NOVEL GLRT PACKET-DATA DETECTORS*

Haoli Qian
Dept. of Electrical Engineering
State Univ. of New York at Buffalo
Buffalo, NY 14260

Stella N. Batalama†
Center for Integrated Transmission and Exploitation
Air Force Research Laboratory, IFGC
Rome, NY 13441

Bruce Suter
Dept. of Engineering Sciences
Harvard University
Cambridge, MA 02138

ABSTRACT

In this paper we propose novel generalized likelihood ratio test (GLRT) packet-data detectors for general multiaccess/multiuser digital communication systems and we develop analytical performance evaluation tools for finite data packet sizes. We evaluate the performance of the proposed GLRT schemes in the context of packet-data CDMA communications.

1. INTRODUCTION

GLRT is a popular detection methodology for RADAR signal detection problems (e.g. [1]–[3]) while it has been less extensively used in the context of multiaccess/multiuser digital communications (e.g. [4], [5]).

In this paper we propose novel GLRT packet-data detectors for general multiaccess/multiuser digital communication systems and we develop analytical performance evaluation tools for finite data packet sizes. In particular, we develop a coherent GLRT packet-data detector for the known channel case, a coherent GLRT pilot assisted detector for the unknown channel case (the channel is estimated implicitly as part of the GLRT formulation while short pilot signaling is used to resolve phase ambiguity) and a differential GLRT detector for differentially encoded packet-data. We also develop analytical performance evaluation tools that provide the bit error rate (BER) that can be reached by a GLRT test for a given finite data packet size as well as the size of a data packet that is necessary for the test to reach a pre-specified BER level. Finally, we demonstrate analytically the BER performance merits of the GLRT test (for finite packet sizes) relative to the popular practice of directly utilizing sample average estimates of the unknown parameters in the LRT formula.

2. SIGNAL MODEL AND BACKGROUND

The discrete-time transmitted signal of interest is given by $\mathbf{x} = \sqrt{E}b\mathbf{s}$, where E is the transmitted power and b is the information bit that modulates a known G-dimension discrete-time complex, in general, signal waveform of unit norm, $\mathbf{s} \in \mathbb{C}^G$ (b takes values ± 1 with equal probability). We assume that the transmitted signal experiences multipath fading. If $\{\mathbf{y}_i\}_{i=1}^N$ denotes the received data packet of size N, then the i-th received observation vector in the packet, $\mathbf{y}_i \in \mathbb{C}^L$, is given by

$$\mathbf{y}_i = b_i\sqrt{E}\mathbf{S}\mathbf{a} + \mathbf{n}_i, \quad i = 1, \ldots, N \tag{1}$$

where b_i is the i-th transmitted bit in the packet, M denotes the number of resolvable multipaths, $L = G + M - 1$ is the dimension of the received vector, and $\mathbf{S}$ is the known signal waveform matrix of size $L \times M$ (we assume that inter-symbol interference (ISI) is negligible and consider each column of $\mathbf{S}$ as a zero-padded-shifted version of the transmitted signal waveform vector). $\mathbf{a} \in \mathbb{C}^M$ is the vector of the (complex) multipath channel coefficients that is assumed to be constant during the transmission of the data packet. $\mathbf{n}_i$ is a zero mean complex colored Gaussian vector with *unknown* covariance matrix $\mathbf{R}_n$, that represents, comprehensively, interference and noise induced by the channel and is independent of b_i. We assume that b_i and b_j, as well as $\mathbf{n}_i$ and $\mathbf{n}_j$ are independent of each other for $i \neq j$.

When $\mathbf{a}$ and $\mathbf{R}_n$ are perfectly known, the optimum rule for the detection of b_i, $i = 1, \ldots N$, is the well known likelihood ratio test (LRT) that selects the maximum-likelihood (ML) bit combination among the finite set of alternatives: The ML test (after some algebraic manipulations) is given by

$$\widehat{b}_{i_{ML}} = \text{sgn}\left[\text{Re}\left(\left(\mathbf{R}_n^{-1}\mathbf{S}\mathbf{a}\right)^H \mathbf{y}_i\right)\right], \quad i = 1, \ldots N, \tag{2}$$

where sgn$[x]$ denotes the sign of the real variable x, and Re(y) extracts the real part of the complex scalar y. When a priori knowledge of the parameters can not be assumed, then we can proceed as follows: *(i)* We either use the parametrically described test in (2) and substitute the unknown parameters/statistics by corresponding estimates, which results in a scheme that exhibits linear complexity in the packet size; *(ii)* or we perform joint detection and parameter estimation, which results in the GLRT scheme that exhibits superior performance than *(i)* at the expense of increased complexity.

3. GLRT DETECTION: KNOWN CHANNEL

Let $\mathbf{v} \triangleq \sqrt{E}\mathbf{S}\mathbf{a}$ and $\mathbf{R}_n$ denote the known channel processed effective signal waveform and the unknown noise covariance matrix, respectively. The GLRT packet-data detector is given by the following Proposition. The proof of all the theorems and propositions are omitted due to lack

*This work was supported in part by the National Science Foundation under Grant CCR-0219903.
†Corresponding author. Email: stella.batalama@rl.af.mil

of space. Let $\mathbf{b} = [b_1, \ldots, b_N]^T$, $\mathbf{Y} \triangleq [\mathbf{y}_1, \ldots, \mathbf{y}_N]$, and $\mathbf{R}_{SA}(N) \triangleq \frac{1}{N}\mathbf{Y}\mathbf{Y}^H$ be the sample average received data correlation matrix. Then:

Proposition 1 *The GLRT test for the detection of the data packet* $\mathbf{b}$ *of size* N *in the presence of complex colored Gaussian noise is*

$$\widehat{\mathbf{b}}_{GLRT} = \arg\max_{\mathbf{b}} \, l_1(\tilde{\mathbf{b}}) \quad (3)$$

where

$$\begin{aligned} l_1(\mathbf{b}) \triangleq & N\mathbf{b}^T\mathbf{Y}^H\left[\mathbf{R}_{SA}(N)\right]^{-1}\mathbf{v} + N\mathbf{v}^H\left[\mathbf{R}_{SA}(N)\right]^{-1}\mathbf{Y}\mathbf{b} \\ & + \left(\mathbf{b}^T\mathbf{Y}^H\left[\mathbf{R}_{SA}(N)\right]^{-1}\mathbf{Y}\mathbf{b}\right)\left(\mathbf{v}^H\left[\mathbf{R}_{SA}(N)\right]^{-1}\mathbf{v}\right) \\ & - \left(\mathbf{b}^T\mathbf{Y}^H[\mathbf{R}_{SA}(N)]^{-1}\mathbf{v}\right)\left(\mathbf{v}^H[\mathbf{R}_{SA}(N)]^{-1}\mathbf{Y}\mathbf{b}\right). \end{aligned} \quad (4)$$

■

The ad-hoc detection scheme obtained by substituting a sample average estimate $\mathbf{R}_{n_{SA}}(K) \triangleq \frac{1}{K}\sum_{k=1}^{K}\mathbf{n}_k\mathbf{n}_k^H$ of $\mathbf{R}_n$ in (2) will be denoted as follows:

$$\widehat{b}_{i_{SMI(K)}} = \mathrm{sgn}\left[\mathrm{Re}\left(\mathbf{v}^H[\mathbf{R}_{n_{SA}}(K)]^{-1}\mathbf{y}_i\right)\right], \; i = 1, \ldots, N \quad (5)$$

When pure disturbance observations (secondary data) are not available, a popular version of the test in (5) utilizes the sample-average correlation matrix of the (*desired-signal-present*) received data, $\mathbf{R}_{SA}(N)$, evaluated using the *same* received data $\mathbf{y}_i$, $i = 1, \ldots, N$, that are processed by the detector. We denote this test by

$$\widehat{b}_{i_{SMI-pre}} = \mathrm{sgn}\left[\mathrm{Re}\left(\mathbf{v}^H[\mathbf{R}_{SA}(N)]^{-1}\mathbf{y}_i\right)\right], \; i = 1, \ldots, N \quad (6)$$

We recall that (5) and (6) converge with probability 1 to the test in (2) as the $K, N \to \infty$. On the other hand recent results on short data record adaptive filtering [6]–[8] indicate that for finite sample support of identical size ($K = N$), the test in (5) outperforms the test in (6) in terms of BER. However, as Theorem 1 shows below, if we utilize the packet-data GLRT detector in (3), we can achieve approximately the average BER performance of the test in (5) without the need for pure disturbance observations (secondary data) that are independent of the received data packet.

Theorem 1 (i) *Let* $\mathbf{b}$ *be the transmitted data packet and* $\widehat{\mathbf{b}}$ *be a data packet estimate that differs from* $\mathbf{b}$ *in* m *bits (i.e. contains* m *bits in error). Then*

$$P\left[l_1(\widehat{\mathbf{b}}) > l_1(\mathbf{b}) \middle| \mathbf{b}\right] = \int_0^1 \frac{Q(\sqrt{2m\gamma x})x^{N-L}(1-x)^{L-2}}{B(N-L+1, L-1)}dx \quad (7)$$

where $Q(x) \triangleq \frac{1}{\sqrt{2\pi}}\int_x^{+\infty} e^{-u^2/2}du$, *and* $\gamma \triangleq \mathbf{v}^H\mathbf{R}_n^{-1}\mathbf{v}$.

(ii) *For sufficiently large transmitted power* E, *the BER of the GLRT detector in (3) is approximately equal to the BER of the scheme in (5) with* $K = N - 1$. ■

Using Theorem 1, we can derive an approximation of the BER of the GLRT detector, and evaluate the packet size that is necessary for the GLRT detector to achieve a given BER performance level as seen in Theorem 2 below.

Theorem 2 (i) *The average BER of the GLRT detector that operates on a data packet of size* $N \geq L + 2$ *is given by*

$$BER_{GLRT}(N) \approx \frac{1}{\pi}\int_0^{\pi/2} M\left(N-L+1, N, \frac{-\gamma}{\sin^2\theta}\right)d\theta \quad (8)$$

$$\approx \frac{2}{3}Q\left(\sqrt{2\mu}\right) + \frac{1}{6}Q\left(\sqrt{2\mu+2\sqrt{3}\sigma}\right) + \frac{1}{6}Q\left(\sqrt{2\mu-2\sqrt{3}\sigma}\right) \quad (9)$$

where $M(\cdot)$ *is the Kummer's confluent hypergeometric function,* $\mu \triangleq \frac{N-L+1}{N}\gamma$, *and* $\sigma^2 \triangleq \frac{(N-L+1)(L-1)}{N^2(N+1)}\gamma^2$.

(ii) *For any given* ν, *the smallest packet size* N_ν, *that guarantees that the BER performance of the GLRT packet-data detector is within* ν *dB from the BER performance of the optimum ML detector in (2) is given by the ceiling of the maximum real root of the following cubic equation*

$$N^3 + \left(1 - \frac{2(L-1)}{1-10^{-\frac{\nu}{10}}}\right)N^2 + \frac{(L-1)^2 - 3(L-1)}{\left(1-10^{-\frac{\nu}{10}}\right)^2}N - \frac{2(L-1)}{1-10^{-\frac{\nu}{10}}}N + \frac{4(L-1)^2}{\left(1-10^{-\frac{\nu}{10}}\right)^2} = 0. \quad (10)$$

■

The roots of (10) can be obtained either numerically or analytically [9]. We note that the result in part *(ii)* of Theorem 2 is independent of the input statistics (i.e. independent of the performance of the optimum ML detector), which is often unknown to the designer.

Since a direct implementation of (3) has exponential complexity, we propose the following suboptimal implementation of (3) with linear complexity in the packet size $\left(O(NL^2) + O(NL) + O(L^3) + O(DPL)\right)$:

<u>Suboptimum GLRT algorithm</u>

Initialization: $\widehat{\mathbf{b}}^{(p)}(0) = \left[\widehat{b}_1(0), \widehat{b}_2(0), \ldots, \widehat{b}_N(0)\right]^T$
Create index sequences $\pi_p(n)$
For n = $1, 2, \ldots D$
  For p = $1, 2, \ldots, P$
    $i = \pi_p(n \bmod N)$
    $\widehat{b}_i^{(p)}(n) = \arg\max_{b_i^{(p)}, \mathbf{R}_n} f\left(\mathbf{Y} \middle| \left\{\widehat{b}_j^{(p)}(n-1)\right\}_{j\neq i}, b_i^{(p)}, \mathbf{v}, \mathbf{R}_n\right)$
    $\widehat{b}_j^{(p)}(n) = \widehat{b}_j^{(p)}(n-1), \quad j \neq i$
  end
end

Details of the method are omitted due to lack of space.

4. GLRT DETECTION: UNKNOWN CHANNEL

When no knowledge of $\mathbf{v}$ (or, equivalently, E and $\mathbf{a}$) is available, the GLRT detector and its pairwise probability of error (in the high SNR region) are given below by Proposition 2 and Theorem 3, respectively.

Proposition 2 *The GLRT test for the detection of the data packet* $\mathbf{b}$ *of size* N *in the presence of complex colored Gaussian noise of unknown covariance matrix* $\mathbf{R}_n$ *is given by*

$$\widehat{\mathbf{b}}_{GLRT} = \arg\max_{\mathbf{b}} \, l_2(\mathbf{b}), \quad (11)$$

where

$$l_2(\mathbf{b}) = \frac{\mathbf{b}^T\mathbf{Y}^H\left[\mathbf{Y}\mathbf{Y}^H\right]^{-1}\mathbf{S}\left(\mathbf{S}^H\left[\mathbf{Y}\mathbf{Y}^H\right]^{-1}\mathbf{S}\right)^{-1}\mathbf{S}^H\left[\mathbf{Y}\mathbf{Y}^H\right]^{-1}\mathbf{Y}\mathbf{b}}{N^2 - N\mathbf{b}^T\mathbf{Y}^H\left[\mathbf{Y}\mathbf{Y}^H\right]^{-1}\mathbf{Y}\mathbf{b}}. \quad (12)$$

■

Theorem 3 *Let* $\mathbf{b}$ *be the transmitted data packet and* $\widehat{\mathbf{b}}$ *be a data packet estimate that differs from* $\mathbf{b}$ *in* m *bits. Then*

$$\lim_{E\to\infty} P\left(l_2(\widehat{\mathbf{b}}) > l_2(\mathbf{b})\Big|\,\mathbf{b}\right) =$$

$$\int_0^1 Q\left(\sqrt{\frac{2m(N-m)}{N}\gamma\cdot x}\right)\frac{x^{N-L-1}(1-x)^{L-2}}{B(N-L,L-1)}dx. \quad (13)$$

■

We observe that both (12) and (13) are phase ambiguous. Phase ambiguity is resolved either by using a pilot sequence or by using differential modulation at the transmitter; the rest of this section deals exactly with these two approaches.

4.1. Pilot assisted GLRT detection

Proposition 3 *Let* $\{b_j\}_{j=1}^J$ *and* $\{b_j\}_{j=J+1}^N$ *denote* J *known pilot bits and* $(N-J)$ *unknown information bits within the data packet* $\mathbf{b}$ *of size* N *that is received in the presence of complex colored Gaussian noise. Then the pilot assisted GLRT detector for* $\{b_i\}_{j=J+1}^N$*, is given by*

$$\left\{\widehat{b}_{i_{GLRT}}\right\}_{i=J+1}^{N} = \arg\max_{b_i, i\geq J+1} l_2(\mathbf{b}). \quad (14)$$

■

It is important to note that the pilot sequence is not used directly in (14) to estimate the phase *explicitly* but is rather incorporated implicitly in the GLRT rule. For a reasonably long pilot sequence, e.g. $J \geq 2$, Theorem 3 implies that we can safely neglect the pairwise probability of error in the high SNR region for $m \geq 2$. The performance of the pilot assisted GLRT detector is evaluated analytically in Corollary 1.

Corollary 1 (i) *The average BER of the pilot assisted GLRT detector for a data packet of size* $N \geq L+3$ *is given by*

$$BER_{GLRT,pilot}(N) \approx BER_{SMI(N-2)}$$

$$\approx \frac{2}{3}Q\left(\sqrt{2\mu}\right)+\frac{1}{6}Q\left(\sqrt{2\mu+2\sqrt{3}\sigma}\right)+\frac{1}{6}Q\left(\sqrt{2\mu-2\sqrt{3}\sigma}\right) \quad (15)$$

where $\mu \triangleq \frac{N-L}{N}\gamma$*, and* $\sigma^2 \triangleq \frac{(N-L)(L-1)}{N^3}\gamma^2$.

(ii) *For any given* ν*, the smallest packet size* N_ν *that guarantees that the BER performance of the GLRT pilot assisted packet-data detector is within* ν *dB from the BER performance of the optimum* coherent *ML detector in (2) is given by the ceiling of the maximum real root of the following cubic equation*

$$N^3 - \frac{2L\cdot N^2}{1-10^{-\frac{\nu}{10}}} + \frac{L^2-3(L-1)}{\left(1-10^{-\frac{\nu}{10}}\right)^2}N + \frac{3L(L-1)}{\left(1-10^{-\frac{\nu}{10}}\right)^2} = 0. \quad (16)$$

■

A suboptimum implementation of the GLRT scheme in (14) that exhibits linear complexity can be obtained by modifying the algorithm presented at the end of Section 3.

4.2. DPSK GLRT detection

Proposition 4 *The DPSK GLRT detector of a differentially encoded data packet* $\mathbf{b}$ *that is received in the presence of complex colored Gaussian noise is given below:*

$$\left\{\widehat{d}_{i_{GLRT}}\right\}_{i=1}^{N-1} = \arg\max_{d_i, i\geq 1} l_2(\mathbf{d}) \quad (17)$$

$$\widehat{b}_{i_{GLRT}} = \widehat{d}_{i-1_{GLRT}}\cdot\widehat{d}_{i_{GLRT}} \quad i=1,\ldots,N-1. (18)$$

where $\mathbf{d} \triangleq [d_0, \ldots d_{N-1}]^T$ *is the differentially encoded transmitted bit sequence.* ■

Using Theorem 3 and the observation that both one bit error and the $(N-1)$-bits error in $\widehat{\mathbf{d}}$ result in 2-bits error in $\widehat{\mathbf{b}}$, we can prove the following corollary.

Corollary 2 (i) *The average BER of the DPSK GLRT detector for a data packet of size* $N \geq L+3$ *is given by*

$$BER_{GLRT,DPSK}(N) \approx BER_{SMI,DPSK(N-2)}$$

$$\approx \frac{4}{3}Q\left(\sqrt{2\mu}\right)+\frac{1}{3}Q\left(\sqrt{2\mu+2\sqrt{3}\sigma}\right)+\frac{1}{3}Q\left(\sqrt{2\mu-2\sqrt{3}\sigma}\right) \quad (19)$$

where $\mu \triangleq \frac{N-L}{N}\gamma$*,* $\sigma^2 \triangleq \frac{(N-L)(L-1)}{N^3}\gamma^2$*, and the subscript "SMI,DPSK(N-2)" identifies the detection scheme that utilizes the* coherent *SMI detector of* $\{d_i\}_{i=1}^N$ *(that requires perfect knowledge of* $\mathbf{a}$ *and utilizes* $K = N-2$ *independent pure disturbance observations) followed by the differential decoding scheme in (18).*

(ii) *For any given* ν*, the smallest packet size* N_ν*, that guarantees that the BER performance of the DPSK GLRT packet-data detector is within* ν *dB from the BER performance of the optimum* coherent *ML DPSK detector, is given by the ceiling of the maximum real root of the following cubic equation*

$$N^3 - \frac{2L\cdot N^2}{1-10^{-\frac{\nu}{10}}} + \frac{L^2-3(L-1)}{\left(1-10^{-\frac{\nu}{10}}\right)^2}N + \frac{3L(L-1)}{\left(1-10^{-\frac{\nu}{10}}\right)^2} = 0. \quad (20)$$

■

5. SIMULATION STUDIES

In this Section we examine the performance of the proposed GLRT detectors as well as the accuracy of the BER expressions derived. The detectors are implemented using the linear complexity algorithm of Section 3 for the known channel case and a modified version of it for the unknown channel case. In Fig. 1 we study a communication system where packet-data are received in the presence of unknown colored Gaussian noise. Figs. 2–4 evaluate the performance

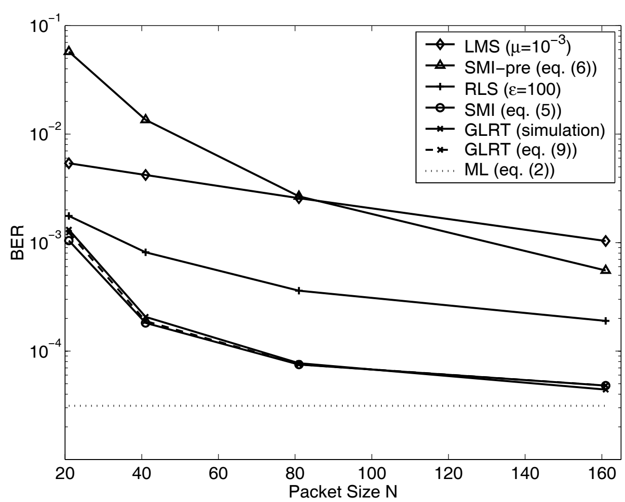

Fig. 1. BER of packet-data detectors in the presence of colored Gaussian noise as function of the packet size N ($L = 9, E = 7dB$, $K = N-1$). $\mathbf{R}_n$ is chosen arbitrarily in this study.

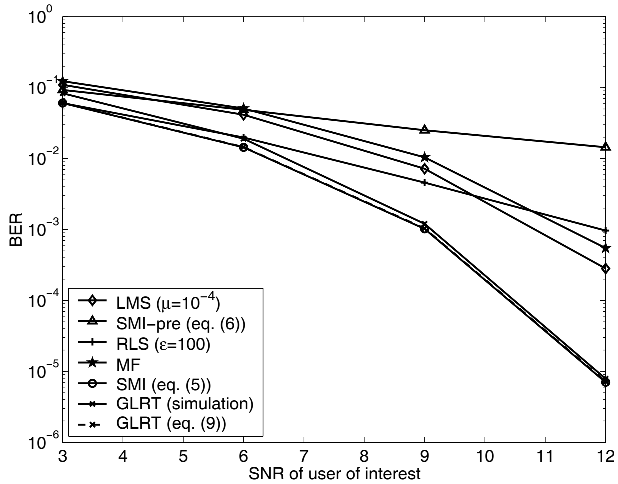

Fig. 2. Synchronous DS-CDMA system: BER as function of the SNR of the user of interest ($N = 127$, $K = N - 1$).

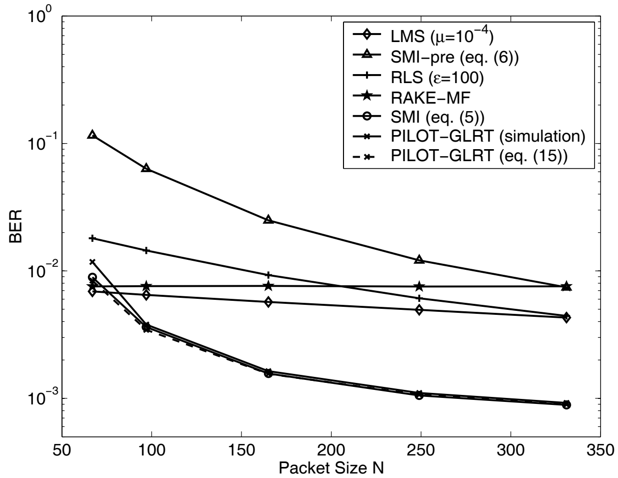

Fig. 3. Asynchronous multipath fading DS-CDMA system: BER as function of the packet size N (SNR of user of interest is fixed to $9dB$, $K = N - 2$).

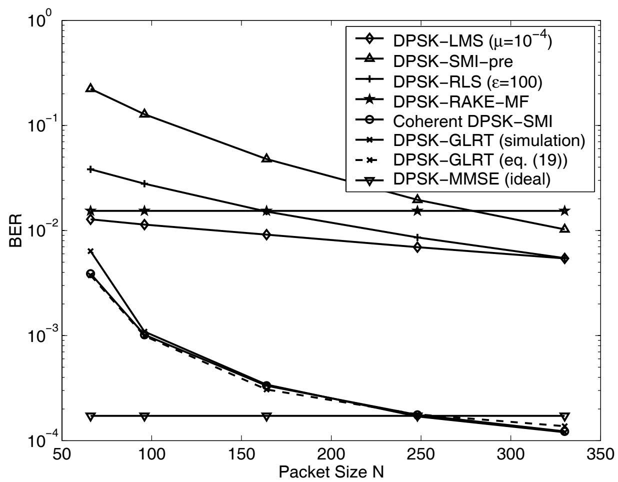

Fig. 4. DPSK asynchronous multipath fading DS-CDMA system (2-symbol DPSK demodulation): BER as function of the packet size N (SNR of user of interest is fixed to $9dB$, $K = N - 2$).

of the proposed GLRT schemes in the context of packet-data CDMA communications[1]. We consider a 10 user DS-CDMA system with processing gain $G = 31$ and Gold codes in AWGN. In Fig. 2, we examine a synchronous system and we assume exact knowledge of the channel. In Figs. 3 and 4 we examine an asynchronous system with 3 multipaths per user and either pilot assisted or DPSK signaling, respectively. In Figs. 3 and 4 the GLRT detector assumes no knowledge of the channel while all other detectors assume either exact knowledge of the channel (Fig. 3) or knowledge of the channel up to an unknown phase (Fig. 4). We compare the BER of the proposed schemes with the BER exhibited by LMS, RLS, matched-filter (MF) and SMI-type (cf. (5) and (6)) detector structures. The ideal DPSK-MMSE noncoherent detector that utilizes perfect knowledge of $\mathbf{R}_n$ is also included in Fig. 4 as a reference. In all simulation studies the SMI detector in (5) assumes availability of K additional pure disturbance observations. The superior performance of the proposed GLRT schemes is evident.

6. CONCLUSIONS

In this paper, we proposed novel GLRT packet-data detectors for general multiaccess/multiuser digital communication systems as well as corresponding suboptimum implementations of linear complexity. We also developed analytical performance evaluation tools that provide the packet size requirements necessary for the GLRT detectors to achieve a given BER performance level.

7. REFERENCES

[1] E. J. Kelly, "An adaptive detection algorithm," *IEEE Trans. Aerosp. Electron. Syst.,* vol. AES-22, pp. 115-127, Jan. 1986.

[2] S. M. Kay, V. Nagesha, and J. Salisbury, "Broad-band detection based on two-dimensional mixed Autoregressive models," *IEEE Trans. on Sig. Proc.,* pp. 2413–2428, July, 1993.

[3] S. Kraut, L. L. Scharf, and L. T. McWhorter, "Adaptive subspace detectors," *IEEE Trans. Sig. Proc.,* pp. 1-16, Jan. 2001.

[4] M. K. Varanasi, "Group detection for synchronous Gaussian code-division multiple-access channels," *IEEE Trans. Info. Theory,* vol. 41, pp. 1083–1096, July 1995.

[5] I. Motedayen and A. Anastasopoulos, "Polynomial-complexity, adaptive symbol-by-symbol soft-decision algorithms with application to non-coherent detection of LDPCC," in Proc. Intern. Conf. Commun. (ICC), New York, pp. 1677–1681, 2002.

[6] I. N. Psaromiligkos and S. N. Batalama, "Data record size requirements for adaptive antenna arrays," in *Proc. SPIE, 14th Annual Int. Symp., Digital Wireless Comm. Conf.*, Orlando, FL, Apr. 2000, vol. 4045, pp. 122-131.

[7] I. N. Psaromiligkos and S. N. Batalama, "Recursive AV and MVDR filter estimation for maximum SINR adaptive space-time processing," to appear in *IEEE Trans. Communications*.

[8] J. Farrell, I. N. Psaromiligkos, and S. N. Batalama, "Desired-signal-present SMI filters with near desired-signal-absent performance in data limited environment," *Conf. on Info. Sc. Syst. (CISS),* Johns Hopkins Univ., Baltimore, MD, Mar., 2003.

[9] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions.* New York, NY: Dover, 1970.

[1]It is noteworthy that the combined effect of DS-CDMA multiple access interference (MAI) and AWGN is Gaussian-mixture distributed. It is interesting to examine how the proposed GLRT detectors perform in such an environment.