}_1 = [\hat{\theta}_{k-1|k-1}, \hat{\theta}_{k-2|k-2}, ..., \hat{\theta}_{k-N_1|k-N_1}]^T - \theta_o[k]\mathbf{e}_1, \quad (11)$$

where $\mathbf{e}_1$ is a vector of length N_1 with all elements being '1,' and $\theta_o[k] = \frac{1}{N}\sum_{l=1}^{N}\hat{\theta}_{k-l|k-l}$. Therefore, the linear minimum mean-squared error (MMSE) estimate of $\theta[k]$ can be found by

$$\hat{\theta}_{k|k-1} = \mathbf{p}_1^T \mathbf{R}_1^{-1} \mathbf{\theta}_1 + \theta_o[k], \quad (12)$$

where $\mathbf{R}_1$ is the autocorrelation matrix of $\mathbf{\theta}_1$, $\mathbf{p}_1$ is the autocorrelation vector between $\mathbf{\theta}_1$ and $\theta[k] - \theta_o[k]$: To calculate $\mathbf{R}_1$ and $\mathbf{p}_1$, $\theta(t)$ around $\theta[k]$ can be expressed as

$$\theta(t) = \theta[k - M] + \phi(t), \quad (13)$$

where $M > N$ and $\phi(t)$ is zero-mean Gaussian with variance $\sigma^2_{\phi(t)} = 4\pi\beta(t - m_o T_s - (k - M)T + t_1)$. The (i,j)$^{\text{th}}$ element of $\mathbf{R}_1$ and the i$^{\text{th}}$ element of $\mathbf{p}_1$ can be shown to be:

$$\begin{aligned}\mathbf{R}_1(i,j) = \sigma^2_\theta[(N+1)(2N+1)/(6N) + (i^2 + i + j^2 + j)/(2N) \\ - \max(i,j)] + [\delta(i-j) - \frac{1}{N}]\frac{\sigma^2_{w1}}{2}E[\frac{1}{|a_n|^2}]\end{aligned} \quad (14)$$

$$\mathbf{p}_1(i) = \sigma^2_\theta[(N+1)(2N+1)/(6N) + (i^2 - i)/(2N) - i]. \quad (15)$$

It is clear from (14) and (15) that $\mathbf{R}_1$ and $\mathbf{p}_1$ depend on neither *k* nor *M*. To avoid an ill-conditioned $\mathbf{R}_1$, *N* should be slightly larger than N_1. The variance of the estimation error $\theta_e[k]$ is given by

$$\sigma_1^2 = E\left[\theta_e^2[k]\right] = \frac{(N+1)(2N+1)}{6N}\sigma_\theta^2 + \frac{1}{2N}\sigma_{w1}^2 E\left(\frac{1}{|a_n|^2}\right) - \mathbf{p}_1^T \mathbf{R}_1^{-1}\mathbf{p}_1. \quad (16)$$

It is noted that the Kalman filter can also be used to estimate Wiener PHN. However, since $\mathbf{R}_1$ and $\mathbf{p}_1$ are time-invariant, the performance of the above Wiener solution can be shown to be comparable to that of a Kalman filter. This paper therefore focuses on the use of the Wiener filter to estimate the Wiener PHN.

In Path II, signal can be viewed as being modulated with a data sequence that is a constant '1,' and can be used in a data-aided smoothing manner [8]. Similar to (10), defining $\varsigma_n = w_2[n]e^{-j(\theta(nT) - \theta(nT - t_2) + \gamma)}$ and assuming $|w_2[n]| << 1$, it can be shown that:

$$\begin{aligned}\varphi[n] &= \arg(r_2[n]) \\ &\approx \gamma + \theta(nT) - \theta(nT - t_2) + \mathrm{Im}[\varsigma_n](1 - \mathrm{Re}[\varsigma_n]).\end{aligned} \quad (17)$$

Unlike $\hat{\theta}_{n|n}$ in (10), $\varphi[n]$ is stationary. Centralized N_2 past values and N_3 future values of $\varphi[n]$ form the observation in Path II, represented in vector form:

$$\mathbf{\theta}_2 = [\varphi[k + N_3], ..., \varphi[k], ..., \varphi[k - N_2]]^T - \gamma\mathbf{e}_2, \quad (18)$$

where $\mathbf{e}_2$ is a vector of the same length as $\mathbf{\theta}_2$ and all elements being '1,' and γ is given by its moving average estimate, the moving average in Path II can be set much longer than Path I, as $\varphi[n]$ is stationary. The joint estimate of $\theta[k]$ based on observations $\mathbf{\theta} = [\mathbf{\theta}_1{}^T \ \ \mathbf{\theta}_2{}^T]^T$ from both Path I and Path II is given by:

$$\hat{\theta}_{k|k-1} = \mathbf{p}^T \mathbf{R}^{-1}\mathbf{\theta} + \theta_o[k] \triangleq \mathbf{w}^T_{opt1}\mathbf{\theta}_1 + \mathbf{w}^T_{opt2}\mathbf{\theta}_2 + \theta_o[k]. \quad (19)$$

In (19), $\mathbf{R}$ is the correlation matrix of $\mathbf{\theta}$, $\mathbf{p}$ is the correlation vector between $\mathbf{\theta}$ and $\theta[k] - \theta_o[k]$. The filter $\mathbf{w}_{opt}$ is divided into two subfilters, $\mathbf{w}_{opt1}$ and $\mathbf{w}_{opt2}$, corresponding to Path I and Path II respectively, as shown in Fig. 2. Similar to (14) and (15), the elements of $\mathbf{R}$ and $\mathbf{p}$ can be shown to be invariant to k and M. The error variance of the PHN estimate using both paths is

$$\sigma^2 = \frac{(N+1)(2N+1)}{6N}\sigma_\phi^2 + \frac{1}{2N}\sigma_{w1}^2 E\left(\frac{1}{|a_n|^2}\right) - \mathbf{p}^T\mathbf{R}^{-1}\mathbf{p}. \quad (20)$$

The SNR of $y[n]$, i.e., SNR after the PHN compensation, is approximately

$$SNR = \frac{1}{\sigma^2 + \sigma_{w1}^2 / E[|a_n|^2]} = \frac{1}{\sigma^2 + 1/SNR_{in1}}, \quad (21)$$

where σ^2_{w1} is the variance of $w_g[n]$ in (8) and $SNR_{in1} = E[|a_n|^2]/\sigma^2_{w1}$ is the input SNR before the received signal is corrupted by the PHN. The SNR after the PHN estimation using Path I only is similarly obtained by replacing σ^2 with σ_1^2 in (16).

To illustrate the benefits of Path II, numerical performance results for a 64-QAM system in the presence of Wiener PHN are presented in Fig. 3(a)-3(d). The symbol period *T* is assumed to be 10^{-6}s and the sampling period T_s=*T*/2. SNR_{in1} in Path I is 29 dB. The SNR in Path II, which is defined as $SNR_{in2} = 1/\sigma^2_{w2}$, σ^2_{w2} being the variance of $w_2[n]$ in (8), is 37.88 dB (6 bits ADC in Path II). There are N_1=8 filter taps in Path I, and 11 taps in Path II, of which N_2=8 taps corre-

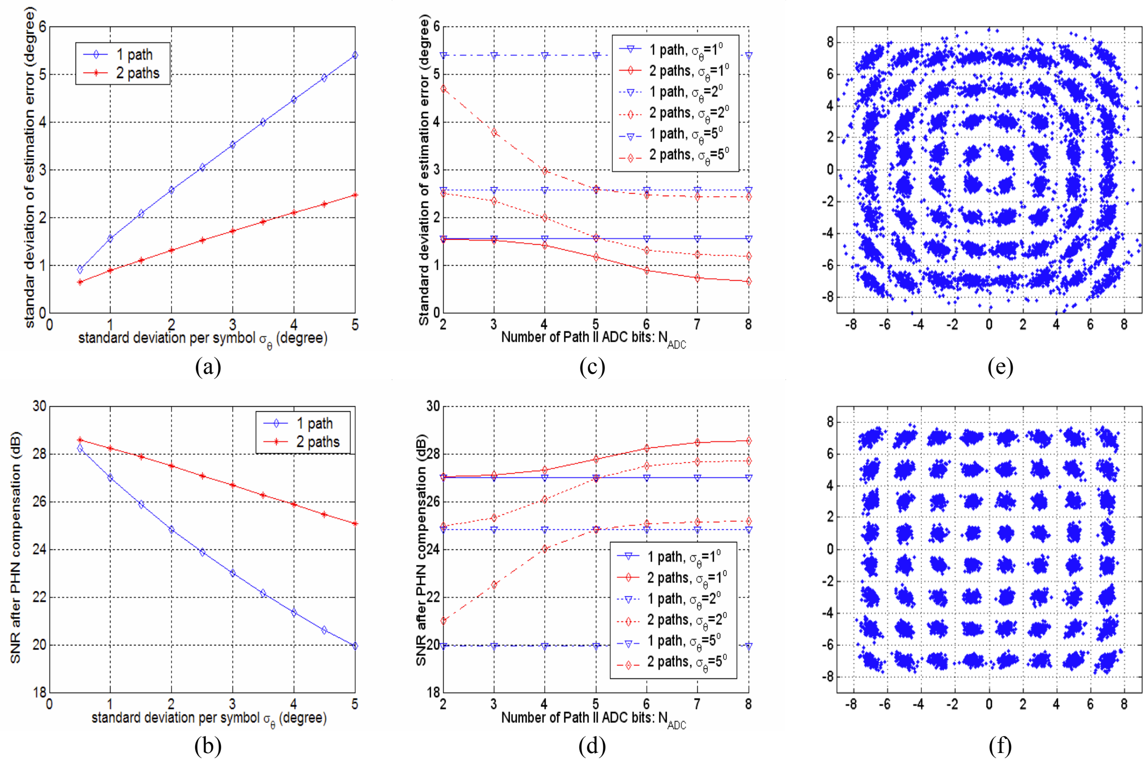

Figure 3. (a) (b): Standard deviation of PHN estimation error (a) and SNR after PHN compensation (b) v.s. standard deviation of PHN. (c)(d): Standard deviation of PHN estimation error (c) and SNR after PHN compensation (d) v.s. number of ADC bits in Path II. (e)(f): Constellation of 64 QAM after PHN compensation using 1-path (e) and 2-paths (f).

spond to 'past' and N_3=2 taps in Path II to the 'future'. The number of averaging in Path I is set to $N=N_1+2$. The delays are $t_1=0.25T$ and $t_2=1.25T$. Fig. 3(a) and 3(b) depict the standard deviation of the estimation error and the associated SNR after compensation vs. σ_θ^2, from which significant improvement can be observed. To reduce the hardware complexity of Path II, the ADC bits in Path II should be as small as possible while being large enough to maintain a reasonable σ_{w2}^2. Fig. 3(c) and 3(d) plot the results for different number of ADC bits in Path II, suggesting a Path II ADC of 5 or 6 bits.

For the phase noise compensation scheme to be practical, the phase noise must be estimated and corrected adaptively, since the statistical properties of the oscillator might be unknown or time-varying. To adaptively tune the weights of the PHN estimation filters, least-mean-square (LMS) and recursive-least-square (RLS) filter can be designed [9]. Fig. 3(e) and 3(f) show the simulation results of QAM signal constellations after PHN compensation for a Wiener PHN of $\sigma_\theta=3^0$, where RLS is employed for both conventional and the "2-paths" approaches.

4. CONCLUSION

This paper analyzes the performance of the "2-paths" PHN compensation scheme in the presence of Wiener PHN. Although non-stationary, Wiener PHN can still be estimated from an MMSE filter by a piece-wise approach. Both analytical and simulation results suggest significant improvement in compensating Wiener PHN by employing an additional signal path.

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