TIME-VARYING CHANNEL MODELING AND VARIABLE-SIZE BURST FOR SPATIO-TEMPORAL INTERFERENCE SUPPRESSION IN DS-CDMA SYSTEMS

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ABSTRACT

Burst-by-burst equalization strategies assume that data in a burst are received under quasi-stationary channel conditions. Hence, using a fixed-size burst results in conservative transmission efficiency, since worst case scenarios must be accounted for. In this work, we investigate the potential of a variable-size burst for efficiency in time-varying environments. First, a channel model explicitly describing environment changes is developed. Analysis of variance is then proposed for tracking the channel stationarity. Based on knowledge of the channel stationarity, the burst size is adapted most advantageously. Obtained results show feasibility of a variable-size burst to improve efficiency compared to a fixed-size counterpart.

1. INTRODUCTION

In high-rate communication systems, with symbol duration less than the delay spread, frequency-selective fading occurs resulting in severe intersymbol interference (ISI) [1]. Channel equalization mitigating ISI is often required for adequate performance in such cases. A standard procedure is to transmit training symbols, to be used by the receiver for equalization. For systems transmitting in bursts, spectral efficiency is inversely proportional to the ratio of training symbols to the total number of symbols in a burst. Moreover, successful equalization requires a minimum number of training symbols [2, 3]. This implies that a short burst is inefficient, since the ratio of training to data symbols might be substantial.

In this work, we investigate the use of a variable-size burst for efficiency. A similar packet optimization was previously reported in the context of ARQ protocols [4], where packet size was optimized based on estimates of bit-error-rate. For burst-by-burst systems considered in this work, the idea is to exploit more stationary operating conditions to utilize longer bursts. Conversely, shorter bursts are employed when the encountered channel is more varying. To this end, a time-varying channel model is first proposed to explicitly describe situations where the coherence times change. Next, analysis of variance (ANOVA) [5] is applied to track long durations of stationarity, enabling a larger burst to be used for efficiency. Obtained results show that a variable-size burst can deliver improved efficiency, while maintaining the same quality of service, compared to a fixed-size burst approach.

This paper is organized as follows. After describing the channel model in Sec. 2, a variable-size burst structure is proposed in Sec. 3. A spatio-temporal system employing DS-CDMA, and a corresponding spatio-temporal equalization strategy using least-squares are then presented in Sec. 4 and Sec. 5 respectively. Stationarity tracking using ANOVA is next given in Sec. 6. Finally, simulation results and concluding remarks are presented in Secs. 7 and 8, respectively.

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2. CHANNEL MODEL

With conventional mobile channel modeling, e.g. Jakes model [1], a specific environment with a particular coherence time is usually described. In such models, while the channel may change with time, its rate of variation is essentially fixed, as determined by an associated coherence time. However, even if the physical environment remains the same, changes in the mobile station's speed can render the effective environment significantly different, since the Doppler spread is modified [1]. In the context of variable-size burst strategy, a channel model explicitly describing such effective environment changes is useful.

For a mobile channel with Rayleigh fading, the coherence time for a particular environment is well characterized using the Jakes model [1]. However, a piecewise constant approximation can also be made, resulting in the so-called block-fading model. In this model, channel coefficients are assumed to be constant or quasi-stationary over some interval, i.e., over a burst duration, and change to another state for the next interval [3, 6].

In order to explicitly account for changing environments, the conventional block-fading model is extended as follows. Assuming that the channel coefficients are quasi-stationary over some *fundamental* period, denote the channel coefficients during the kth fundamental period as $\mathbf{p}_k$. Then the channel changes between fundamental periods as

$$\mathbf{p}_k = \nu_k\big(\eta_v\,\mathbf{p}_{k-1} + \mathbf{u}_v\big) + \big(1-\nu_k\big)\big(\eta_q\,\mathbf{p}_{k-1} + \mathbf{u}_q\big) \qquad (1)$$

where ν_k is a Bernoulli random variable (equal to 1 with probability p), $\mathbf{u}_q \sim \mathcal{CN}(\mathbf{0}, \sigma_q^2\mathbf{I})$ and $\mathbf{u}_v \sim \mathcal{CN}(\mathbf{0}, \sigma_v^2\mathbf{I})$. Hence, depending on the value of ν_k, the channel can change according to either model-q, specified by a correlation constant η_q (between 0 and 1) and variance σ_q^2, or model-v, specified by η_v and σ_v^2.

By setting η_q to values closer to 1 (high correlation to previous stage) and σ_q^2 to be very small (little random change), model-q has the interpretation of a *quasi-stationary state*. Likewise, setting η_v to values closer to 0 and σ_v^2 to be larger, model-v is the representation of a *varying state*. The value of p specifies the probability that the varying state will occur.

It is easy to see that, with appropriate choices of parameters, this model includes the conventional block-fading model as a special case. Furthermore, we note the following:

- With $\nu_k = 1$ and the fundamental period being a single symbol, this model reduces to a Gauss-Markov model [2], used as an approximation of the Jakes model [6].

- As a two-state characterization, this model is analogous to the Gilbert-Elliott channel in [4], with the good state being the quasi-stationary and the bad state being the varying.
- In essence, this model is the usual block-fading model with the block size now being variable (multiples of the fundamental period).

Hence the proposed model can characterize scenarios where stationarity conditions change. The smallest coherence time it can account for is equal to the fundamental period. We shall henceforth refer to this model as the variable-size block-fading (VSBF) model.

The time evolution of the VSBF model can be represented by defining a transition sequence Ξ of the values ν_k. For example, $\Xi = [10111111001\ldots]$ represents a more varying channel, since it has longer runs of ones. Likewise, for a multi-user or multi-antenna scenario, a transition matrix can be considered, with each row characterizing the time evolution of the channel for the respective user and antenna.

3. VARIABLE-SIZE BURST STRUCTURE

A variable-size burst is constructed by taking advantage of portions in the transition sequence Ξ with runs of zeros. Over these durations, the channel is more stationary, thus enabling a longer burst to be valid. Define a *fundamental* burst $\mathbf{B}_f$ (corresponding to the fundamental period) to be a burst consisting of $N_B^{(f)}$ data symbols, (this implies quasi-stationarity over any fundamental burst). All other *accumulated* bursts to be decoded are constructed from fundamental bursts. Hence, an accumulated burst $\mathbf{B}$ has size $N_B = \rho N_B^{(f)}$ where ρ is an integer. Training symbols are sent at the start of each fundamental burst, $N_t^{(\xi)}$ training symbols for the ξth fundamental burst (see Fig. 1).

A variable-size accumulated burst is attained by accumulating a maximum possible number of existing fixed-size fundamental bursts, while satisfying quasi-stationarity, for decoding. Note that this strategy can be performed entirely at the receiver, and relies on the receiver's ability to track the corresponding transition sequence (to be described in Sec. 6). This approach has the advantage of not requiring significant protocol changes, e.g. feedback, in existing transmitters and receivers designed for regular fixed-size bursts.

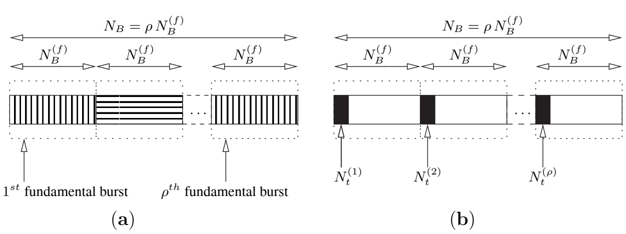

Fig. 1. (a) Accumulated burst of ρ fundamental bursts; (b) Training sequences ($N_t^{(\xi)}$) at start of every (ξth) fundamental burst

4. SIGNAL MODEL

In this section, we establish the signal model for a spatio-temporal multi-user scenario similar to that originally described in [7], with A total antennas and M active users. In a DS-CDMA system with chip duration T_c, denote the overall equivalent discrete-time channel (sampled at chip rate) for the mth user at the ath receiver element as [7]:

$$p_m^a[l] = p_m^a(lT_c - \tau_m) \tag{2}$$

where τ_m is the propagation delay. It is assumed that the channel is FIR of order q_m. Let $p_m^a[l]|_{[\xi]}$ be the channel coefficients during the ξth fundamental burst. The FIR nature of the channel is emphasized by defining $\mathbf{p}_\xi^{m,a} = \left[p_m^a[0]|_{[\xi]}, \ldots, p_m^a[q_m]|_{[\xi]}\right]^T$. Then, using the VSBF channel model in Sec. 2,

$$\mathbf{p}_\xi^{m,a} = \left[\nu_\xi \left(\eta_v \, \mathbf{p}_{\xi-1}^{m,a} + \mathbf{u}_v\right)\right] + \left[(1-\nu_\xi)\left(\eta_q \, \mathbf{p}_{\xi-1}^{m,a} + \mathbf{u}_q\right)\right] \tag{3}$$

For the mth user with normalized spreading sequence $\{c_m[n]\}_{n=0}^{N-1}$, consider transmitting an accumulated burst $\mathbf{B}$ of N_B data symbols, with data symbols $b_m[k]$ taking values from a 4-QAM alphabet. Then the baseband discrete-time transmitted signal for the mth user is $x_m[n] = \sum_{k=0}^{N_B-1} b_m[k]c_m[n-kN]$, $n = 0, \cdots, NN_B - 1$.

Highlighting the time-varying nature of the channel, a partitioning in terms of fundamental bursts (similar to overlap-add block-convolution) is performed yielding

$$x_{m,\xi}[n] = \begin{cases} x_m[n + \xi N N_B^{(f)}], & 0 \le n \le NN_B^{(f)} - 1 \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

Then at the ath antenna, the received signal due to the ξth fundamental burst is $r_{a,\xi}[n] = \sum_{m=1}^{M} x_{m,\xi}[n] * (\mathbf{p}_\xi^{m,a})^T$, and the total received signal due to the entire accumulated $\mathbf{B}$ is

$$r_a[n] = \sum_{\xi=0}^{\rho} r_{a,\xi}[n - \xi N N_B^{(f)}] + v_a[n] \tag{5}$$

where $v_a[n]$ is complex additive white Gaussian noise (AWGN).

When $\mathbf{B}$ satisfies quasi-stationarity, the total received signal reduces to [7]

$$r_a[n] = \sum_{m=1}^{M} \sum_{k=0}^{N_B-1} b_m[k] g_m^a[n-k] + v_a[n] \tag{6}$$

where

$$g_m^a[k] = \sum_{n=0}^{N-1} c_m[n] p_m^a[k-n] \tag{7}$$

is the equivalent channel with spreading code. Finally, as shown in [7], the entire spatio-temporal system also admits the following high-level vector representation for estimating the kth symbol

$$\mathbf{r}_\mu(k) = \mathbf{G}_\mu \mathbf{b}_\mu(k) + \mathbf{v}_\mu(k) \tag{8}$$

where μ is the smoothing factor, $\mathbf{G}_\mu$ a matrix of channel coefficients, $\mathbf{b}_\mu(k)$ a vector of transmitted symbols and $\mathbf{v}_\mu(k)$ a vector representing the AWGN.

5. LEAST-SQUARES INTERFERENCE SUPPRESSION

The objective of interference suppression is to jointly mitigate the effects of ISI and multi-access interference. With the signal model given by (8), when m is the desired user, the received signal $\mathbf{r}_\mu(k)$ is filtered with a spatio-temporal weight vector $\mathbf{W}_m$. This weight vector is chosen to minimize the MSE cost function

$$J_{\text{MSE}}(\mathbf{W_m}) = E\left\{\left|b_m[k] - \mathbf{W}_m^H \mathbf{r}_\mu(k)\right|^2\right\} \tag{9}$$

With the classical least squares (LS) method [2], time-averaging approximates the ensemble average in (9) using the N_t available training symbols, producing the following solution

$$\mathbf{W}_{m,\text{LS}} = \left(\frac{1}{N_t} \sum_{k=0}^{N_t-1} \mathbf{r}_\mu(k)\mathbf{r}_\mu^H(k) \right)^{-1} \left(\frac{1}{N_t} \sum_{k=0}^{N_t-1} b_m^*[k]\mathbf{r}_\mu(k) \right) = \hat{\mathbf{R}}_{N_t}^{-1}\hat{\mathbf{P}}_{N_t}^m \quad (10)$$

6. STATIONARITY TRACKING

For a constituent Gauss-Markov channel coefficient at the ξ fundamental burst in the VSBF model [2]

$$\text{Var}(p_m^a[k]|_{[\xi]}) = \eta_{\text{GM}}^2 \text{Var}(p_m^a[k]|_{[\xi-1]}) + \sigma_{\text{GM}}^2 \quad (11)$$

where η_{GM} and σ_{GM} assume values in their respective q-state or v-state, depending on the channel state. Hence, depending on the VSBF model parameters, the variance can change significantly in the v-state, while remaining close to invariant in a q-state. In such cases, detection of whether the channel state has changed, at the breakpoint of two consecutive bursts, can be performed by tracking the variances of the channel coefficients.

To enable variance tracking, a channel estimate is first obtained. Assuming independence of data symbols transmitted by different users, then the cross-correlation $\mathbf{P}_m = E\{b_m^*[k]\ \mathbf{r}_\mu(k)\}$ encapsulates the overall channel with spreading codes, from (7), for the desired mth user since

$$\mathbf{P}_m = \begin{cases} \left[g_m^1[0], \cdots, g_m^A[LN-1], \cdots, 0, \cdots, 0\right]^T, & \mu > L \\ \left[g_m^1[0], \cdots, g_m^A[\mu N-1]\right]^T, & \mu \le L \end{cases} \quad (12)$$

where L is the channel support. Furthermore, $\mathbf{P}_m$ can be estimated using training symbols to obtain $\hat{\mathbf{P}}_{N_t}^m$ as shown in (10).

Denote the channel estimate for the ξth burst which has training symbols $N_t^{(\xi)}$ (see Fig. 1) as $\hat{\mathbf{P}}_{(\xi)}^m$. Noting that each such estimate is a complex random vector, with $AN\mu$ i.i.d. elements, an associated sample variance S_ξ^2 can be found as [5]

$$S_\xi^2 = \frac{1}{AN\mu - 1} \sum_{i=1}^{AN\mu} \left| \hat{\mathbf{P}}_{(\xi)}^m[i] - \frac{1}{AN\mu} \sum_{j=1}^{AN\mu} \hat{\mathbf{P}}_{(\xi)}^m[j] \right|^2. \quad (13)$$

Then Neyman-Pearson hypothesis testing of the associated variance σ_ξ^2 is performed as follows [5, 8]. For a cluster of ξ consecutive fundamental bursts consecutive, with associated variances $\sigma_1^2, \cdots, \sigma_\xi^2$, the hypothesis testing problem is

$$H_0 : \sigma_1^2 = \cdots = \sigma_\xi^2 \quad \text{vs.} \quad H_1 : \sigma_1^2 \neq \cdots \neq \sigma_\xi^2 \quad (14)$$

which is the well known homoscedasticity testing problem in ANOVA [5]. The maximum F-ratio test can be used to reduce this multi-sample problem to a two-sample problem by considering the maximum and minimum variances σ_{max}^2 and σ_{min}^2,

$$H_0 : \sigma_{\text{max}}^2/\sigma_{\text{min}}^2 = 1 \quad \text{vs.} \quad H_1 : \sigma_{\text{max}}^2/\sigma_{\text{min}}^2 > 1 \quad (15)$$

Define the statistic

$$F = S_{\text{max}}^2/S_{\text{min}}^2 \quad (16)$$

then if the null hypothesis H_0 is true, F has a complex F-distribution with complex degrees of freedom $n = m = AN\mu - 1$ and probability density function [8]

$$f_F(x) = \frac{(n+m-1)!}{(n-1)!(m-1)!} \frac{x^{n-1}}{(1+x)^{n+m}} U(x) \quad (17)$$

Hence, given a level of statistical significance specified by α [5], the critical value F_α for decision is [5]

$$\text{Prob}[F > F_\alpha] = \int_{F_\alpha}^{\infty} f_F(x)\, dx = \alpha. \quad (18)$$

Then the decision is to accept H_0 if $F < F_\alpha$ (quasi-stationarity accepted). Otherwise accept the alternative hypothesis H_1 (non-stationarity detected).

Evidently, the above procedure allows clustering and segmenting received bursts to construct maximum-length accumulated bursts that satisfy quasi-stationarity. The LS weight vector $\mathbf{W}_{m,\text{LS}}$ in (10) can now be calculated and applied to each such accumulated burst created. A conceptual algorithmic description is summarized in Table 1.

N_{total} : total number of fundamental bursts to be processed s: starting fundamental burst of the current accumulated burst

I. *Initialization*: Set $s = 1$
II. *Iteration*
for $i = 2, 3, \cdots, N_{\text{total}}$
if ($s \le N_{\text{total}}$)
$S_{\text{min}}^2 = \min\{S_s^2, \cdots, S_i^2\}$; $S_{\text{max}}^2 = \max\{S_s^2, \cdots, S_i^2\}$
$F = S_{\text{max}}^2/S_{\text{min}}^2$
if ($F \ge F_\alpha$)
1. Set current accumulated burst = all fundamental bursts from s to $i-1$
2. Decode the current accumulated burst
3. Reset $s = i$

Table 1. Stationarity tracking using ANOVA

7. SIMULATION RESULTS AND DISCUSSION

Unless otherwise indicated, the following parameters remain the same: fundamental burst size $N_B^{(f)} = 150$, spreading factor $N = 16$, SNR=15 dB, number of active users $M = 8$, smoothing factor $\mu = 2$, FIR order $q_m = 15$. Each channel tap coefficient was independently initialized from an identical complex Gaussian distribution $\mathcal{CN}(0, 2)$. For each simulation, 80 Monte-Carlo runs were conducted, each run corresponding to a transition sequence of 20 fundamental bursts.

Channel I (Moderately varying VSBF channel): The following VSBF model parameters from Sec. 2 were used: $p = 0.4$, $\eta_q = 0.95$, $\sigma_q^2 = 0.002$, $\eta_v = 0.7$, $\sigma_v^2 = 0.1$. Fig. 2 shows the MSE performance using $A = 2$ antennas (left) and $A = 4$ (right). For each simulation, two different levels of significance specified by $\alpha = 0.05$ and $\alpha = 0.2$ are applied.

The fixed-burst performance was obtained using equalization without channel tracking, i.e., all bursts considered were fundamental. Also shown for reference is the pseudo-ideal burst performance, obtained using perfect knowledge of the transition sequence for the desired user. It is not necessarily a lower bound, e.g. if the q-state in reality corresponds to a highly varying state, using the known transition sequence for decoding is disastrous.

For $A = 2$, it is seen that $\alpha = 0.05$ results in faster convergence. However, residual excess MSE is higher. While a smaller $\alpha = 0.05$ results in less false alarms, it has a higher chance of missing the alternative hypothesis, i.e., Type II error is higher than Type I error [5]. Increasing $\alpha = 0.2$ increases the power of the test [5] (ability to detect the alternative hypothesis when it occurs)

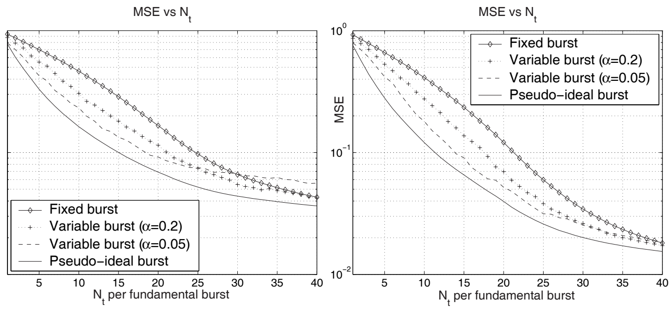

Fig. 2. Channel I MSE performance with increasing training: (left) $A = 2$; (right) $A = 4$.

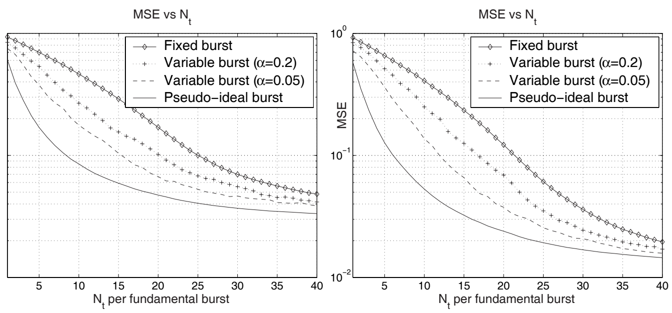

Fig. 3. Channel II MSE performance: (left) $A = 2$; (right) $A = 4$.

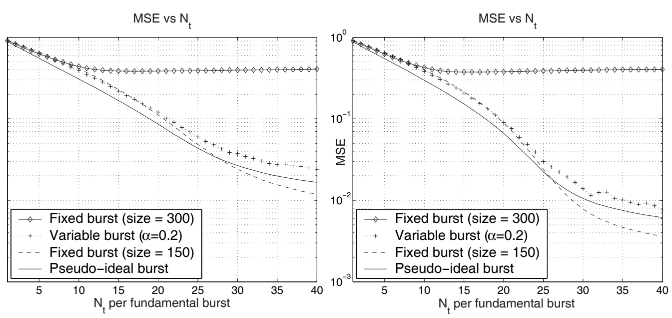

Fig. 4. Channel III MSE performance: (left) SNR=15 dB; (right) SNR=20 dB

at the cost of reducing the convergence speed. Hence the choice of the most appropriate value for α depends on the encountered channel.

Alternatively, the power of test can be improved by increasing the degrees of freedom. In a spatio-temporal setting, this can be achieved by increasing A. As seen in Fig. 2 (right) for $A = 4$, the excess MSE is now less significant when using $\alpha = 0.05$.

The above results also show that, using a variable-size structure, the number of training symbols per fundamental burst can be reduced, while still attaining the same interference suppression performance compared to the fixed-size burst. Hence, the overall spectral efficiency can be higher when using a variable-size burst compared to a fixed-size counterpart.

Channel II (Less varying VSBF channel): A more stationary scenario is next considered with VSBF parameters: $p = 0.2$, $\eta_q = 0.98$, $\sigma_q^2 = 0.001$, $\eta_v = 0.7$, $\sigma_v^2 = 0.1$. Fig. 3 shows the corresponding MSE performance for $A = 2$ and $A = 4$. In this case, it is possible and more advantageous to utilize $\alpha = 0.05$, because the penalty for a missed detection is less severe in this channel. And as in Channel I, spectral efficiency can be increased when using a variable-size burst compared to a fixed-size burst.

Channel III (Highly varying VSBF channel): Finally, a highly varying Channel III is considered: $p = 0.9$, $\eta_q = 0.7$, $\sigma_q^2 = 0.2$, $\eta_v = 0.1$, $\sigma_v^2 = 0.8$. Here, the penalty for missed detection is severe. To appreciate the severity of the incurred penalty, the performance of an overvalued fixed-size burst, twice the size of the fundamental burst, is also shown. Fig. 4 shows the resulting performances for $A = 4$, $\alpha = 0.2$, with SNR=15 dB and SNR=20 dB. Indeed, the error floor for an overvalued fixed-size burst is large. Increasing the SNR does not improve performance, since equalization is not successful with stationarity violation, resulting in severe residual interferences and thus the error floor. This is a convincing motivation for a variable-size burst, which can account for such worst-case scenario, corresponding to an operating instant where the environment is highly distorting. When the operation returns to a more benign environment, the variable-size burst can again take advantage of longer quasi-stationarity periods for better performance.

As shown in these scenarios, at worst (Fig. 4), there is no significant performance loss when using a variable-size burst compared to a fixed-size burst. On the other hand, there is a possibility for increased spectral efficiency in more benign situations (Figs. 2 and 3). The added cost would be that of tracking the stationarity.

8. CONCLUDING REMARKS

In this paper we presented a strategy for constructing a variable-size burst that adapts to the operating environment. The burst size used for decoding is longer when the channel is more stationary, and shorter when the channel is more varying. Obtained results show that, compared to a fixed-size burst, a variable-size burst is more flexible and can yield good performance, when the former case may prove inadequate, in a wider range of operating conditions.

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