Application of Auto-Regressive Spectrum Estimation to Joint CDMA Code Detection and Interference Cancellation

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Abstract— **The CDMA forward link consists of multiple users' signals transmitted synchronously by a base station. In current CDMA systems, the mobile user has knowledge only of its own spreading code; therefore, blind interference suppression (IS) algorithms are required to cancel the multiple access interference (MAI). In this paper, an algorithm using auto-regressive spectrum estimation (ARSE) methods is introduced to allow the mobile to determine all of the codes that the base station is transmitting. The technique is based on computing an orthogonal code spectrum, analogous to the traditional Fourier spectrum, by correlating each code with the received signal. This technique allows a simple, single stage, joint interference cancellation (JIC) algorithm to be applied to remove the effects of the other users. It is shown that this technique can improve forward link performance by up to 2 dB over interference suppression (IS) techniques such as minimum mean square error (MMSE) with the added advantage of lower sample support requirements which allows fast computation and adaptivity to a changing environment. Application of the technique to asynchronous data is also discussed.**

I. Introduction

Code Division Multiple Access (CDMA) has been proposed as the technique of choice for future generation wireless communications systems. This method of multiple access allows for an increase in system capacity over other approaches such as Time or Frequency Division Multiple Access. However, systems employing CDMA are interference limited due to the non-orthogonal multiplexing of users that results from multipath induced by the channel.

In the forward link of a CDMA system, the mobile handset only has knowledge of its own spreading code, and so blind interference suppression (IS) must be done. For future generation systems, higher capacity is required, and thus better interference mitigation techniques are sought. In this paper, an algorithm based on auto-regressive spectrum estimation (ARSE) is presented which will enable the mobile to quickly estimate the other spreading codes that are being transmitted by the base station, thus enabling a superior, joint interference cancellation (JIC) algorithm to be applied.

An outline of the paper is as follows: In Section II, the CDMA signal model and problem are introduced. Section III describes the ARSE approach to code detection. Simulation results are presented in Section IV using probability of (code) detection (P_D) and bit error rate (BER) as the performance measures. In Section V, the JIC algorithm and its performance is shown. Advantages of the new algorithm over the conventional minimum mean square error (MMSE) approach, which performs blind IS, are discussed. Extension to asynchronous CDMA is described in VI. Concluding remarks are provided in Section VII.

II. CDMA Signal Model

First consider the forward link, assumed to be a synchronous direct sequence (DS) CDMA system. The transmitted baseband signal corresponding to user k is [1]

$$x_k(t) = \sum_i A_k b_k(i) s_k(t - iT - \tau_k), \tag{1}$$

where $b_k(i)$ is the symbol transmitted by user k at time i, $s_k(t)$ is the associated spreading code, and A_k and τ_k are the real amplitude and delay, respectively. In the synchronous case, each $\tau_k = 0$. The spreading codes are assumed to be short Walsh-Hadamard (WH) codes, as in IS-95, and the symbols $b_k(i) \in$ (-1,+1) are independent. The spreading sequence can be written as

$$s_k(t) = \sum_{i=1}^{N-1} a_k[i] \Psi(t - iT_c), \tag{2}$$

where $a_k[i] \in (\frac{+1}{\sqrt{N}}, \frac{-1}{\sqrt{N}})$. The processing gain is $N = \frac{T}{T_c}$, where T_c is the chip period and T is the symbol period. The chip sequence $\Psi(t)$ has a rectangular pulse shape.

Define the sampled transmitted signal $\mathbf{x}(i)$ as the N-vector composed of synchronous combinations of the data for each user multiplied with its respective spreading sequence. Using IS-95 as an example, the mobile first synchronizes to the short pseudo-random (PN) spreading code in the modulator with the aid of a pilot sequence [2]. Then, the mobile extracts the sync channel, whose frames are aligned with the short PN codes. With the sync channel information, the mobile synchronizes to the set of $N = 64$ WH spreading codes, up to 55 of which are reserved for the traffic channel. Assuming, without loss of generality, that user one is the user of interest, the transmitted signal at time i may be written as

$$\mathbf{x}(i) = A_1 b_1(i) \mathbf{s}_1 + \sum_{k=2}^{K} A_k b_k(i) \mathbf{s}_k. \tag{3}$$

Here, $\mathbf{s}_k$ is the $N \times 1$ spreading sequence associated with user k, and K users are present in the system. Note that the mobile only has knowledge of its own code, $\mathbf{s}_1$.

Assume that the signal is distorted by a fading, multipath channel. The received signal is written as

$$\mathbf{y}(i) = \hat{\mathbf{x}}(i) + \mathbf{n}(i), \tag{4}$$

where $\hat{\mathbf{x}}(i) = \mathbf{x}(i) * \mathbf{h}$, $\mathbf{h} = [h_1, h_2, ..., h_L]$ is the vector of channel coefficients, L is the delay spread of the channel, and $\mathbf{n}(i)$ are AWGN samples. So write

$$\mathbf{y}(i) = A_1 b_1(i)\hat{\mathbf{s}}_1 + \sum_{k=2}^{K} A_k b_k(i)\hat{\mathbf{s}}_k + \mathbf{n}(i), \tag{5}$$

where $\hat{(\cdot)}$ denotes convolution of the operand with the channel vector $\mathbf{h}$ throughout the paper. The channel vector is usually estimated with the pilot sequence, and for the remainder of the paper it is assumed known.

Now, the problem at hand is to optimally detect the bits transmitted by user one, while suppressing the interference induced by the other $K-1$ users. However, the spreading codes of the interfering users are typically unknown to the mobile and thus IS techniques are used. In the next section, ARSE techniques are employed to determine which codes are being transmitted by the other $K-1$ users.

III. Joint Code Detection Using Auto-Regressive Spectrum Estimation (ARSE) Techniques

ARSE techniques perform spectrum estimation by searching the signal space to determine which signals are most correlated with the incoming stream to determine if that signal is present. This is a signal dependent approach that uses some known information about the received process. In forward link CDMA, the set of spreading codes is the a priori knowledge that can be exploited. Thus, an orthogonal code spectrum is obtained in place of the conventional orthogonal frequency spectrum.

Note that the MMSE solution in the presence of multipath can be written as [4]

$$\mathbf{c}_{MMSE} = \mathbf{Y}^{-1}\hat{\mathbf{s}}_1 = E[\mathbf{y}(i)\mathbf{y}(i)^H]^{-1}\hat{\mathbf{s}}_1, \tag{6}$$

where $\mathbf{Y}$ is the covariance matrix of the received signal $\mathbf{y}(i)$ and $\hat{\mathbf{s}}_1$ is the channel matched filter (rake), given by [4]

$$\hat{\mathbf{s}}_1 = \mathbf{s}_1 * \mathbf{h}. \tag{7}$$

In conventional ARSE, the value of a process at time i is formed from a linear combination of the previous M samples of the process by the augmented Wiener-Hopf equation [5]. This could be, for example, the value of a linearly spaced array at element $M+1$, where the problem is defined in terms of plane waves impinging the array from different orthogonal angles, θ, and the goal is to determine those angles. In the current context, this is received signal at time $M+1$, and the goal is to determine the different orthogonal users that are present. Assume that M samples of the process are collected to form a set of snapshots. Thus, write the estimate of the received process at time i as

$$\tilde{\mathbf{y}}(i) = -\sum_{k=1}^{M} a_M(k)\mathbf{y}(i-k) \tag{8}$$

Comparing Eq. (8) to Eq. (5), it is seen that the bits b_k can be absorbed in the coefficient a_M, so that the estimate of the process contains the spreading codes $\mathbf{s}_k$ as desired. In matrix form,

$$\mathbf{Y}_M^H \mathbf{a}_M = -\mathbf{y}, \tag{9}$$

where $\mathbf{Y}_M$ is an $(N+L-1)\times M$ matrix whose columns contain the $(N+L-1)\times 1$ chips multiplied with the bit at each time instance i, and whose rows contain M bits of data starting at time i. It is implicitly assumed that the codes do not change within the block of M bits. Furthermore, $\mathbf{a}_M$ and $\mathbf{y}$ are $(N+L-1)\times 1$ and $M\times 1$ vectors, respectively. The vector $\mathbf{y}$ is the sample of the process, at time i, to be estimated from the previous M samples. The solution to this problem is

$$\mathbf{a}_M = -\mathbf{R}_M^{-1}\mathbf{r}_{Yy}, \tag{10}$$

where $\mathbf{R}_M = \mathbf{Y}_M\mathbf{Y}_M^H$ is the correlation matrix of the process, and $\mathbf{r}_{Yy} = \mathbf{Y}_M\mathbf{y}$ is a cross-correlation vector.

The augmented Wiener-Hopf solution is obtained by manipulation of the above equation, and yields [5]

$$\mathbf{a}_{M+1} = \parallel \epsilon \parallel^2 \mathbf{R}_{M+1}^{-1}\mathbf{u}_{M+1}, \tag{11}$$

where $\parallel \epsilon \parallel^2$, defined as the mean-square prediction error (MSPE), is given by

$$\parallel \epsilon \parallel^2 = \sigma_y^2 + \mathbf{r}_{Yy}^H\mathbf{a}_M, \tag{12}$$

$\mathbf{R}_{M+1}$ is an $(N+L-1)\times(N+L-1)$ correlation matrix, and $\mathbf{u}_{M+1}$ is an $(N+L-1)\times 1$ unit vector.

The conventional AR power spectral density (PSD), in terms of spatial frequencies, is written as

$$P_y(\theta) = \frac{\parallel \epsilon \parallel^2}{\parallel \mathbf{G}_\theta^H \mathbf{a}_{M+1} \parallel^2}, \tag{13}$$

where $\mathbf{G}_\theta = [\mathbf{g}_{\theta_1}\ \mathbf{g}_{\theta_2}, ...,\ \mathbf{g}_{\theta_N}]$ is a matrix of steering vectors. Each steering vector $\mathbf{g}_{\theta_k}$ is beamformed to point in the direction of angle θ_k. For the CDMA problem, spatial frequencies θ_k are replaced by codes, equivalently each $\mathbf{g}_{\theta_k}$ is replaced by $\mathbf{s}_k$ and thus angle space is mapped to code space. More specifically, the channel matched spreading codes are used after the channel is estimated with a pilot, so that $\hat{\mathbf{s}}_k$ is used and all dimension N vectors or matrices become dimension $N+L-1$. So, one can write

$$\mathbf{G}_\theta \Rightarrow \hat{\mathbf{S}}_N = [\hat{\mathbf{s}}_1\ \hat{\mathbf{s}}_2\ ...\ \hat{\mathbf{s}}_N], \tag{14}$$

Note that this steering matrix includes all of the N codes present in the set, since it is not known, but to be determined, which K codes (users) are present in the correlation

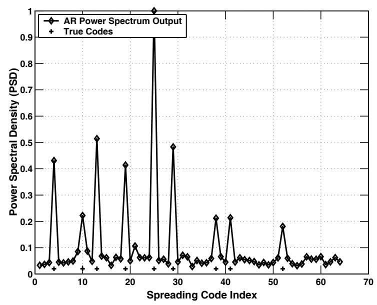

Fig. 1. Power Spectral Density (PSD) vs. Spreading Code Index; Synchronous CDMA; $K = 9$ Users

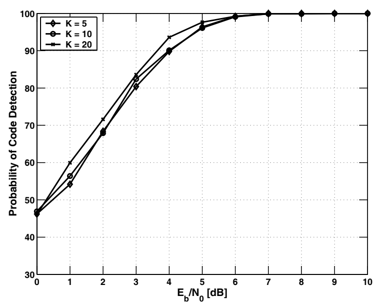

Fig. 2. E_b/N_0 vs. P_D; $M = 1000$; $K = 5, 10, 20$

matrix. The PSD as a function of code number from 1 to N can now be written as

$$P_y(N) = \frac{\parallel \epsilon \parallel^2}{\parallel \hat{\mathbf{S}}_N^H \mathbf{a}_{M+1} \parallel^2}, \tag{15}$$

IV. Simulation Results

Numerical examples are now presented to illustrate the performance of the ARSE algorithm. In Fig. 1, $N = 64$ (WH codes as in the IS-95 forward link), $E_b/N_0 = 5$ dB, $K = 9$ randomly chosen users, $M = 1000$ bits, and an $L = 5$ tap Rayleigh fading multipath channel are used. A detection results if the PSD exceeds a threshold, set equal to three times the noise floor. It is seen that the algorithm correctly identifies all 9 codes present in the signal.

Fig. 2 shows probability of code detection (P_D) vs. E_b/N_0 for a block size of $M = 1000$ bits, and number of users $K = 5, 10$, and 20, in which many Monte Carlo trials have been averaged. Probability of false alarm(P_{FA}) remains fairly constant as a function of E_b/N_0 and can be made arbitrarily low by choosing large enough E_b/N_0 or M. It is interesting to note that performance is nearly the same as the number of users changes, and is in fact slightly better for the larger K.

Fig. 3 shows P_D vs. E_b/N_0 for $K = 10$ users, and block sizes $M = 500, 1000$, and 2000. For larger M, the P_D starts off lower as a function of E_b/N_0 but converges faster. In all cases, at $E_b/N_0 = 6$ dB, a P_D greater than 99 percent is achieved. Note that under realistic operating conditions,

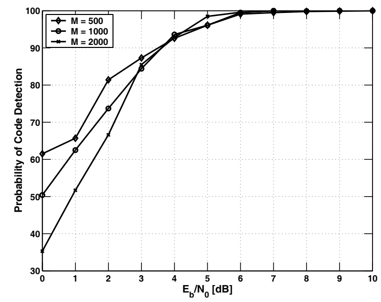

Fig. 3. E_b/N_0 vs. P_D; $K = 10$; $M = 500, 1000, 2000$

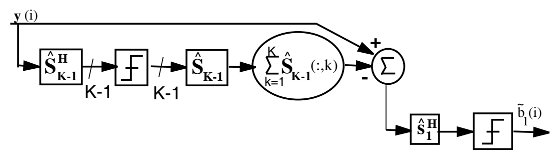

Fig. 4. Joint Interference Canceller (JIC)

e.g. $P_D > 99$ percent, the algorithm does not depend on sample support, a very desirable result, which is not true of the MMSE algorithm. Also, for moderate E_b/N_0s, the ARSE algorithm does not require large M, unlike MMSE, because the N chips to be estimated are small compared to M. With MMSE, M bits are being blindly estimated.

V. Joint Interference Cancellation (JIC)

Joint interference cancellation (JIC) can be performed after the ARSE code detection algorithm is applied since now the mobile has identified the interfering users' codes. First form a matrix matched filter containing the spreading codes of the $K-1$ users that were detected. That is, form the $(N + L - 1) \times (K - 1)$ matrix

$$\hat{\mathbf{S}}_{K-1} = [\hat{\mathbf{s}}_2 \; \hat{\mathbf{s}}_3 \; ... \; \hat{\mathbf{s}}_K], \tag{16}$$

where each $\hat{\mathbf{s}}_k$, $k = 2, 3, ..., K$ is one of the $K - 1$ interfering codes in the set $\hat{\mathbf{S}}_N$. The simplest JIC applies this matrix matched filter at the first stage to de-spread the interfering users' codes, estimates the bits, re-spreads the estimates and subtracts the sum of the result from the received signal. This produces the desired cancellation of the interference term in Eq. (5). Finally, user one is de-spread and a hard decision is made to obtain the final bit estimate. A diagram of the canceller is shown in Fig. 4.

Fig. 5 presents the BER performance improvement using the ARSE technique followed by the JIC and compares it to the conventional MMSE approach, which treats the interfering users as noise. Is is seen that $1.5 - 2$ dB gain is achieved in the 10^{-4} to 10^{-5} BER range by applying knowledge of the interfering users obtained using the ARSE approach. A sample support of $M = 500$ is chosen for the JIC algorithm, but this could be made arbitrarily low

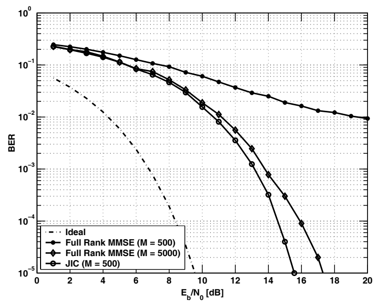

Fig. 5. E_b/N_0 vs. Bit Error Rate (BER); $K = 10$

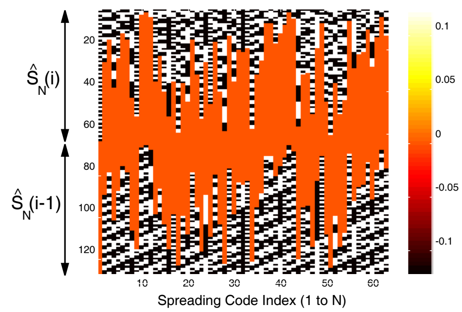

Fig. 6. $2N \times 2N$ Asynchronous Spreading Code Matrix $\hat{\mathbf{S}}_N$ Structure; Spreading Code Length $N = 64$

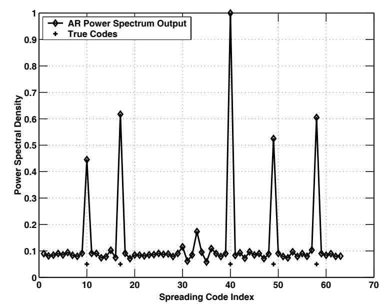

Fig. 7. Power Spectral Density (PSD) vs. Spreading Code Index; Asynchronous CDMA; $K = 5$ Users

since the algorithm uses a matched filter which does not depend on sample support. As shown in the plot, this is not true for the MMSE algorithm. Note that only the simplest cancellation algorithm has been presented here. A more sophisticated algorithm, such as that described in [3], can provide an additional $2 - 4$ dB improvement.

VI. Extension to Asynchronous CDMA

The ARSE technique is most applicable to synchronous links, since asynchronous links are detected and demodulated non-coherently. Here, detection of asynchronous spreading codes is briefly discussed. Again, it is assumed that short spreading codes are used. The first issue is that the relative time offsets of each mobile must be pre-assigned and known by the BS. This is necessary so that the steering matrix $\hat{\mathbf{S}}_N$ can be formed. The second issue is that the BS needs to synchronize to the matrix of codes somehow; this could be done through the use of a preamble transmitted by any given user. The third important fact is that due to the asynchronous transmissions, each steering vector in the steering matrix now spans two bits. Denote the steering vector, of dimension $2(N + L - 1) \times 1$, as $\hat{\mathbf{s}}_a, k$ and write

$$\hat{\mathbf{s}}_{a,k} = [\hat{\mathbf{s}}_k^-; \hat{\mathbf{s}}_k^+], \tag{17}$$

where $\mathbf{s}_k^-$ and $\mathbf{s}_k^+$ are the $N \times 1$ spreading codes for user k in the previous bit $i - 1$ and current bit i, respectively. A sample plot of the structure of $\mathbf{S}_N$, for $L = 1$, is shown in Fig. 6. The covariance matrix, $\mathbf{R}_{M+1}$ is formed over the same two bit interval and then Eq. (15) is applied.

An example of the ARSE PSD for the asynchronous case is shown in Fig. 7. Length $N = 64$ codes, $E_b/N_0 = 5$ dB, $K = 5$ users, $M = 1000$ bits, and an $L = 5$ tap channel are used. The algorithm correctly identifies all 5 codes. Note that for better performance when asynchronous users are present in multipath, codes with better cross-correlation properties, such as Gold codes, must be used.

VII. Conclusion

In CDMA systems, the mobile on the synchronous forward link only has knowledge of its own spreading code and therefore must perform blind interference suppression. In this paper, a technique based on auto-regressive spectrum estimation (ARSE) is described which allows the mobile or base station to quickly estimate which users' codes are present, thereby allowing more powerful joint interference cancellation (JIC) techniques to be applied. This technique is more efficient than MMSE and provides up to 2 dB improvement over MMSE. The technique is most applicable to the synchronous forward link, but could be applied to the reverse link as well.

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