A SOFT-DETECTOR BASED ON MULTIPLE SYMBOL DETECTION FOR DOUBLE DIFFERENTIAL MODULATION

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ABSTRACT

We consider the problem of soft-detection for communication systems employing double differential modulation and forward error correction codes, for example, a convolutional code. We propose a soft-detector with low computational complexity. Simulation results are provided to show the superior performance of the new soft-detector.

1. INTRODUCTION

Double differential (DD) modulation/detection is a means of implementing frequency offset insensitive communication systems conveying information via multiple phase-shift-keying (MPSK) symbols [1, 2]. A number of hard-detectors have been proposed in the past [1, 2, 3, 4, 5], which compromise between detection performance and implementation complexity. Among these hard-detectors, the multiple symbol heuristic detector (MSHD) of [1, 5], which can be efficiently implemented via the fast algorithm of [6], is especially attractive. The MSHD significantly outperforms the single symbol detector of [1, 2] and has a complexity on the order of $N \log_2 N$ with N being the number of received data samples used for multiple symbol detection; on the other hand, it is orders of magnitude more efficient than the multiple symbol detector of [4] at the cost of only a slight performance degradation.

For practical communication systems, forward error correction codes, such as convolutional codes, are often used to lower the transmission error rate to an acceptable level by adding redundancy in the transmission. As a result, soft-detectors are preferable at the receiver, since they can deliver soft-information—in the form of a bit metric—to the decoder, such as a Viterbi decoder, which can lead to a better decoding performance. A single symbol detection (SSD)-based soft-detector was proposed in [7], which, as expected, can lead to a better decoding performance compared to the SSD-based hard-detector. Due to the advantage of joint symbol detection, even the MSHD-based hard-detector can outperform the SSD-based soft-detector in terms of decoding performance. Here we propose a MSHD-based soft-detector. This soft-detector exploits the detection results of the MSHD in a simple way and thus has a low computational complexity. Simulation results are provided to show the superior performance of the new soft-detector.

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2. BACKGROUND

Consider a communication system employing DD modulation/detection and convolutional coding, as shown in Figure 1. At the transmitter, the *c*onvolutional en*c*oder (CC), which has a constraint length K_C, takes a block (also called frame) of bits $\mathbf{d} = [d_1, d_2, \ldots, d_K]^T \in \{-1, +1\}^K$ [with $(K_C - 1)$ (-1)'s at the tail to reset the CC] as its input and gives a larger block of bits $\mathbf{u} = \mathrm{C}(\mathbf{d}) = [u_1, u_2, \ldots, u_{\breve{K}}]^T \in \{-1, +1\}^{\breve{K}}$ as its output, where $(\cdot)^T$ stands for the transpose and -1 and $+1$ denote the binary digits 0 and 1, respectively. The CC coding rate is then defined as $R_C = K/\breve{K}$. We can puncture the CC output block $\mathbf{u}$ to obtain a smaller block of bits $\mathbf{v} = [v_1, v_2, \ldots, v_{\tilde{K}}]^T \in \{-1, +1\}^{\tilde{K}}$ ($\tilde{K} < \breve{K}$) to increase the transmission data rate. The puncturing rate is $R_P = \tilde{K}/\breve{K}$, and the coding rate of the punctured CC is $R = R_C/R_P = K/\tilde{K}$. The output $\mathbf{v}$ of the (punctured) CC is then fed to the interleaver whose output is denoted as $\mathbf{b} = [b_1, b_2, \ldots, b_{\tilde{K}}]^T \in \{-1, +1\}^{\tilde{K}}$. The mapper maps the interleaved bits into data symbols through the mapping $f : \{-1, +1\}^B \to \mathcal{C}$, where $\mathcal{C}$ denotes the MPSK constellation and $B = \log_2 |\mathcal{C}|$ is the number of bits comprised in a data symbol. Let $\bar{K} = \tilde{K}/B$ be an integer, which is the number of data symbols in a block. Then the mapper output can be expressed as $\mathbf{z} = [z_1, z_2, \dots, z_{\bar{K}}]^T \in \mathcal{C}^{\bar{K}}$, the elements of which are double differentially modulated to obtain the symbols to be transmitted, $\mathbf{s} = [s_{-1}, s_0, s_1, \dots, s_{\bar{K}}]^T \in \mathcal{C}^{\bar{K}+2}$, by

$$s_t = s_{t-1} p_t, \quad p_t = p_{t-1} z_t \tag{1}$$

with all symbols being unit magnitude, i.e., $|z_t| = |p_t| = |s_t| = 1$. Here t denotes the time (in units of symbol intervals), which, for notational convenience, can be considered to start from -1. The initial conditions are $s_{-1} = 1$ and $p_0 = 1$ (and thus $s_0 = 1$).

At the receiver, the received signal can be expressed as

$$r_t = h e^{j\omega t} s_t + n_t, \quad t = -1, 0, 1, \dots, \bar{K}, \tag{2}$$

where h is the unknown complex channel gain, ω is the unknown frequency offset (in radians) due to a Doppler shift or instabilities associated with the transmit and receive carrier oscillators, and $n_t \sim \mathcal{N}(0, \sigma^2)$ is the additive zero-mean white circularly symmetric complex Gaussian noise with variance σ^2. Here h and ω are assumed to be constant during the entire block (or frame). (We only need h and ω to be constant during a sub-block of length N used in the MSHD, but we made the previous stronger assumption for notational convenience.) Using the input $\mathbf{r} = [r_{-1}, r_0, r_1, \dots, r_{\bar{K}}]^T \in \mathbb{C}^{\bar{K}+2}$, the soft-detector generates a sequence of bit metrics $\hat{\mathbf{b}} = [\hat{b}_1, \hat{b}_2, \dots, \hat{b}_{\tilde{K}}]^T \in \mathbb{R}^{\tilde{K}}$, with $\hat{b}_i$ being the bit metric corresponding to b_i, $i = 1, 2, \dots, \tilde{K}$. Passing the above bit metric sequence $\hat{\mathbf{b}}$ through the deinterleaver, we obtain the deinterleaved bit metric sequence $\hat{\mathbf{v}} = [\hat{v}_1, \hat{v}_2, \dots, \hat{v}_{\tilde{K}}]^T \in \mathbb{R}^{\tilde{K}}$. For the punctured CC codes, we also need the bit metric for each punctured bit before using the Viterbi algorithm. This can be done easily by using zero as the bit metric for each punctured bit. Once we get the bit metric sequence $\hat{\mathbf{u}} = [\hat{u}_1, \hat{u}_2, \dots, \hat{u}_{\check{K}}]^T \in \mathbb{R}^{\check{K}}$ corresponding to the CC output $\mathbf{u}$, we can use the Viterbi algorithm to obtain the estimate $\hat{\mathbf{d}} = [\hat{d}_1, \hat{d}_2, \dots, \hat{d}_K]^T \in \{-1, +1\}^K$ of the source bit sequence $\mathbf{d}$.

In the sequel, we focus on the calculation of the bit metrics based on the detection results of the MSHD.

3. SOFT-DETECTION

Consider a sub-block of received data with length N, denoted as $\mathbf{r}_t = [r_{t-N+1}, r_{t-N+2}, \dots, r_t]^T \in \mathbb{C}^N$. Let $\mathbf{b}_t = [b_{(t-N+2)B+1}, b_{(t-N+2)B+2}, \dots, b_{tB}]^T \in \{-1, +1\}^{(N-2)B}$ be the data bits corresponding to $\mathbf{r}_t$. Then, for the ith bit, $i = (t-N+2)B+1, (t-N+2)B+2, \dots, tB$, the bit metric (also known as the L-value) can be defined as

$$\hat{b}_i = \sigma^2 \log \frac{P(b_i = +1 | \mathbf{r}_t, h, \omega)}{P(b_i = -1 | \mathbf{r}_t, h, \omega)}, \tag{3}$$

where the scalar σ^2 is immaterial for decoding. Assuming equal prior probability for each data bit and using the Bayes' theorem, the bit metric can be written as (with the presence of the unknown parameters h and ω, which will be concentrated out later by performing optimizations for detection over them):

$$\hat{b}_i = \sigma^2 \log \frac{\sum_{\mathbf{b}_t \in \mathcal{B}_{t,i,+1}} P(\mathbf{r}_t | \mathbf{b}_t, h, \omega)}{\sum_{\mathbf{b}_t \in \mathcal{B}_{t,i,-1}} P(\mathbf{r}_t | \mathbf{b}_t, h, \omega)}, \tag{4}$$

where $\mathcal{B}_{t,i,+1}$ and $\mathcal{B}_{t,i,-1}$ are the sets of $2^{(N-2)B-1}$ bit vectors $\mathbf{b}_t$ with b_i being $+1$ and -1, respectively.

Under the assumption of additive zero-mean white circularly symmetric complex Gaussian noise for the received data, the above equation can be written as

$$\hat{b}_i = \sigma^2 \log \frac{\sum_{\mathbf{b}_t \in \mathcal{B}_{t,i,+1}} e^{-\frac{1}{\sigma^2} \|\mathbf{r}_t - h \mathbf{s}_\omega(\mathbf{b}_t)\|^2}}{\sum_{\mathbf{b}_t \in \mathcal{B}_{t,i,-1}} e^{-\frac{1}{\sigma^2} \|\mathbf{r}_t - h \mathbf{s}_\omega(\mathbf{b}_t)\|^2}}, \tag{5}$$

which, by using the max-log approximation [8], can be approximated as

$$\begin{aligned} \hat{b}_i \approx\ & \sigma^2 \max_{\mathbf{b}_t \in \mathcal{B}_{t,i,+1}} \left\{ -\frac{1}{\sigma^2} \|\mathbf{r}_t - h \mathbf{s}_\omega(\mathbf{b}_t)\|^2 \right\} \\ & -\sigma^2 \max_{\mathbf{b}_t \in \mathcal{B}_{t,i,-1}} \left\{ -\frac{1}{\sigma^2} \|\mathbf{r}_t - h \mathbf{s}_\omega(\mathbf{b}_t)\|^2 \right\} \\ =\ & \min_{\mathbf{b}_t \in \mathcal{B}_{t,i,+1}} \|\mathbf{r}_t - h \mathbf{s}_\omega(\mathbf{b}_t)\|^2 \\ & - \min_{\mathbf{b}_t \in \mathcal{B}_{t,i,-1}} \|\mathbf{r}_t - h \mathbf{s}_\omega(\mathbf{b}_t)\|^2, \end{aligned} \tag{6}$$

where

$$\mathbf{s}_\omega(\mathbf{b}_t) = \begin{bmatrix} e^{j\omega(t-N+1)} s_{t-N+1} \\ e^{j\omega(t-N+2)} s_{t-N+2} \\ \vdots \\ e^{j\omega t} s_t \end{bmatrix} \in \mathbb{C}^N. \tag{7}$$

(This vector is written as a function of $\mathbf{b}_t$ to stress its dependence on $\mathbf{b}_t$.)

In what follows, we consider a further approximation of (6); in particular, we show how h and ω can be concentrated out from (6).

First, we simplify the calculation of the bit metric by restricting it to appropriate subsets of $\mathcal{B}_{t,i,+1}$ and $\mathcal{B}_{t,i,-1}$, since (6) is a computationally formidable task even for a

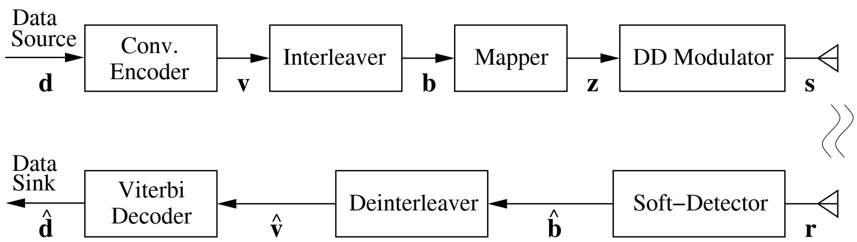

Fig. 1. Diagram of a communication system with DD modulation/detection and convolutional coding.

moderate values of N. Let

$$\mathbf{z}_t = \begin{bmatrix} z_{t-N+3} \\ z_{t-N+4} \\ \vdots \\ z_t \end{bmatrix} \in \mathcal{C}^{N-2} \tag{8}$$

be the data symbols corresponding to $\mathbf{b}_t$. Due to the fact that $\mathbf{s}_\omega(\mathbf{b}_t)$ can be expressed as $\mathbf{s}_\omega(\mathbf{z}_t)$, we note that

$$\begin{aligned} &\min_{\mathbf{z}_t \in \mathcal{C}^{N-2}} \|\mathbf{r}_t - h\mathbf{s}_\omega(\mathbf{z}_t)\|^2 \\ &= \min \Big\{ \min_{\mathbf{b}_t \in \mathcal{B}_{t,i,+1}} \|\mathbf{r}_t - h\mathbf{s}_\omega(\mathbf{b}_t)\|^2, \\ &\qquad \min_{\mathbf{b}_t \in \mathcal{B}_{t,i,-1}} \|\mathbf{r}_t - h\mathbf{s}_\omega(\mathbf{b}_t)\|^2 \Big\}, \end{aligned} \tag{9}$$

which means that one term of (6) can be obtained by computing

$$\min_{\mathbf{z}_t \in \mathcal{C}^{N-2}} \|\mathbf{r}_t - h\mathbf{s}_\omega(\mathbf{z}_t)\|^2. \tag{10}$$

As for the other term, it cannot be calculated efficiently; however, it can be approximately computed by searching within $\mathcal{B}^C_{t,i,+1} \subset \mathcal{B}_{t,i,+1}$ or $\mathcal{B}^C_{t,i,-1} \subset \mathcal{B}_{t,i,-1}$, two small sets of $\mathbf{b}_t$, the elements of which are largely determined by $\hat{\mathbf{z}}_t$ (the detection results for $\mathbf{z}_t$) as specified in the sequel.

Second, we show how to calculate $\min_{\mathbf{z}_t \in \mathcal{C}^{N-2}} \|\mathbf{r}_t - h\mathbf{s}_\omega(\mathbf{z}_t)\|^2$ with h and ω being concentrated out and we also specify $\mathcal{B}^C_{t,i,+1}$ and $\mathcal{B}^C_{t,i,-1}$. It is shown in [5] that, by using a generalized likelihood ratio criterion, $\min_{\mathbf{z}_t \in \mathcal{C}^{N-2},h,\omega} \|\mathbf{r}_t - h\mathbf{s}_\omega(\mathbf{z}_t)\|^2$ is equivalent to

$$\max_{\mathbf{z}_t \in \mathcal{C}^{N-2},\alpha} \left| \sum_{n=0}^{N-1} r^*_{t-n} e^{j\alpha(N-1-n)} \prod_{m=0}^{N-3-n} z^{m+1}_{t-n-m} \right|^2, \tag{11}$$

where $e^{j\alpha} = e^{j\omega} p_{t-N+2}$ and $(\cdot)^*$ denotes the complex conjugate. Here the product is deemed to have unit value if the lower limit exceeds the upper one. It is further shown in [5] that (11) can be approximated by the MSHD

$$\max_{\mathbf{z} \in \mathcal{C}^{N-2}} \left| \sum_{n=0}^{N-2} y_{t-n} \prod_{m=0}^{N-3-n} z^*_{t-n-m} \right|^2, \tag{12}$$

where $y_{t-n} \triangleq r_{t-n} r^*_{t-n-1}$. Let

$$x_{t-n} = x \prod_{m=0}^{N-3-n} z^*_{t-n-m}, \tag{13}$$

where x may be any member in $\mathcal{C}$ due to the ambiguity caused by the unknown channel parameters. Then (12) can be rewritten, in terms of $\mathbf{x}_t = [x_{t-N+2}, x_{t-N+3}, \ldots, x_t]^T \in \mathcal{C}^{N-1}$, as

$$\max_{\mathbf{x}_t \in \mathcal{C}^{N-1}} \left| \sum_{n=0}^{N-2} y_{t-n} x_{t-n} \right|^2, \tag{14}$$

which can be calculated efficiently by the fast algorithm of [6]. Let $\hat{\mathbf{x}}$ be the detection result for $\mathbf{x}$ by using (14). Then we can obtain the elements of the detected data symbol vector $\hat{\mathbf{z}}_t$ via

$$\hat{z}_{t-n} = \hat{x}^*_{t-n} \hat{x}_{t-n-1}. \tag{15}$$

We can see from the above equation that $\hat{z}_{t-n}$ is determined by two consecutive symbols $\hat{x}_{t-n}$ and $\hat{x}_{t-n-1}$. As a result, for $i = (t-n-1)B+1, (t-n-1)B+2, \ldots, (t-n)B$, i.e., the data bits corresponding to z_{t-n}, $\mathcal{B}^C_{t,i,+1}$ can be formed by varying x_{t-n-1} and x_{t-n}, respectively, within $\mathcal{C}$ while keeping the other elements in $\hat{\mathbf{x}}_t$ fixed. Therefore, $\mathcal{B}^C_{t,i,+1}$ has $2 \times 2^{B-1}$ elements, half of which come from altering x_{t-n-1} and the others from altering x_{t-n}. Similarly, we can obtain $\mathcal{B}^C_{t,i,-1}$.

By letting $N = 3$, we obtain the SSD-based soft-detector. In this case, however, $\mathcal{B}^C_{t,i,+1} = \mathcal{B}_{t,i,+1}$ and $\mathcal{B}^C_{t,i,-1} = \mathcal{B}_{t,i,-1}$ have 2^{B-1} instead of $2 \times 2^{B-1}$ elements due to the ambiguity in $\mathbf{x}_t$.

4. SIMULATION RESULTS

We provide a numerical example to demonstrate the performance of the new MSHD-based soft-detector. The simulation conditions are as follows: (a) the CC is the industrial standard constraint length $K_C = 7$, rate 1/2 encoder, which has generation polynomials $g_0 = (133)_8$ and $g_1 = (171)_8$; (b) the channel is additive white Gaussian noise channel with ω being uniformly distributed over $[-\pi, +\pi]$; (c) K

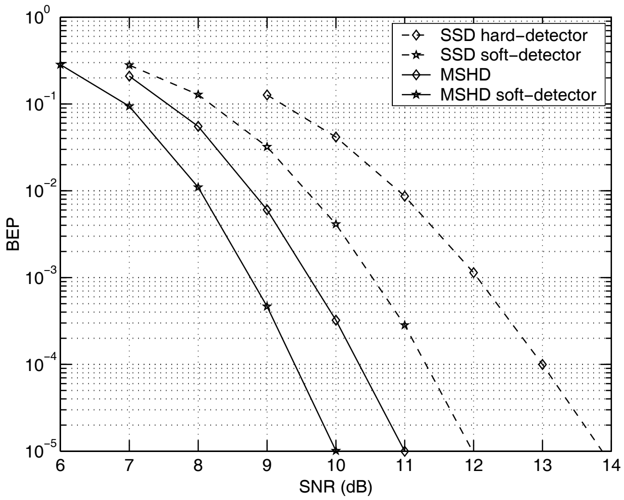

Fig. 2. A comparison of the average BEP performance of the SSD- and MSHD-based hard- and soft-detectors.

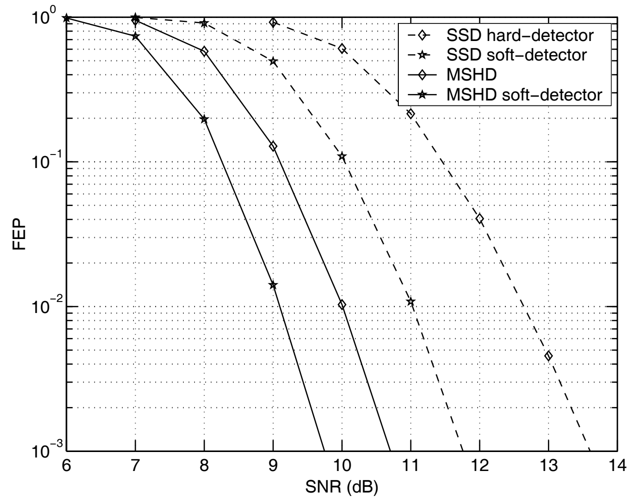

Fig. 3. A comparison of the average FEP performance of the SSD- and MSHD-based hard- and soft-detectors.

= 256, and N = 34, where the sub-blocks of received data are overlapped by two; (d) the constellation is QPSK; and (e) the SNR used herein is the symbol to noise power ratio. Figures 2 and 3, respectively, show the simulation results for the average bit-error probability (BEP) and frame-error probability (FEP) for the new soft-detector, obtained via 4×10^5 Monte-Carlo trials. The corresponding curves for the MSHD-based hard-detector and SSD-based soft- and hard-detectors are also given for comparison. We can see from the figures that the new soft-detector is superior to the other detectors.

5. CONCLUDING REMARKS

We have proposed a simple soft-detector for double differential modulation, which exploits the detection results of the computationally efficient MSHD. Simulation results have been provided to show the superior performance of the new soft-detector. Although this soft-detector was proposed here for double differential modulation, it can be readily extended to the single differential modulation case.

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