PRECONDITIONED CONJUGATE GRADIENT BASED FAST COMPUTATION OF INDIRECT DECISION FEEDBACK EQUALIZER

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ABSTRACT

In this paper, we use the Preconditioned Conjugate Gradient (PCG) method to rapidly compute the tap weights of a minimum mean-square error (MMSE) decision feedback equalizer (DFE). The equalizer setting is computed indirectly after channel estimation. According to the Toeplitz block structure of the MMSE DFE equation $\mathbf{Rw} = \mathbf{r}$, the preconditioner $\mathbf{P}$ is chosen to be a block diagonal matrix with circulant blocks along its diagonal. The spectral clustering property of the preconditioner matrix is analyzed. It is shown that the eigenvalues of $\mathbf{P}^{-1}\mathbf{R}$ are clustered around unity except for a small number of outliers when the number of the equalizer taps becomes large. The preconditioner can be inverted via Fast Fourier Transform (FFT) with complexity $O(N \log N)$. Since the PCG method converges in a small number of steps, the total complexity for computing the DFE setting is proportional to $O(N \log N)$. The proposed scheme is also suitable for "smart" initialization which can further reduce the computational burden by decreasing the number of iteration steps. Simulations of a DFE for Digital TV channels demonstrates superior performance of the scheme.

1. INTRODUCTION

The MMSE DFE is widely used in modern communication systems. It can effectively mitigate intersymbol interference caused by frequency selective channels. Recently, the Indirect DFE has received considerable attention [1][2][3]. While the traditional adaptive DFE's adapt their setting directly from the training sequence or symbol decisions, the Indirect DFE's tap weights are computed from the channel estimate. This indirect approach has been shown to exhibit a better performance and tracking ability over the direct DFE[1].

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In the Digital TV application, the DFE is required to have a large number of taps in both the feedforward (FF) and feedback (FB) filter. For example, the DFE used in Digital TV systems might have as much as 512 FF taps and 512 FB taps. For such cases, the large computational complexity of computing the DFE weights from the channel is a major difficulty for implementation and real-time adaptation of such equalizers. Several methods have been proposed to reduce the computational burden of Indirect DFE. In [2], the inverse of circulant matrix is used to approximate the inverse of a Toeplitz matrix. In [3], a generalized Schur algorithm is used to do the complex Cholesky factorization to solve the matrix equation.

In this paper, we tackle the problem by using the Preconditioned Conjugate Gradient method, an iterative scheme, to solve the MMSE DFE equation. Comparing with the standard Conjugate Gradient method, which has recently been widely used in adaptive filtering and image restoration [4], the PCG method effectively accelerates the convergence rate by using a preconditioning matrix $\mathbf{P}$. Since the number of iteration steps needed for CG depends on the number of distinct eigenvalues of the system matrix $\mathbf{R}$, a good preconditioner is (1) a matrix $\mathbf{P}$ which clusters the eigenvalues of $\mathbf{P^{-1}R}$ around 1 except a small number of ouliers and (2) a matrix for which the product $\mathbf{P^{-1}b}$ can be computed efficiently. According to the Toeplitz block structure of the DFE matrix $\mathbf{R}$, we choose the preconditioner $\mathbf{P}$ to be a block diagonal matrix with circulant blocks along its diagonal. It is shown that under certain condition, the eigenvalues of $\mathbf{P^{-1}R}$ are clustered around 1 except for a small number of outliers when the number of the equalizer taps becomes large. By using FFT, the preconditioner can be inverted with complexity $O(N \log N)$. Since the number of outliers doesn't depend on N, the total complexity for computing the DFE setting is proportional to $O(N \log N)$. Moreover, the proposed scheme is suitable for "smart" initialization, which can effectively utilize the previous filter setting or some preliminary approximation to further reduce the computational burden of adaptation. The simulation results on DFE for digital TV channels illustrate the fast convergence of the method and the improved performance.

The outline of the paper is as follows. Section II describes the MMSE DFE equation and the PCG method. The preconditioner and its spectral clustering property are analyzed in Section III. The simulation results are presented in Section IV. Section V contains a concluding discussion.

2. MMSE DFE AND THE PCG METHOD

2.1. MMSE DFE

The structure of the DFE is shown in Fig. 1. In this model, $h(k)$ is a "composite" channel impulse response that combines the effects of transmitting pulse, the multipath channel response, and receiver match filter. We assume that the $h(k)$ extends over $0 <= k <= L$, the noise $n[k]$ is additive white Gaussian with σ_n^2 and the symbol is i.i.d. with energy ε_s^2. Both the $h[k]$ and the σ_n^2 are already known from channel estimation. The equalizer is symbol spaced but the result can be easily extended to the fractionally spaced case.

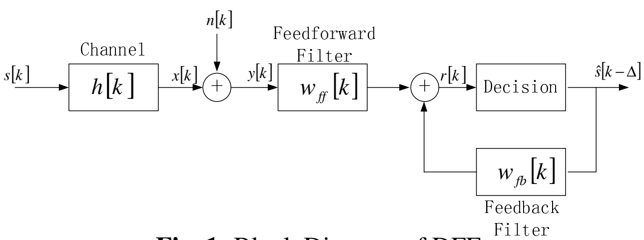

Fig. 1. Block Diagram of DFE

We denote the N_{ff} taps FF filter by $\mathbf{w}_{ff} = [w_{ff}[0], w_{ff}[1], \ldots, w_{ff}[N_{ff}-1]]^T$, the N_{fb} taps FB filter by $\mathbf{w}_{fb} = [w_{fb}[1], w_{fb}[2], \ldots, w_{fb}[N_{fb}]]^T$. Let $s[k]$ be the transmitted symbols, $y[k]$ be the input to the DFE. Their relation in vector form is given by:

$$\mathbf{y}_k = \mathbf{H}\mathbf{s}_k + \mathbf{n}_k = \mathbf{x}_k + \mathbf{n}_k \tag{1}$$

where

$$\mathbf{y}_k = [y[k],\, y[k-1],\, \ldots,\, y[k-N_{ff}+1]]^T,$$

$$\mathbf{s}_k = [s[k],\, s[k-1],\, \ldots,\, s[k-N_{ff}-L+2]]^T,$$

$$\mathbf{n}_k = [n[k],\, n[k-1],\, \ldots,\, n[k-N_{ff}+1]]^T,$$

and $\mathbf{H} = \begin{bmatrix} h[0] & h[1] & \ldots & h[L] & 0 & \ldots & 0 \\ 0 & h[0] & h[1] & \ldots & h[L] & \ldots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & h[0] & h[1] & \ldots & h[L] \end{bmatrix}$.

Assuming the decisions are correct and with delay Δ, which is optimally selected to be N_{ff}, the error is defined as

$$\begin{aligned} e[k] &= s[k-\Delta] - r[k] \\ &= s[k-\Delta] - \mathbf{w}_{ff}^*\mathbf{y}_k - \mathbf{w}_{fb}^*\mathbf{s}[k-\Delta-1] \quad (2) \\ &= s[k-\Delta] - \mathbf{w}^*\mathbf{u}_k \end{aligned}$$

where $\mathbf{u}_k = [\mathbf{y}_k^T,\, \mathbf{s}_{k-\Delta-1}^T]^T$, $\mathbf{s}_{k-\Delta-1} = [s[k-\Delta-1], s[k-\Delta-2], \ldots, s[k-\Delta-N_{fb}]]^T$, and $\mathbf{w} = [\mathbf{w}_{ff}^T,\, \mathbf{w}_{fb}^T]^T$.

Using the orthogonal principle, we obtain the DFE equation:

$$\mathbf{R}\mathbf{w} = \mathbf{r} \tag{3}$$

where $\mathbf{R} = E\{\mathbf{u}_k\mathbf{u}_k^*\}$ and $\mathbf{r} = E\{\mathbf{u}_k s[k-\Delta]^*\}$. It is helpful to partition $\mathbf{R}$ into blocks:

$$\mathbf{R} = \begin{bmatrix} \mathbf{R_{yy}} & \mathbf{R_{ys}} \\ \mathbf{R_{sy}} & \mathbf{R_{ss}} \end{bmatrix} \tag{4}$$

Notice that $\mathbf{R_{yy}} = E\{\mathbf{y}_k\mathbf{y}_k^*\}$ is symmetric Toeplitz and $\mathbf{R_{ss}} = E\{\mathbf{s}_{k-\Delta-1}\mathbf{s}_{k-\Delta-1}^*\}$ is diagonal, while $\mathbf{R_{ys}} = \mathbf{R_{sy}^*} = E\{\mathbf{y}_k\mathbf{s}_{k-\Delta-1}^*\}$ usually is not square matrix and becomes Toeplitz only when the FF and FB filters have the same tap length. So generally, $\mathbf{R}$ is only a matrix with Toeplitz blocks along its diagonal.

2.2. The PCG Method

The widely used CG method solves the equation $\mathbf{R}\mathbf{w} = \mathbf{r}$ by minimizing $\frac{1}{2}\mathbf{w}^*\mathbf{R}\mathbf{w} - \mathbf{w}^*\mathbf{r}$ over $\mathbf{w}$. The convergence of the CG depends on the eigenvalue spectrum of $\mathbf{R}$. In particular, the number of iterative steps needed depends on the number of distinct eigenvalues. Thus, to accelerate the convergence, the *preconditioning* is typically used to cluster the spectrum, which leads to the PCG method.

In the PCG method, an easily invertible matrix $\mathbf{P}$ which makes the eigenvalues of $\mathbf{P}^{-1}\mathbf{R}$ around 1, or in other words $\mathbf{P}^{-1}\mathbf{R} \approx \mathbf{I}$, is constructed. Then the CG algorithm is then implicitly applied to the system $\mathbf{M}^{-1}\mathbf{R}\mathbf{M}^{-*} = \mathbf{M}^{-1}\mathbf{r}$, with $\mathbf{w} = \mathbf{M}^{-*}\mathbf{z}$ and $\mathbf{P} = \mathbf{M}\mathbf{M}^*$. The PCG steps are listed in Table 1.

Table 1. PCG algorithm

Initialization: $\mathbf{b}_0 = \mathbf{r} - \mathbf{R}\mathbf{w}_0$; $k = 0$
While $\|\mathbf{b}_0\| > \delta$:
$\mathbf{z}_k = \mathbf{P}^{-1}\mathbf{b}_k$
$k = k + 1$
$\beta_k = \mathbf{z}_{k-1}^*\mathbf{r}_{k-1}/\mathbf{z}_{k-2}^*\mathbf{r}_{k-2}$, $(\beta_1 \equiv 0)$
$\mathbf{q}_k = \mathbf{z}_{k-1} + \beta_k\mathbf{q}_{k-1}$, $(\mathbf{q}_1 \equiv \mathbf{z}_0)$
$\alpha_k = \mathbf{z}_{k-1}^*\mathbf{r}_{k-1}/\mathbf{q}_k^*\mathbf{R}\mathbf{q}_k$
$\mathbf{w}_k = \mathbf{w}_{k-1} + \alpha_k\mathbf{q}_k$
$\mathbf{b}_k = \mathbf{r}_{k-1} - \alpha_k\mathbf{R}\mathbf{q}_k$

Each iterative step of the PCG is more expensive than CG, since it needs to computer the matrix-vector product $\mathbf{P}^{-1}\mathbf{b}_k$. However, the convergence can be significantly improved; hence the total cost of solving $\mathbf{R}\mathbf{w} = \mathbf{r}$ can be much less. Usually for point Toeplitz system, a circulant matrix is selected to the preconditioner, because it can be easily inverted via FFT and it converges to the Toeplitz matrix asymptotically.

3. THE PRECONDITIONER AND ITS CLUSTERING PROPERTY

Considering the $\mathbf{R}$ matrix in the DFE equation, if we substitute the channel estimation $\mathbf{H}$, noise energy σ_n^2 and the symbol energy ε_s^2 into (4), we get

$$\mathbf{R}=\begin{bmatrix}\mathbf{R_{yy}} & \mathbf{R_{ys}}\\ \mathbf{R_{sy}} & \mathbf{R_{ss}}\end{bmatrix}=\begin{bmatrix}\varepsilon_s^2\mathbf{H}\mathbf{H}^*+\sigma_n^2\mathbf{I}_{N_{ff}} & \varepsilon_s^2\mathbf{H}\mathbf{J}_\Delta\\ \varepsilon_s^2\mathbf{J}_\Delta^*\mathbf{H}^* & \varepsilon_s^2\mathbf{I}_{N_{fb}}\end{bmatrix} \quad (5)$$

where $\mathbf{J}_\Delta = E\{\mathbf{s}_k\mathbf{s}_{k-\Delta-1}^*\}$. Generally the blocks on the anti-diagonal of $\mathbf{R}$ are not square; but it contains Toeplitz blocks along its diagonal. So it is natural to let the preconditioner $\mathbf{P}$ to have the same block structure as $\mathbf{R}$ and be defined as:

$$\mathbf{P}=\begin{bmatrix}c(\mathbf{R_{yy}}) & \mathbf{0}\\ \mathbf{0} & c(\mathbf{R_{ss}})\end{bmatrix}=\begin{bmatrix}c(\mathbf{R_{yy}}) & \mathbf{0}\\ \mathbf{0} & \varepsilon_s^2\mathbf{I}_{N_{fb}}\end{bmatrix} \quad (6)$$

where $c(.)$ denotes some operator which constructs a circulant matrix from a Toeplitz matrix. For diagonal matrix $\mathbf{R_{ss}}$, $c(.)$ has no effect. There are a lot of $c(.)$ operators for point Toeplitz matrices in the literature. Here we use the one proposed in [4]. For any symmetric Toeplitz $N\times N$matrix $\mathbf{T}$, we have

$$c(\mathbf{T})=\begin{bmatrix} t_0 & t_1+t_{N-1} & t_2+t_{N-2} & \dots & t_{N-1}+t_1\\ t_1+t_{N-1} & t_0 & \dots & \dots & \vdots\\ t_2+t_{N-2} & \ddots & \ddots & \dots & \vdots\\ \vdots & \ddots & \ddots & \ddots & \vdots\\ t_{N-1}+t_1 & \dots & \dots & \dots & t_0\end{bmatrix} \quad (7)$$

Note that all elements of $\mathbf{T}$ are used to construct $c(\mathbf{T})$ and the following property can be easily derived from the results in [4].

Lemma 1. Assuming the generating sequence of the positive definite Toeplitz matrix sequence $\mathbf{T}_n$ is absolute summable, $\sum_{i=-\infty}^{+\infty}|t_i|\le\infty$; then for large n, $c(\mathbf{T}_n)-\mathbf{T}_n$ is the sum of two matrices $\mathbf{L}_n$ and $\mathbf{V}_n$. $\mathbf{L}_n$ is a low rank matrix whose rank doesn't depend on n, and the largest eigenvalue of $\mathbf{V}_n$ satisfies $\lambda_{max}(\mathbf{V}_n)\le\varepsilon$.

For the preconditioner $\mathbf{P}$ defined in (6), the following lemma gives its asymptotic characteristic.

Lemma 2. Let $\mathbf{R}$ be as in (5) and $\mathbf{P}$ as in (6), then for large N_{ff} and N_{fb}, the matrix $\mathbf{R}-\mathbf{P}$ can be decomposed into a sum of a low rank matrix whose rank doesn't depend on N_{ff} and N_{fb}, and a small norm matrix whose largest eigenvalue $\lambda_{max}\le\varepsilon$.

Proof: It is obvious that

$$\mathbf{R}-\mathbf{P}=\begin{bmatrix}\mathbf{R_{yy}}-c(\mathbf{R_{yy}}) & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}+\begin{bmatrix}\mathbf{0} & \mathbf{R_{ys}}\\ \mathbf{R_{sy}} & \mathbf{0}\end{bmatrix} \quad (8)$$

Since the channel sequence is finite length, the generating sequence of $\mathbf{R_{yy}}$ is always absolute summable. By using Lemma 1, the first matrix in the right side of (8) is the sum of a low rank matrix with rank not depending on the size of $\mathbf{R}$ and a small norm matrix. When the N_{ff} and N_{fb} are both large, we have

$$\mathbf{J}_\Delta=\begin{bmatrix}\mathbf{0}_{(\Delta+1)\times(L-2)} & \mathbf{0}_{(\Delta+1)\times(N_{fb}-L+2)}\\ \mathbf{I}_{(L-2)\times(L-2)} & \mathbf{0}_{(L-2)\times(N_{fb}-L+2)}\end{bmatrix}$$

So the second matrix in the right side of (8) is also a low rank matrix with rank not depending on N_{ff} and N_{fb}. By combining the above, we complete the proof.

Finally, the clustering property of $\mathbf{P}$ is described in the following theorem.

Theorem 1: Let $\mathbf{R}$ be as in (5) and $\mathbf{P}$ as in (6), then for large N_{ff} and N_{fb}, most of the eigenvalues of $\mathbf{P}^{-1}\mathbf{R}$ is clustered between $(1-\epsilon, 1+\epsilon)$, and the number of the outliers doesn't depend on N_{ff} and N_{fb}.

The theorem can be easily proved by combining Lemma 2 and the Cauchy Interlacing theorem [5].

From Theorem 1, we can see that by using the preconditioner $\mathbf{P}$, for large large N_{ff} and N_{fb}, only a small number of iteration steps are needed to solve the DFE equation. Due to its circulant block structure, the matrix-vector product $\mathbf{P}^{-1}\mathbf{b}$ can be computed via FFT with complexity $O(N\log N)$,$N=N_{ff}+N_{fb}$. So the total complexity for computing the DFE setting is proportional to $O(N\log N)$.

4. SIMULATION RESULTS

The DFE in a 8-VSB Digital TV system is simulated. The Digital TV channel is assumed to be known for the purposes of constructing the MMSE DFE equation. The first channel we use for simulation is a standard testing Digital TV channel (Brazil-C). It contains echoes at $-0.96T_s$, $0T_s$, $3.55T_s$, $15.25T_s$, $24.03T_s$, and $29.16T_s$ with corresponding gains 0.73, 1, 0.64, 0.98, 0.74, and 0.86. We also tested our scheme on randomly generated channels with length 128, which model some dense channels. The result is averaged over 5000 randomly generated channels. The channel impulse is the convolution of the path rays with the raised cosine function having excess bandwith 0.115. The phase of each path gain was set to $2\pi f_c T_s d_i$ where is the carrier frequency f_c of 50MHz, T_s is the symbol period $91.9ns$, and d_i is the relative path delay.

In the simulation, both the FF part and FB part of the DFE have 512 taps, the delay of the DFE is optimally set to be 512, and the receiver has a SNR of 30dB, which is defined as $SNR=\varepsilon_s^2\|h\|^2/\sigma_n^2$. The DFE equation is constructed from the channel and the noise parameters. The MSE of the equalizer output is calculated as $MSE=\varepsilon_s^2+\mathbf{w}^*\mathbf{R}\mathbf{w}-2\mathbf{w}^*\mathbf{r}$. Fig. 2 shows the iterative curve of PCG solving the DFE for Brazil-C. In Fig. 3, we can see the averaged iteration curve for dense channel and the comparison with the regular CG method.

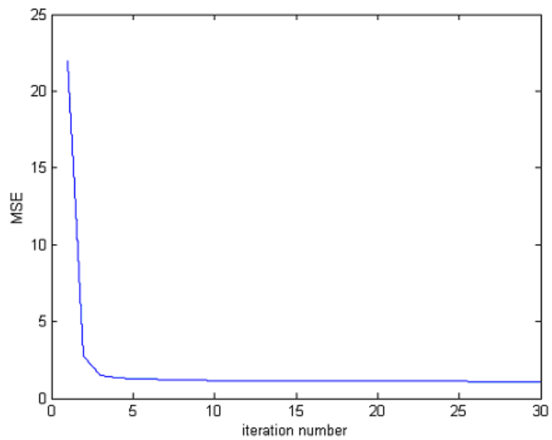

Fig. 2. iterative curve for Brazil-C

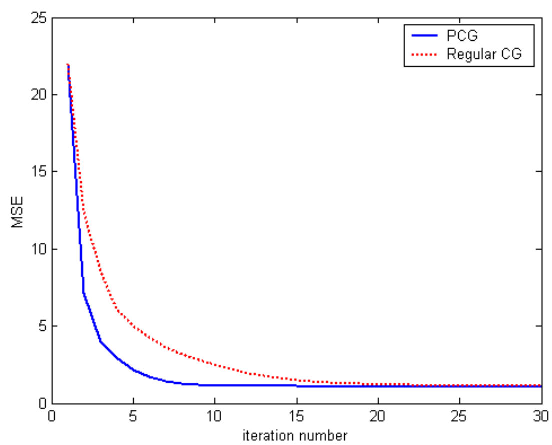

Fig. 3. iterative curve for random dense channels

It can be seen that the PCG only needs less than 10 iterations to achieve the minimum MSE DFE weights. The easy initialization feature makes the PCG method very suitable for tracking time variations via Indirect DFE.

We also simulated the PCG with "smart" initialization. In the Digital TV scenario, the channel variation during each frame period is relatively small. So by initializing the PCG method with the previous DFE tap values reduces the number of PCG steps needed to get very close to the minimum MSE. A time varying channel is constructed from Brazil-C by inducing a 100Hz Doppler shift. The channel estimation is updated at each frame and the PCG is initialized by the previous DFE weights. The iteration curve with "smart" initialization is shown in Fig. 4. Clearly, the "smart" initialization greatly decreases the computational burden and thus enhances the adaptation ability of the DFE.

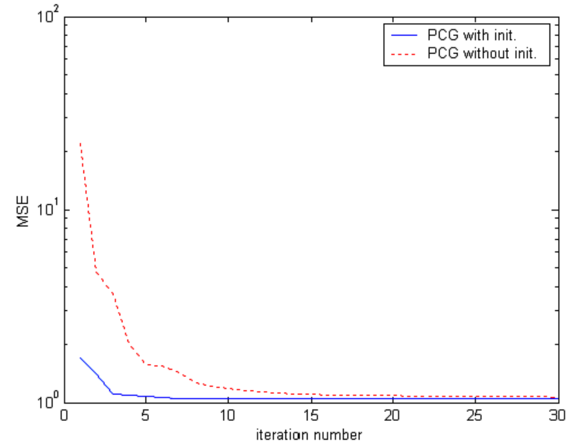

Fig. 4. iteration curve of PCG with initialization

5. CONCLUSION

The PCG method is used for fast computation of indirect DFE. The preconditioner $\mathbf{P}$ is chosen to be a block diagonal matrix with circulant blocks on its diagonal. The spectral clustering property of the preconditioner matrix is analyzed. It is shown that the eigenvalues of $\mathbf{P}^{-1}\mathbf{R}$ are clustered around unity except a small number of outliers. The proposed scheme is also suitable for "smart" initialization which can further reduce the computational burden by decreasing the number of steps needed. The simulations on DFE for digital TV channels show the fast convergence property of the scheme.

6. REFERENCES

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