NEAR-FAR RESISTANT MULTI-USER DETECTOR USING ENERGY CONTOURS

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ABSTRACT

A multi-user detector with scalable complexity that achieves the maximum likelihood (ML) solution for two users and gives good sub-optimal performance for a higher number of users is proposed. The key idea is to construct a look-up table based on the geometric structure of the signal constellation, and then perform fast decoding based on the look-up table. The proposed detector is near-far resistant and its performance is consistently better than existing sub-optimal detectors when the number of users is greater than the number of dimensions. The robustness of the detector against noise can be controlled at the expense of higher complexity.

1. INTRODUCTION

The goal of multi-user detection is to correctly demodulate the information bits of mutually interfering users in a noisy communication system. The performance bound for joint detection is given by the maximum likelihood (ML) detector, which determines the most likely bits sent over the channel. While the ML detector (sometimes called the optimal detector [1]) achieves the lowest probability of error for joint detection, it has a complexity that is exponential in the number of users. A number of popular approaches to low-complexity sub-optimal multi-user detection include interference cancellation and several decision-driven detectors ([2],[3],[4]) have been developed based on this idea. In this work, rather than subtract the interference of individual signals, we have adopted a geometric approach that exploits the overall structure of the joint signal constellation. The proposed detector is near-far resistant in the sense that it gives perfect decoding for every user in high SNR or noiseless situations, regardless of the relative energies of the interfering users. The near-far resistant property will continue to hold in oversaturated situations where the number of users is greater than the number of dimensions. The linear minimum mean squared error (LMMSE) detector [1] and related schemes [5] are not near-far resistant in oversaturated systems where the user signals form a linearly dependent set. In noisy channels, we show how the performance of the proposed detector can be enhanced at the expense of a reasonable increase in detection complexity.

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2. ENERGY CONTOUR (EC) DETECTOR

We assume a synchronous K-user system in which the users transmit BPSK modulated signals $\{s_i(t)\}_{i=1}^{K}$ over an AWGN channel. The received signal can be written in equivalent discrete-time form as

$$y[t] = \sum_{i=1}^{K} b_i s_i[t] + n[t], \;\; \{b_i\}_{i=1}^{K} \in \{-1,+1\}^K \qquad (1)$$

In equivalent signal space notation, Equation (1) can be rewritten as

$$\mathbf{y} = \mathbf{S}\mathbf{b} + \mathbf{n} \qquad (2)$$

where $\mathbf{y}$ is the received vector, $\mathbf{b}$ is the bit vector or K-tuple sent, $\mathbf{S} = [\mathbf{s}_1\mathbf{s}_2 \ldots \mathbf{s}_K]$ is the $(N \times K)$ signal matrix, $\{\mathbf{s}_i\}_{i=1}^{K}$ are the signal vectors, N is the dimension of signal space, and $\mathbf{n}$ is the noise vector modeled as $\mathbf{n} \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$, i.e., as a zero-mean Gaussian N-dimensional random vector with variance σ^2 along each dimension.

The K-tuples $\{\mathbf{b}_j\}_{j=1}^{2^K}$ generate 2^K constellation points in signal space. It is easy to verify that constellation points corresponding to bitwise complements $\mathbf{b}$ and $\bar{\mathbf{b}}$ will have same energy $||\mathbf{Sb}||^2$ (where $||\cdot||^2$ denotes the Euclidean norm). The energy contours of different constellation points will, therefore, be concentric hyperspheres centered at the origin. A constellation point corresponding to $\mathbf{b}$ and its bitwise complement $\bar{\mathbf{b}}$ will lie along opposite ends of the diameter of the same energy contour. Such a pair of constellation points will henceforth be called a complement pair. It is possible for complement pairs to share the same energy contour.

We can set up an energy contour (EC) table that stores the constellation points sorted in ascending order of their energy. Since we need to store only one member of a complement pair, the EC table will have (2^{K-1}) entries. The i^{th} entry in the table stores the i^{th} signal energy r_i in the sorted list, and the bit vector $\mathbf{b}_i$ from a complement pair that have that energy. Once the table is set up, the basic EC detector works according to the following algorithm:

1. Measure the energy r_y of the received signal.

2. Compare r_y against the $(\frac{2^{K-1}}{2})^{th}$ entry of the EC table. If $r_y > r_{2^{K-2}}$ we need to search only the bottom half of the EC table. Else, we need to search the top half. The selected half of the EC table containing 2^{K-2} entries is called the reduced EC table.

3. Repeat Step 2 recursively on the reduced EC table obtained in the last recursion, until we reach a reduced EC table with only two entries. This is equivalent to a binary search ([6]) of the EC table and in $(K-1)$ steps we can find a signal energy r_i such that

$$r_1 \leq r_2 \leq \cdots \leq r_i \leq r_y \leq r_{i+1} \leq \cdots \leq r_{2^{(K-1)}}$$

4. Perform a local ML (LML) detection over the two pairs of constellation points $(\mathbf{b}_i, \bar{\mathbf{b}}_i)$ and $(\mathbf{b}_{i+1}, \bar{\mathbf{b}}_{i+1})$ that correspond to r_i and r_{i+1}. If $i = 2^{K-1}$, then perform LML detection over $(\mathbf{b}_i, \bar{\mathbf{b}}_i)$ and $(\mathbf{b}_{i-1}, \bar{\mathbf{b}}_{i-1})$. If $r_y \leq r_1$, then perform LML detection over $(\mathbf{b}_1, \bar{\mathbf{b}}_1)$ and $(\mathbf{b}_2, \bar{\mathbf{b}}_2)$. The LML solution is the decoded result.

2.1. Complexity

The EC table has 2^{K-1} entries already sorted in ascending order of energy. Therefore, performing a binary search we can reach the right entry in $\log_2(2^{K-1})$ or $(K-1)$ comparisons. Finding the LML solution for two complement pairs requires $4N$ subtractions (to determine the errors $e(\mathbf{y}, \mathbf{b}) = \mathbf{y} - \mathbf{S}\mathbf{b}$), $4N$ multiplies and $4(N-1)$ additions to determine the error energies, and finally, 4 comparisons to choose the LML solution. Measuring the received signal's energy is basically another inner product operation requiring N multiplies and $(N-1)$ additions. The total complexity of the basic EC detector is therefore $(K-1)+4$ compares, $4N$ subtractions, $5(N-1)$ additions and $5N$ multiplies.

However, to set up the table, we first need to measure the energy of every complement pair, which in total requires $2^{K-1}N$ multiplies and $2^{K-1}(N-1)$ additions (since we can leave out the complements of every bit vector considered). Sorting the 2^{K-1} energies in ascending order using a quick-sort algorithm (see [6] for details) will require $(K-1)2^{(K-1)}$ comparisons.

2.2. Decision regions of the EC detector

Consider the constellation point with bit vector $\mathbf{b}_i$ corresponding to the i^{th} entry of the EC table. The decision region of $\mathbf{b}_i$, denoted as $\Omega(\mathbf{b}_i)$, is given by the union of two separate regions:

(i) The intersection of the Voronoi partitioning (or LML decision region) of the constellation point $\mathbf{b}_i$ with respect to its complement $\bar{\mathbf{b}}_i$ and the complement pair $(\mathbf{b}_{i-1}, \bar{\mathbf{b}}_{i-1})$, and the hyper-annulus enclosed between the energy contours corresponding to the i^{th} and $(i-1)^{th}$ entry of the EC table. If $\Lambda(\mathbf{b}_i; \mathbf{b}_j, \mathbf{b}_k, \mathbf{b}_l)$ denotes the overall Voronoi partition of $\mathbf{b}_i$ with respect to $\mathbf{b}_j$, $\mathbf{b}_k$ and $\mathbf{b}_l$, and C_i denotes the volume enclosed by the i^{th} energy contour, then this part of the decision region of $\mathbf{b}_i$ can be mathematically expressed as:

$$\Omega_1(\mathbf{b}_i) = \Lambda(\mathbf{b}_i; \bar{\mathbf{b}}_i, \mathbf{b}_{i-1}, \bar{\mathbf{b}}_{i-1}) \cap (C_i \cap \bar{C}_{i-1})$$

(ii) The intersection of the Voronoi partition of the constellation point $\mathbf{b}_i$ with respect to its complement $\bar{\mathbf{b}}_i$ and the complement pair $(\mathbf{b}_{i+1}, \bar{\mathbf{b}}_{i+1})$, and the hyper-annulus enclosed between the energy contours corresponding to the i^{th} and $(i+1)^{th}$ entry of the EC table. Using similar notation we express this part of the decision region of $\mathbf{b}_i$ as

$$\Omega_2(\mathbf{b}_i) = \Lambda(\mathbf{b}_i; \bar{\mathbf{b}}_i, \mathbf{b}_{i+1}, \bar{\mathbf{b}}_{i+1})) \cap (C_{i+1} \cap \bar{C}_i)$$

Note that $C_{i-1} \subseteq C_i \subseteq C_{i+1}$. Then $\Omega(\mathbf{b}_i)$ can be expressed as

$$\Omega(\mathbf{b}_i) = \Omega_1(\mathbf{b}_i) \cup \Omega_2(\mathbf{b}_i)$$

A property of Voronoi partitioning is that the constellation point always lies within its decision region. Therefore, by construction every constellation point will always lie within its decision region for the EC detector. Of course, these decision regions will not be the ML decision regions and hence, the detector will not give optimal decisions in the presence of noise. However, in the absence of noise or in sufficiently high SNR situations, it will give perfect decoding for every user, regardless of the energies of the interfering users, i.e., the EC detector is near-far resistant.

3. NOISE-ROBUST EC DETECTOR

The basic EC detector is near-far resistant and gives good performance in high SNR situations. However, in noisy channels, the received point can be thrown outside the right hyper-annulus between energy contours, especially when the radii of the energy contours are close enough. This will lead to erroneous detection. It is prudent to search within a window of W energy contours in the EC table where W is decided upon dynamically based on $\mathbf{y}$ and the noise variance σ^2. The modified algorithm will then be as follows:

1. Measure the energy r_y of the received signal.

2. Compute the energies $r_{min} = (\sqrt{r}_y - n_r)^2$ and $r_{max} = (\sqrt{r}_y + n_r)^2$, where n_r is the radius of noise perturbation chosen depending on the noise statistics. For example, for an AWGN channel, we can choose $n_r = \alpha\sigma\sqrt{N}$, where α is a controllable parameter.

3. Perform two separate binary searches through the EC table and in $2(K-1)$ steps determine the signal energies r_{i_1} and r_{i_2} such that

   $$r_1 \leq r_2 \leq \cdots \leq r_{i_1} \leq r_{min} \leq r_{i_1+1} \leq \cdots \leq r_{2^{(K-1)}}$$

   and

   $$r_1 \leq r_2 \leq \cdots \leq r_{i_2} \leq r_{max} \leq r_{i_2+1} \leq \cdots \leq r_{2^{(K-1)}}$$

   Because the EC table is already sorted, we must have $r_{i_1} \leq r_{i_2}$.

4. Perform LML detection over the constellation pairs that belong to the reduced EC table:

   $$\{r_{\max(1,i_1)}, \ldots, r_{\min(2^{K-1}, i_2+1)}\}$$

   .

5. The LML solution to the $M = 2(i_2 - i_1 + 1)$ constellation points is the decoded result. Note that $W = \frac{M}{2}$.

3.1. Complexity of the modified EC detector

The complexity of setting up the EC table will still be same. But the complexity of the EC detector will increase with M. We will need only $2(K-1)$ comparisons to reach the right r_{i_1} and r_{i_2} in the EC table, but we will need additional M comparisons to reach the LML solution. Therefore we will need $(M+1)N$ multiplies, $(M+1)(N-1)$ additions (to measure r_y and to find the squared error from each of the M constellation points), MN subtractions (to compute the errors) and M compares.

3.2. Decision regions of the modified detector

The decision region of every constellation point will be given by the intersection of

(i) the LML decision region with respect to its complement and the $(M-2)$ points in the surrounding $(W-1)$ contours, and

(ii) the hyper-annulus between the energy contours corresponding to r_{i_1} and r_{i_2+1}.

Therefore, by of Voronoi partitioning, every point will always lie in its decision region. The modified detector will still be near-far resistant and we can expect the robustness against noise to improve as we increase n_r. But increasing n_r will increase M and the complexity will go up. In the limiting case, when $M = 2^K$, the EC detector becomes the ML detector and has exponential complexity in K but also gives ML performance.

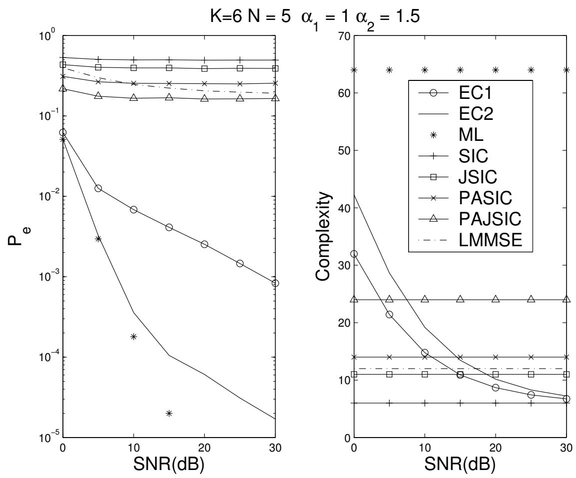
Though related, our approach is different from sphere decoding [7]. The LML search is performed over a hyper-annulus defined by $\mathbf{y}$ and n_r whereas the basic sphere decoding algorithm searches over a sphere centered on the received point. Our approach is also different from the class of LML detectors proposed in [8]. We perform LML search over constellation points classified according to geometric proximity, rather than bitwise proximity.

4. SIMULATION RESULTS

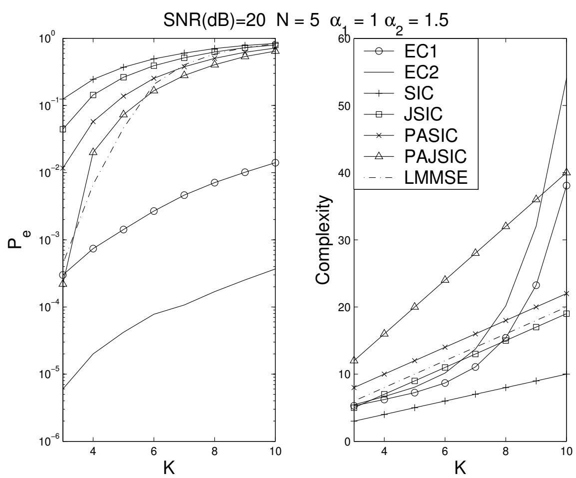
The performance of the EC detector was tested against that of the standard serial interference canceller (SIC) [1], the linear minimum mean squared error detector (LMMSE) [1], joint serial interference canceller (JSIC) [4], parallel arbitrated SIC (PASIC) [3] and parallel arbitrated JSIC (PAJSIC) [4] detectors. The PASIC detector runs the basic SIC detection algorithm separately on L different orderings of the same signal set and then does LML detection over the L different SIC solutions obtained. PAJSIC is the same as PASIC except that it uses JSIC detection, for each of the L decoding stages. In our simulations, we chose $L = 2$.

Simulations were run over randomized user signatures $\{\mathbf{s}_i\}_{i=1}^K$, where $\mathbf{s}_i \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$. The performance of the EC detector has been tested for $\alpha_1 = 1$ (EC_1) and $\alpha_2 = 1.5$ (EC_2). The number of dimensions, N, was chosen to be 5. Figure 1(a) shows the probability of error for sequence detection (i.e. $P_e(\hat{\mathbf{b}} \neq \mathbf{b})$) for all the detectors plotted against increasing SNR for $K = 6$. The performance of the ML detector is plotted as a lower bound. Figure 1(a) also shows a comparison of the complexity of the different detectors for the same set of simulations. The SIC detector performs K inner products, LMMSE performs $2K$ inner products (K projections to get the matched filter outputs and another K projections to determine the bits) JSIC performs $(2(K-1)+1)$ inner products, PASIC with $L = 2$ performs $2K$ inner products and then 2 error comparisons. Similarly, the complexity of the PAJSIC detector is $2(2(K-1)+1)+2 = 4K$. Figure 1(b) shows the probability of error and detection complexity for the different detectors plotted against the number of users for SNR = 20dB.

We observe that the EC detectors consistently outperform the other sub-optimal detectors in oversaturated situations ($K > N$). It is also observed that the performance of the EC detector improves with higher α with a correspond-



(a) P_e and detection complexity plotted against SNR



(b) P_e and detection complexity plotted against K

ing increase in complexity. This is expected because the EC detector with the higher α searches over a larger hyper-annulus and hence over a larger set of constellation points. We also observe that its expected complexity for fixed K decreases with increasing SNR. This is because the noise radius n_r decreases with increase in SNR, and hence, the EC detector needs to perform LML detection over a smaller set of constellation points.

5. CONCLUSION

We have proposed a multi-user detector that has scalable complexity and is near-far resistant, even when the number of users is greater than the number of dimensions. For low SNR situations, the basic EC detector can be adjusted to give higher robustness against noise at the cost of higher complexity of detection. A trade-off between decoding complexity and detection error can be achieved by controlling the noise radius about the received point. The proposed detector will give much better performance than existing sub-optimal detectors in oversaturated situations, i.e., where the number of users is higher than the number of dimensions. The proposed detector assumes perfect knowledge of the user signals. An interesting future direction will be to analyze how imperfect knowledge of the user signals affects detector performance. Future work will also focus on extending the detection algorithm to the asynchronous communication case.

6. REFERENCES

[1] S. Verdú, *Multiuser Detection*, Cambridge University Press, 1998.

[2] E. Dahlman and K. Jamal, "Multi-stage serial interference cancellation for ds-cdma," in *IEEE* 46^{th} *Vehicular Technology Conference*, Atlanta, GA, May 1996.

[3] G. Barriac and U. Madhow, "Pasic: A new paradigm for low-complexity multiuser detection," in *Proc. CISS 2001*, Baltimore, MD, March 21-23 2001.

[4] A. Sen Gupta and A.C. Singer, "Linear complexity multiuser detector using joint successive interference cancellation," to appear in *Proc. Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, Nov. 9-12, 2003.

[5] U. Madhow and M.L. Honig, "Error probability and near-far resistance of minimum mean squared error interference suppression schemes for cdma," in *Global Telecommunications Conference*, Dec. 1992, vol. 3, pp. 1339–1343.

[6] T. H. Cormen et al, *Introduction to Algorithms*, MIT Press, second edition, 2001.

[7] E. Viterbo and J. Boutros, "A universal lattice code decoder for fading channel," *IEEE Trans. Information Theory*, vol. 45, pp. 1639–1642, July 1999.

[8] Y. Sun, "Local maximum likelihood multiuser detection," in *Proc. CISS 2001*, Baltimore, MD, March 21-23 2001, pp. 7–12.