ITERATIVE CHANNEL ESTIMATION FOR TURBO RECEIVERS IN DS-CDMA

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ABSTRACT

This work considers the problem of channel estimation in the iterative reception of pilot-aided signals in DS-CDMA systems. The performance of classical training-based schemes is severely degraded in highly frequency selective channels due to the codemultiplexing of traffic and pilot signals. Thus, estimation algorithms that rely on the presence of the pilot signal but also consider the information signal structure are preferred. In this paper we present a bayesian channel estimation algorithm for turbo receivers that effectively exploits the available soft information about the symbols (to model the traffic signal) in order to iteratively improve the channel estimation. Simulation results in realistic frequency selective test cases reveal only a moderate degradation compared to the perfect channel knowledge case.

1. INTRODUCTION

The introduction of training sequences or reference signals for channel estimation purposes is a quite common practice in mobile communication systems. Traditionally the pilot or training signal has been transmitted time-multiplexed with the traffic data. Nevertheless, in many of the terrestrial mobile communication standards proposed today -such as UMTS- the pilot signal is codemultiplexed and sent at the same time as the traffic signal. Although the traffic and the pilot signal are in principle perfectly orthogonal codes, the high frequency selective nature of the mobile radio channel destroys this orthogonality at the reception stage. For this reason, the use of channel estimation algorithms based on the sole knowledge of the training signal results in a very poor performance. Thus, the natural solution to the problem of estimating the channel impulse response in this case is the use of semi-blind techniques, i.e. estimation algorithms that rely on the presence of the pilot signal but also consider the traffic signal structure [1].

This work considers a high rate MIMO system using channel coding and space-time (ST) coding as well as an iterative receiver based on the turbo principle, which employs error control coding to aid in the symbol estimation algorithm. In a turbo receiver the soft information about the symbols available at each iteration can be used to model the traffic signal in the channel estimation algorithm. Therefore, we propose a bayesian channel estimator that effectively exploits this iterative structure of the turbo receiver.

The remainder of this paper is organized as follows. The signal model and the turbo receiver structure are described in Section



Fig. 1. Transmitter Structure.

2 and 3, respectively. Section 4 presents the demodulator structure and focuses on the proposed channel estimator. Simulation results in realistic frequency selective Third Generation Partnership Project (3GPP) test cases are provided in Section 4. Finally, concluding remarks are summarized in Section 5.

2. SIGNAL MODEL

We consider a transmission system (see Fig. 1) in which the binary information bits are first encoded via a turbo or a convolutional encoder, interleaved and then modulated to symbols using a complex constellation. We will denote this set of bits as a coding block. In the modulation stage, the bits to symbols mapping must be done in such a form that the in-phase and in-quadrature components of each symbol can be treated as independent. Afterwards, the modulated symbols are encoded with a ST encoder, spreaded with a spreading factor F and transmitted along with the pilot signal by the M transmit antennas. Since the ST encoding process is linear the transmitted signal can be decomposed in B ST blocks that depend linearly on the in-phase and in-quadrature components of the modulated symbols. Each ST block is transmitted during Tsymbol periods using the Linear Dispersion Codes formulation [2]. In this case the chip-level transmitted signal for the b'th block can be represented by the vector $\overline{\mathbf{x}}(b) \in C^{MTF \times 1}$ equal to

$$\overline{\mathbf{x}}(b) = \overline{\mathbf{D}}\mathbf{s}(b) + \overline{\mathbf{p}}(b) \quad b = 1, \dots, B,$$
(1)

where $\overline{\mathbf{p}}(b)$ is the spreaded pilot signal, $\overline{\mathbf{D}}$ is a $MTF \times 2Q$ matrix representing the ST encoding and spreading processes and $\mathbf{s}(b)$ is the vector formed by the in-phase and in-quadrature components of the Q complex symbols corresponding to the b'th ST block.¹

The MIMO physical channel is modelled as a set of linear filters with L taps. Each tap $\overline{\mathbf{H}}^{(l)} \in \mathcal{C}^{N \times M}$ is a complex time varying matrix containing the gains of all the possible paths between

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¹In the rest of this paper we consider the following notation: $\overline{\mathbf{a}}$ denotes a complex vector or matrix and \mathbf{a} the real vector or matrix formed as $\mathbf{a} = [\operatorname{Re}\{\overline{\mathbf{a}}\}^T \operatorname{Im}\{\overline{\mathbf{a}}\}^T]^T$.

the N receive antennas and the M transmit antennas. The received signal corresponding to the b'th transmitted ST block, denoted by the vector $\overline{\mathbf{y}}(b) \in \mathcal{C}^{NTF \times 1}$ is given then by

$$\overline{\mathbf{y}}(b) = \overline{\mathbf{H}}(b) \left(\overline{\mathbf{Ds}}(b) + \overline{\mathbf{p}}(b)\right) + \mathrm{IBI} + \overline{\mathbf{w}}(b), \qquad (2)$$

where $\overline{\mathbf{H}}(b)$ is the $NTF \times MTF$ convolution matrix of the MIMO channel, the term IBI denotes the interblock interference and $\overline{\mathbf{w}}(b)$ is the additive spatially and temporally white Gaussian noise vector.

In order to design the channel estimator it is more convenient to express the term of the received signal due to the transmitted pilot as a function of the channel vector defined as

$$\overline{\mathbf{h}}(b) = \left[\overline{\mathbf{H}}^{(1)}(1,:)^{\mathrm{T}} \dots \overline{\mathbf{H}}^{(1)}(N,:)^{\mathrm{T}} \dots \overline{\mathbf{H}}^{(L)}(N,:)^{\mathrm{T}}\right]^{\mathrm{T}}.$$
(3)

Neglecting the IBI for the moment (as justified later), the received signal model in (2) can be then rewritten as

$$\overline{\mathbf{y}}(b) = \overline{\mathbf{H}}(b)\overline{\mathbf{D}}\mathbf{s}(b) + \overline{\mathbf{G}}_p(b)\mathbf{h}(b) + \overline{\mathbf{w}}(b), \tag{4}$$

with $\overline{\mathbf{G}}_p(b)\mathbf{h}(b) = \overline{\mathbf{H}}(b)\overline{\mathbf{p}}(b)$. The vector $\mathbf{h}(b)$ is directly obtained from the definition in (3) and $\overline{\mathbf{G}}_p(b)$ is a known $NTF \times 2MNL$ matrix constructed from the pilot signal $\overline{\mathbf{p}}(b)$ as described in the appendix. In addition, since the unknown vectors $\mathbf{s}(b)$ and $\mathbf{h}(b)$ are real by definition, equation (4) can be decoupled by stacking its real part over its imaginary part as

$$\mathbf{y}(b) = \mathbf{G}_s(b)\mathbf{s}(b) + \mathbf{G}_p(b)\mathbf{h}(b) + \mathbf{w}(b).$$
(5)

3. TURBO RECEIVER STRUCTURE

Even if the channel impulse response is known, the optimal receiver for the signal model in (5) is impractical, because it involves a Viterbi Algorithm of the effective super-trellis associated with the concatenated encoders and the frequency selective channel. This high complexity can be mitigated by appealing to the turbo principle. A turbo receiver (see Fig. 2) decouples the demodulation stage and the decoding stage and iterates the corresponding algorithms exchanging soft information between iterations. The soft information is expressed using log likelihood ratios (LLRs) defined as

$$L(d) = \log \frac{P(d=+1)}{P(d=-1)}.$$
(6)

The basic building blocks of a turbo receiver are the soft-input/softoutput (SISO) demodulator and the SISO channel decoder. The demodulation algorithm calculates the a posteriori LLR of each coded bit (\mathbf{L}_{D1}) as a function of the received samples and the a



Fig. 2. Receiver Structure.

priori LLR of all other coded bits (\mathbf{L}_{A1}). Similarly, the decoder algorithm obtains the a posteriori LLR of each coded bit (\mathbf{L}_{D2}) as a function of the a priori LLR of all other coded bits (\mathbf{L}_{A2}) and the correlations between them induced by the encoder's trellis. Of course, the decoder also computes posterior information bits probabilities, which yield the overall output of the combined algorithm. Since the decoder structure is well known in the literature, e.g. [3], now we concentrate on the demodulator.



Fig. 3. Demodulator Structure.

4. DEMODULATOR STRUCTURE

The turbo receiver structure shown in Fig. 2 processes coding blocks, whereas the demodulator (see Fig. 3) works individually on each of the *B* ST blocks that form a coding block. Due to the highly frequency selective nature of the MIMO channels, these received ST blocks are correlated and therefore, the performance of the channel estimator and the symbol estimator degrades. Hence, before applying the demodulation algorithm itself, we use a soft decision feedback equalizer (DFE) which tries to remove from the present block the contributions from the previous ST block (see dashed part of Fig. 3). The IBI can be consequently neglected and thus the resulting signal $\mathbf{y}(b)$ for the *b*'th received block corresponds with the received signal model presented in (5).

After the DFE, the channel estimation algorithm obtains the estimated channel impulse response $\hat{\mathbf{h}}(b)$ as a function of the received samples, $\mathbf{y}(b)$, and the a priori information about the channel and the symbols. The a priori channel information consists on the second order statistics, η_h and C_{hh} , obtained from the channel estimation of the precedent ST block. Similarly, the a priori symbol information is given by the second order statistics, η_s and C_{ss} , obtained by transforming the a priori LLRs L_{A1} of the symbols delivered by the decoder in the previous iteration. Once the current channel has been estimated, the associated mean η_h and covariance matrix C_{hh} are updated and the symbol detection algorithm begins. This algorithm also uses the soft information of the symbols and the already updated channel estimation as well as its second order statistics to improve the symbols estimations of the b'th ST block. The estimated symbols for the b'th ST block are transformed again into LLRs exploiting the fact that each symbol estimate is normally distributed as $\hat{s}_q \sim N(s_q, \sigma_{\hat{s}_q}^2)$. This process is repeated in a serial fashion for each of the B ST blocks, so that the resulting LLRs for all the ST blocks are the a posteriori LLRs \mathbf{L}_{D1} delivered by the demodulator. In the next sections we focus on the channel and the symbol estimation algorithms.

4.1. Channel Estimator

The proposed channel estimator is derived from the following signal model

$$\mathbf{y}(b) = \mathbf{G}_s(b)\mathbf{s}(b) + \mathbf{G}_p(b)\mathbf{h}(b) + \mathbf{w}(b).$$
(7)

as the bayesian estimate of the channel vector $\mathbf{h}(b)$ by assuming it to be Gaussian distributed with mean $\eta_{\mathbf{h}}$ and covariance matrix $\mathbf{C}_{\mathbf{h}\mathbf{h}}$. In order to include the soft information available about the information symbols we also consider the symbol vector $\mathbf{s}(b)$ as Gaussian with mean $\eta_{\mathbf{s}}$ and covariance matrix $\mathbf{C}_{\mathbf{ss}}$. With the purpose of obtaining an analytically tractable expression, the equivalent channel matrix \mathbf{G}_s is considered deterministic, although it depends on $\mathbf{h}(b)$. Thus, \mathbf{G}_s is formed by replacing the channel matrix $\mathbf{H}(b)$ with its existing estimate.

Taking into account the aforementioned assumptions the expression of the proposed channel estimator is given by [4]

$$\hat{\mathbf{h}}(b) = \mathcal{E}\{\mathbf{h}(b)|\mathbf{y}(b)\}$$

$$= \eta_{\mathbf{h}}(b-1) + \mathbf{C}_{\mathbf{h}\mathbf{h}}(b-1)\mathbf{G}_{p}(b)^{\mathrm{T}}\mathbf{C}_{\mathbf{y}\mathbf{y}}(b)^{-1}$$

$$(\mathbf{y} - \mathbf{G}_{s}(b)\eta_{\mathbf{s}}(b) - \mathbf{G}_{p}(b)\eta_{\mathbf{h}}(b-1)),$$
(8)

where

 $\mathbf{C}_{\mathbf{yy}}(b) = \mathbf{G}_s(b)\mathbf{C}_{\mathbf{ss}}(b)\mathbf{G}_s^{\mathrm{T}}(b) + \mathbf{G}_p(b)\mathbf{C}_{\mathbf{hh}}(b-1)\mathbf{G}_p^{\mathrm{T}}(b) + \sigma_w^2 \mathbf{I},$

and σ_w^2 is the noise variance. It is important to remark that the required channel statistics are obtained from the estimation corresponding to the previous ST block instead of using the statistics of the current block obtained in the previous iteration. Yet, they provide a good initialization (although the physical channels may change significantly from block to block) and speed up the convergence of the iterative channel estimator. As a consequence, after each estimation the available channel statistics are updated as

$$\eta_{\mathbf{h}}(b) = \mathbf{h}(b)$$
$$\mathbf{C}_{\mathbf{h}\mathbf{h}}(b) = \mathbf{C}_{\mathbf{h}\mathbf{h}}(b-1) \left(\mathbf{I} - \mathbf{G}_{p}^{\mathrm{T}}(b) \mathbf{C}_{\mathbf{y}\mathbf{y}}^{-1}(b) \mathbf{G}_{p}(b) \mathbf{C}_{\mathbf{h}\mathbf{h}}(b-1) \right).$$

4.2. Symbol Estimator

The symbol estimator is based on the linear estimator described in [5]. It is derived from (7) as the Maximum Likelihood (ML) estimate of current symbol assuming it to be a deterministic parameter while taking all other symbols as Gaussian random variables. This estimator also considers the channel vector $\mathbf{h}(b)$ to be Gaussian distributed. For simplicity we omit the block index and use the following model

$$\mathbf{y} = \mathbf{G}_s \mathbf{s} + \mathbf{G}_p \mathbf{h} + \mathbf{w}$$

= $\mathbf{G}_s \mathbf{e}_q s_q + \mathbf{G}_s \left(\mathbf{I} - \mathbf{e}_q \mathbf{e}_q^{\mathrm{T}} \right) \mathbf{s} + \mathbf{G}_p \mathbf{h} + \mathbf{w},$ (9)

where e_q is the q'th unity vector. It can be shown (see [4]) that the ML estimate of s_q assuming the signal model of equation (9) is

$$\hat{s}_{q} = \left(\mathbf{g}_{q}^{\mathrm{T}} \mathbf{C}_{q}^{-1} \mathbf{g}_{q}\right)^{-1} \mathbf{g}_{q}^{\mathrm{T}} \mathbf{C}_{q}^{-1}$$
$$\left(\mathbf{y} - \mathbf{G}_{s} \left(\mathbf{I} - \mathbf{e}_{q} \mathbf{e}_{q}^{\mathrm{T}}\right) \eta_{\mathbf{s}} - \mathbf{G}_{p} \eta_{\mathbf{h}}\right), \qquad (10)$$

where we define

$$\begin{split} \mathbf{g}_{q} &= \mathbf{G}_{s} \mathbf{e}_{q}, \\ \mathbf{C}_{q} &= \mathbf{G}_{s} \left(\mathbf{I} - \mathbf{e}_{q} \mathbf{e}_{q}^{\mathrm{T}} \right) \mathbf{C}_{\mathrm{ss}} \left(\mathbf{I} - \mathbf{e}_{q} \mathbf{e}_{q}^{\mathrm{T}} \right) \mathbf{G}_{s}^{\mathrm{T}} \\ &+ \mathbf{G}_{p} \mathbf{C}_{\mathrm{hh}} \mathbf{G}_{p}^{\mathrm{T}} + \sigma_{w}^{2} \mathbf{I}. \end{split}$$

This estimator should be applied to the 2Q unknown real symbols in each ST block, requiring for each of them the inversion of the associated C_q matrix. Fortunately, it can be shown that these estimators can be jointly derived as [5]

$$\hat{\mathbf{s}} = \eta_{\mathbf{s}} + \tilde{\mathbf{C}}_{\mathbf{ss}} \mathbf{C}_{\mathbf{yy}}^{-1} \left(\mathbf{y} - \mathbf{G}_s \eta_{\mathbf{s}} - \mathbf{G}_p \eta_{\mathbf{h}} \right)$$
(11)

where

$$\mathbf{C}_{\mathbf{y}\mathbf{y}} = \mathbf{G}_s \mathbf{C}_{\mathbf{ss}} \mathbf{G}_s^{\mathrm{T}} + \mathbf{G}_p \mathbf{C}_{\mathbf{h}\mathbf{h}} \mathbf{G}_p^{\mathrm{T}} + \sigma_w^2 \mathbf{I}.$$

 $\tilde{\mathbf{C}}_{ss}$ is a diagonal matrix with the elements $1/\lambda_q$, where λ_q are the diagonal elements of the matrix $\mathbf{G}_s^T \mathbf{C}_{yy}^{-1} \mathbf{G}_s$. Thus, only one inversion of the matrix \mathbf{C}_{yy} is required.

5. SIMULATIONS

Computer simulations have been carried out to analyze the performance of the aforementioned system in terms of frame error rate (FER) in the scope of the High Speed Downlink Packet Access (HSDPA) service, that has been recently standardized in 3GPP for UMTS. Consequently, most of the parameters for the simulation are based on the specifications of HSDPA, e.g. [6].

The transmitter is a space time generalization of the standardized UMTS downlink transmitter. Following the specifications, we define a Transmission Time Interval (TTI) as an interval of $N_{\rm chips} = 7680$ chips in which a frame of bits is transmitted. Due to the restrictions of the channel encoder, this frame is divided into coding blocks of up to $N_{\rm bits} \leq 5115$ information bits. Each coding block is encoded using a turbo encoder with rate 1/2 (constraint length 4, and polynomial generators 15 and 13), interleaved, and modulated to symbols using QPSK or 16-QAM. The symbols are then multiplexed into P = 10 parallel ST encoders by using 10 different spreading codes $\{\mathbf{c}(i)\}_{i=1}^{P}$ with a spreading factor of F = 16. The antennas transmit the sum of these parallel branches and the pilot signal using a chip rate of 3.84 Mcps. The pilot signal is code multiplexed with the information signal and is generated according to the standardized Common Pilot Channel (CPICH) of UMTS [7].

In the simulations we consider the frequency selective fading ITU Generalized Pedestrian A and Vehicular A channel models proposed in [8], in which all signal power arrives through L = 3 and L = 10 fading taps, respectively. The speed of the user equipment is assumed to be 3 km/h in the Pedestrian A channel and 50 km/h in the Vehicular A channel. The temporal variation of the channel is modeled with a time resolution equal to the chip period.

We provide results for the Alamouti scheme [9] with M = 2 transmit antennas and N = 1 receive antennas using both modulating schemes. The FER vs. EbNo results for 4 iterations of the receiver considering perfect channel knowledge and the proposed channel estimation algorithm are provided in Fig. 4 for the Pedestrian A channel and in Fig. 5 for the Vehicular A channel.

The results for the Generalized Pedestrian A channel illustrate only a minor degradation in the FER when using the proposed channel estimation scheme as compared to the results assuming perfect channel knowledge, specially for QPSK and in the FER range of interest for HSDPA services (between 1% and 10%). The degradation in the FER results is greater in the Vehicular channel, in which the higher Doppler rate makes channel tracking more difficult, but still moderate when using QPSK modulation.



Fig. 4. FER of Alamouti 2x1 in Pedestrian A Channel.



Fig. 5. FER of Alamouti 2x1 in Vehicular A Channel.

6. CONCLUSIONS

This paper presents an iterative bayesian channel estimator for pilot-aided coded DS-CDMA MIMO systems over frequency selective channels. As opposed to classical channel estimation techniques that are based on the sole knowledge of the training signal, the proposed algorithm also considers the presence of the traffic signal by taking profit of the turbo receiver structure. With the inherent soft-decision feedback available in a turbo receiver the channel estimation accuracy as well as the information symbol reliability is improved in a few iterations. Simulation results in the scope of the HSDPA service using a realistic stochastic MIMO channel model show promising performance at high transmission rates.

A. APPENDIX

In this appendix, we provide the details of the matrix $\overline{\mathbf{G}}_p(b)$ used in the derivation of the signal model in equation (4). $\overline{\mathbf{G}}_p(b)$ is the $NTF \times 2MNL$ matrix given by

$$\begin{split} \overline{\mathbf{G}}_{p}(b) &= \begin{bmatrix} \overline{\mathcal{G}}_{p}(b) & j\overline{\mathcal{G}}_{p}(b) \end{bmatrix} \\ \overline{\mathcal{G}}_{p}(b) &= \\ \begin{bmatrix} \overline{\mathbf{P}}_{b}(1) & 0 & \cdots & 0 \\ \overline{\mathbf{P}}_{b}(2) & \overline{\mathbf{P}}_{b}(1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\mathbf{P}}_{b}(L) & \overline{\mathbf{P}}_{b}(L-1) & \cdots & \overline{\mathbf{P}}_{b}(1) \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\mathbf{P}}_{b}(FT) \overline{\mathbf{P}}_{b}(FT-1) \cdots & \overline{\mathbf{P}}_{b}(FT-L+1) \end{bmatrix}, \end{split}$$

defining

$$\overline{\mathbf{P}}_b(k) = \mathbf{I}_N \otimes \begin{bmatrix} \overline{\mathbf{p}}_{(k-1)M+1}(b) & \cdots & \overline{\mathbf{p}}_{kM}(b) \end{bmatrix}.$$

where \otimes denotes the Kronecker product and $\overline{\mathbf{p}}_i(b)$ is the *i*'th component of vector $\overline{\mathbf{p}}(b)$.

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