A SIMPLE METHOD FOR COMPUTING PARTIAL CANCELLATION FACTORS IN CDMA USING PPIC RECEIVER

Mohsen Ghotbi and M. Reza Soleymani

Concordia University, Montreal, Quebec, Canada H3G 1M8
email: {mohsen,msoleyma}@ece.concordia.ca

ABSTRACT

In this paper, a new closed-form expression for the *Partial Cancellation Factor* (PCF) in a multistage *Partial Parallel Interference Cancellation* (PPIC) receiver is introduced. This approach has several advantages compared to a previously proposed method. First of all, PCFs are found directly from the moments of the eigenvalues of the correlation matrix. This results in less complexity in circuitry. In addition, there is no need that the PCFs be ordered. Therefore, a monotonically improving performance is obtained with even more reduced complexity. Finally, it is not necessary to know the number of interference cancellation stages a priori. It will be shown that by choosing these PCFs, the performance of PPIC receiver will converge to that of the *Minimum Mean-Squared Error* (MMSE) receiver.

1. INTRODUCTION

Code Division Multiple Access (CDMA) is an efficient air interface for wireless communication where all users share the same bandwidth simultaneously. In CDMA, each user is assigned a specific signature sequence. When these signature sequences are not orthogonal, each user will suffer from the *Multiple Access Interference* (MAI) originating from other users. In order to cancel or mitigate the effect of MAI, *Multiuser Detection* (MUD) is proposed where the information of other users is employed in order to recover a more accurate version of users' data to be detected or fed to a soft error-control decoder. Several multiuser receivers have been proposed in the literature (e.g., [1]-[4]). In [1], Verdu devised an optimal *Maximum Likelihood* (ML) receiver. The complexity of the ML multiuser receiver grows exponentially with the number of users. As a result, this type of receiver becomes too complex to be implemented. Therefore, the goal of suppressing or mitigating MAI has been compromised in order to find schemes that sacrifice optimality for the sake of implementation feasibility. As a result of such endeavors, several suboptimal multiuser detectors have been introduced.

An example of suboptimal multiuser receivers is the class of interference-cancellation-based receivers [2]. *Successive Interference Cancellation* receiver is the simplest scheme of interference cancellation based receivers. In this scheme, the MAI is cancelled sequentially. This makes SIC scheme easy to implement. However, when the number of users is from moderate to high, this scheme becomes too slow providing all users' outputs sequentially. In order to avoid this issue, *Parallel Interference Cancellation* (PIC) receiver is proposed as a better alternative under the class of interference cancellation based receivers. In PIC receiver, the interference of all other users on any given user is removed at once. This makes this scheme more attractive due to its higher speed while its complexity grows linearly in the number of active users. However, in PIC receiver, there is the risk of wrong interference cancellation. This is due to the fact that in the early stages of interference cancellation, information is not reliable and complete cancellation of interference in most cases results in poor performance. In order to resolve this problem, an improved version of PIC receiver called the partial parallel interference cancellation (PPIC) receiver was proposed in [4] and [5]. In this scheme, instead of cancelling the interference completely, it is cancelled partially by introducing a partial cancellation factor (PCF). It has been shown that the performance of PPIC receiver is considerably better than that of pure PIC. As a result, PPIC is a more promising alternative for multiuser detection considering complexity, performance, and processing delay. In addition, if this receiver is employed in a multistage format, its performance will tend to that of well-known linear multiuser receivers such as decorrelating or minimum mean-squared error (MMSE) receivers usually used as a reference to gauge the performance of the linear multistage PPIC receiver ([6] and [7]). It is noted that the performance of PPIC receiver depends strongly on how to choose PCFs. This means that if PCFs are not selected properly, the performance of PPIC could be inferior to PIC receiver.

In [8], [9], and [10], an interesting method for finding the optimum PCFs was proposed. It was shown that the empirical and theoretical results are almost the same. However, there is a problem that two stages of interference cancellation is not sufficient specially for the cases that the system load is high (i.e., $\beta = K/N \to 1$). In [7] an expression for PCF when the number of interference cancellation stages is known was derived. The criterion to find the expression for the optimum PCF was to minimize the *Mean Squared Error* (MSE) between the mth stage output and the transmitted data. Therefore, in this case, the PCFs for all stages will be ready at once. This approach has a few problems. First of all, the number of cancellation stages should be known a priori. This is not always possible to know the number of cancellation stages. Secondly, even having known the number of stages, there will be a performance fluctuation in the middle stages as it will be seen from numerical simulations. Therefore, in order to have a monotonic improvement in performance, the PCFs should be ordered. This is another issue to make the circuitry more complex. Finally, PCFs are found using some matrix inversion. This even results in more complexity. In order to overcome these issues, PCFs will be found using another method. In this method, the MSE at any given stage is minimized using the PCFs of the previous stages. The expression for this method is also given in [7]. Our contribution in this paper is to manipulate the given expression to find an explicit formula for PCF that is a function of the moments of the eigenvalues of correlation matrix. By employing the method presented in this paper, there is no need to know the number of interference cancellation stages a priori. Moreover, the calculated PCFs are applied as they are without any need for ordering. Finally, since the PCFs are direct functions of moments of the eigenvalues of the correlation matrix, the circuitry will be even simpler.

This paper is continued as follows. The system model is introduced in Section 2. The closed-form expression of optimal PCF is found in Section 3. Section 4 presents some numerical results; and this paper is concluded by Section 5.

2. SYSTEM MODEL

A K-user synchronous CDMA system with a binary antipodal modulation and processing gain of N is assumed. The baseband sampled received signal is shown as

$$\mathbf{r} = \mathbf{A}\mathbf{b} + \mathbf{n} \tag{1}$$

where $\mathbf{r}$ is an $N \times 1$ received signal vector as $\mathbf{r} = [r_1, r_2, \ldots, r_N]^t$, $\mathbf{A} = [a_1\mathbf{s}_1, a_2\mathbf{s}_2, \ldots, a_K\mathbf{s}_K]$ is an $N \times K$ matrix including the amplitude a_k and signature sequence $\mathbf{s}_k$ for each user that is defined as $\mathbf{s}_k = 1/\sqrt{N}[s_{k1}, s_{k2}, \ldots, s_{kN}]^t$, and $\mathbf{b} = [b_1, b_2, \ldots, b_K]^t$ is the $K \times 1$ transmitted data vector. The $N \times 1$ additive white Gaussian noise vector is $\mathbf{n} = [n_{1,2}, \ldots, n_N]^t$ with zero mean and variance σ^2. The linear multistage PPIC receiver (Figure 1) applies a linear transformation on the received signal such that the output at the mth stage is $\mathbf{b}^{(m)} = \mathbf{C}_m^t.\mathbf{r}$ where $\mathbf{C}_m^t$ is an $K \times N$ filter

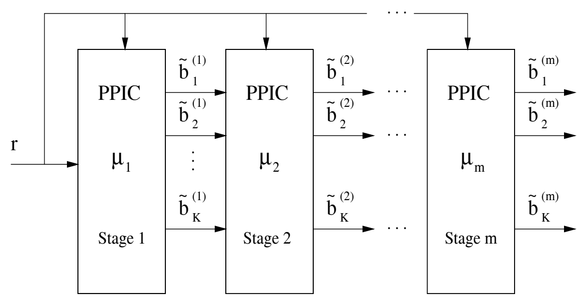

Fig. 1. An m-stage PPIC receiver.

defined as [5]

$$\mathbf{C}^t(m) = \left[\mathbf{I} - \prod_{i=1}^{m} \left(\mathbf{I} - \mu_i(\mathbf{R} + \sigma^2\mathbf{I})\right)\right] (\mathbf{R} + \sigma^2\mathbf{I})^{-1}\mathbf{A}^t \tag{2}$$

where μ_i is the PCF at stage i and the matrix $\mathbf{R} = \mathbf{A}^t\mathbf{A}$ is called the correlation matrix. It can be shown that

$$\mathbf{R} = \mathbf{U}\Lambda\mathbf{U}^H \tag{3}$$

with $\mathbf{U}\mathbf{U}^H = 1$ and $\Lambda = \text{diag}[\lambda_1, \lambda_2, \ldots, \lambda_K]$ where λ_k are the eigenvalues of the correlation matrix for $k = 1, 2, \ldots, K$. If $\mathbf{R}$ is Hermitian and positive semi-definite, then all eigenvalues of $\mathbf{R}$ are non-negative. If $\mathbf{R}$ is full rank, which corresponds to the spreading codes from all K users are linearly independent, then R has only positive eigenvalues.

3. DERIVATION OF PCF

Suppose that the PCF for an m-stage PPIC receiver is $\mu_1, \mu_2, \ldots, \mu_m$. By assuming that the PPIC receiver will converge to the MMSE receiver, The optimum PCF expression found in [7] is simplified to

$$\mu_i = \frac{E\left[\sum_{k=1}^{K} \lambda_k . \left|t_k^{(i)}\right|^2\right]}{E\left[\sum_{k=1}^{K} \lambda_k \varphi_k . \left|t_k^{(i)}\right|^2\right]} = \frac{E\left[\frac{1}{K}\sum_{k=1}^{K} \lambda_k . \left|t_k^{(i)}\right|^2\right]}{E\left[\frac{1}{K}\sum_{k=1}^{K} \lambda_k \varphi_k . \left|t_k^{(i)}\right|^2\right]} \tag{4}$$

where λ_k are the eigenvalues of the correlation matrix for $k = 1, 2, \ldots, K$. Define

$$t_k^{(i)} \stackrel{\text{def}}{=} 1 + \sum_{j=1}^{i-1} x_j \varphi_k^j, \tag{5}$$

and

$$x_i \stackrel{\text{def}}{=} (-1)^i \sum_{j=1}^{m} \prod_{i} \mu_j \tag{6}$$

where $\sum_{j=1}^{m} \prod_i \mu_j$ denotes the sum of the i-ary product of μ_j for $j = 1, 2, \ldots, m$. This means that

$$
\begin{aligned}
x_1 &= (-1)\left(\mu_1+\mu_2+\ldots\mu_m\right) \\
x_2 &= (-1)^2\left(\mu_1\mu_2+\mu_1\mu_3+\ldots+\mu_{m-1}\mu_m\right) \\
&\vdots \\
x_m &= (-1)^m\left(\mu_1\mu_2\ldots\mu_{m-1}\mu_m\right)
\end{aligned}
\tag{7}
$$

furthermore,

$$\varphi_k \stackrel{\text{def}}{=} \lambda_k+\sigma^2 \tag{8}$$

It can be shown that

$$\prod_{j=1}^{m}\left(1-\mu_j\varphi_k\right)=1+\sum_{i=1}^{m}x_i\varphi_k^i \tag{9}$$

using Equation (5), we get

$$
\begin{aligned}
\left|t_k^{(i)}\right|^2 &= \left(1+\sum_{j=1}^{i-1}x_j\varphi_k^j\right)^2 \\
&= 1+2\sum_{j=1}^{i-1}x_j\varphi_k^j+\sum_{j=2}^{2i-2}x'_j\varphi_k^j
\end{aligned}
\tag{10}
$$

where

$$x'_j=\sum_{l=1}^{j-1}x_{j-l}x_l. \tag{11}$$

The numerator of Equation (4) becomes

$$
\begin{aligned}
&E\left[\frac{1}{K}\sum_{k=1}^{K}\lambda_k.\left|t_k^{(i)}\right|^2\right] \\
&=E\left\{\frac{1}{K}\sum_{k=1}^{K}\lambda_k.\left[1+2\sum_{j=1}^{i-1}x_j\varphi_k^j+\sum_{j=2}^{2i-2}x'_j\varphi_k^j\right]\right\} \\
&=E\left\{\frac{1}{K}\sum_{k=1}^{K}\lambda_k\left[1+2\sum_{j=1}^{i-1}x_j\left(\lambda_k+\sigma^2\right)^j+\sum_{j=2}^{2i-2}x'_j\left(\lambda_k+\sigma^2\right)^j\right]\right\} \\
&=E\left[\frac{1}{K}\sum_{k=1}^{K}\lambda_k\right]+2\sum_{j=1}^{i-1}x_j\sum_{l=0}^{j}\binom{j}{l}\sigma^{2(j-l)}E\left[\frac{1}{K}\sum_{k=1}^{K}\lambda_k^{l+1}\right] \\
&\quad+\sum_{j=2}^{2i-2}x'_j\sum_{l=0}^{j}\binom{j}{l}\sigma^{2(j-l)}E\left[\frac{1}{K}\sum_{k=1}^{K}\lambda_k^{l+1}\right]
\end{aligned}
\tag{12}
$$

By defining the rth moment of the eigenvalue of the correlation matrix as

$$M_r=E\left\{\frac{1}{K}\sum_{k=1}^{K}\lambda_k^r\right\} \tag{13}$$

and

$$M'_j=\sum_{l=0}^{j}\binom{j}{l}\sigma^{2(j-l)}M_{l+1}, \tag{14}$$

the numerator of Equation (4) eventually becomes

$$E\left[\sum_{k=1}^{K}\lambda_k.\left|t_k^{(i)}\right|^2\right]=M_1+2\sum_{j=1}^{i-1}x_jM'_j+\sum_{j=2}^{2i-2}x'_jM'_j \tag{15}$$

by doing the same manipulation in the denominator of Equation (4) the equation for PCF at ith stage is found as

$$\mu_i=\frac{\eta_i}{\nu_i} \tag{16}$$

where

$$\eta_i=M_1+2\sum_{j=1}^{i-1}x_jM'_j+\sum_{j=2}^{2i-2}x'_jM'_j \tag{17}$$

and

$$
\begin{aligned}
\nu_i &= M_2+\sigma^2M_1+2\sum_{j=1}^{i-1}x_j\left(M''_j+\sigma^2M'_j\right) \\
&\quad+\sum_{j=2}^{2i-2}x'_j\left(M''_j+\sigma^2M'_j\right)
\end{aligned}
\tag{18}
$$

with

$$M''_j=\sum_{l=0}^{j}\binom{j}{l}\sigma^{2(j-l)}M_{l+2} \tag{19}$$

For a large-system scenario, as N and K tend to infinity but their ratio remains finite (*i.e.*, $N,K\to\infty,\ \beta=K/N<\infty$), the asymptotic *Probability Density Function* (p.d.f) for eigenvalues of the correlation matrix becomes [11]

$$f(\lambda)=\frac{1}{2\pi\beta\lambda}\sqrt{(\lambda-\lambda_{min})(\lambda_{max}-\lambda)} \tag{20}$$

where $\lambda_{min}<\lambda<\lambda_{max}$ with $\lambda_{min}=(1-\sqrt{\beta})^2$ and $\lambda_{max}=(1+\sqrt{\beta})^2$. The rth moment is shown to be a polynomial in N and K that can be expressed as

$$M_r=\sum_{j=0}^{r-1}\frac{1}{j+1}\binom{r}{j}\binom{r-1}{j}\beta^j \tag{21}$$

4. NUMERICAL RESULTS

Some numerical results are presented in this section. In all simulations, purely random signature waveforms are assumed. All simulations are run for 1000 samples. In order to find out the physical meaning of a large system, in Figure 2, the analytical p.d.f. of the eigenvalues of the correlation matrix is compared with the simulation results for $K=16, N=32$ (i.e., $\beta=K/N=0.5$). From this figure it is seen that the difference between analytical and simulation results is negligible. Therefore, for the case of $N\geq 32$, the system can be considered as a large system. This point has been discussed in [11] taking a different approach.

Figure 3 compares the performance improvement of the method proposed in [5] without ordering (hereafter, being called the first method) with the method that was calculated in this paper (hereafter, being called the second method). As is seen from this figure, there is no fluctuation in performance for the second method. Moreover, by defining a proper stopping criterion, e.g., the difference between two consecutive PCFs, we can proceed to any number of stages of interference cancellation. In the first method, a matrix inversion is applied for the calculation of the PCFs. In addition, the PCFs should be ordered to obtain an optimum performance in the middle stages of interference cancellation.

Since the second method needs non of the above mentioned tasks, it turns out to have a much simpler circuitry.

Figure 4 compares the convergence of performance of the PPIC receiver using two methods to that of the MMSE receiver. For the method one to produce the same smooth convergence as that of second method, the ordering has a complexity of $O(m^3)$ where m is the number of interference cancellation stages. It should be noted that the difference between performances of two methods at final stage of interference cancellation is due to the simulation result that was obtained for $N = 31$ which results in a negligible error to model a large system as Figure 2 proves this matter. Therefore, using larger processing gains (*e.g.,* $N \geq 64$) will overcome the issue. However, considering the simulation time length, the choice of $N = 32$ is an acceptable solution.

5. CONCLUSIONS

A new closed-form expression for the optimum partial cancellation factor (PCF) for partial parallel interference cancellation receiver was introduced in this paper. Comparing the proposed method with a previously proposed method, one can point out some advantages. Firstly, it is less complex due to the fact that the calculated PCFs are direct functions of the moments of the eigenvalues of correlation matrix. Secondly, there is no need to know the number of interference cancellation stages a priori. Finally, there is no need that the PCF to be ordered. This last advantage results in even more simplicity in circuitry.

6. REFERENCES

[1] S. Verdu, “Multiuser Detection,” Cambridge, U.K.: Cambridge Univ. Press, 1998.

[2] S. Moshavi, “Multi-user detection for DS-CDMA communications,” *IEEE Commun. Mag.*, pp. 124-136, Oct. 1996.

[3] D. Koulakiotis and A. H. Aghvami, “Data detection techniques for DS/CDMA mobile systems: A review,” *IEEE Personal Commun.*, vol. 7, pp. 24-34, June 2000.

[4] D. Divsalar, M. K. Simon, and D. Raphaeli, “Improved parallel interference cancellation for CDMA,” *IEEE Trans. Commun.*, vol. 46, pp. 258-268, Feb. 1998.

[5] D. Guo, L.K.Rasmussen, and T. J. Lim, “Linear parallel interference cancellation in long-code CDMA multiuser detection,” *IEEE JSAC*, vol. 17, pp. 2074 -2081, Dec. 1999.

[6] M. Ghotbi and M. R. Soleymani, “Partial parallel interference cancellation of DS-CDMA satellite signals with amplitude and phase estimation,” *IEEE GLOBECOM 02*, vol. 3, pp. 2882 -2886, Nov. 2002, Taipei, Taiwan.

[7] D. Guo, “Linear parallel interference cancellation in CDMA,” M.Eng. thesis, Nat. Univ. Singapore, 1998.

[8] P. G. Renucci, “optimization of soft interference cancellation in DS-CDMA receivers,” Master’s thesis, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, May 1998.

[9] A. N. Fawzy, A. W. Fayez, and M. M. Riad, “Optimization of partial parallel interference cancellation (PPIC) factor in CDMA systems,” *Proc. IEEE sys., man, cyber. 2000*, vol. 4, pp. 2375-2380.

[10] Y. T. Hsieh and W. R. Wu, “An optimum two-stage partial parallel interference canceller for CDMA systems,” *IEEE GLOBECOM ’02*, vol. 1, pp. 499 -503, Nov. 2002.

[11] D. Guo, S. Verdu, and L. K. Rasmussen, “Asymptotic normality of linear multiuser receiver outputs,” *IEEE Trans. Inform Theory*, vol. 48, pp. 1-16, Dec. 2002.

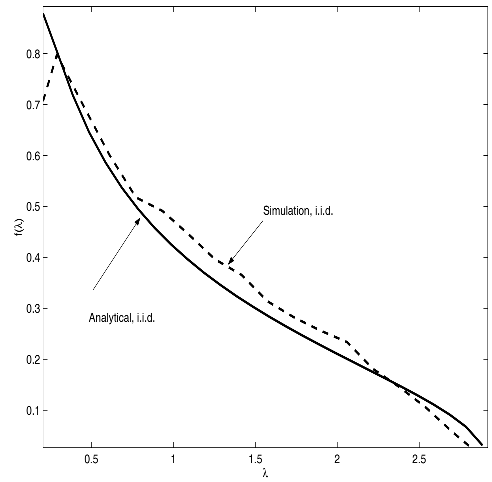

Fig. 2. The p.d.f of eigenvalues of the correlation matrix for the two cases of analytical i.i.d and simulation i.i.d. entries with $N = 32$ and $\beta = K/N = 0.5$.

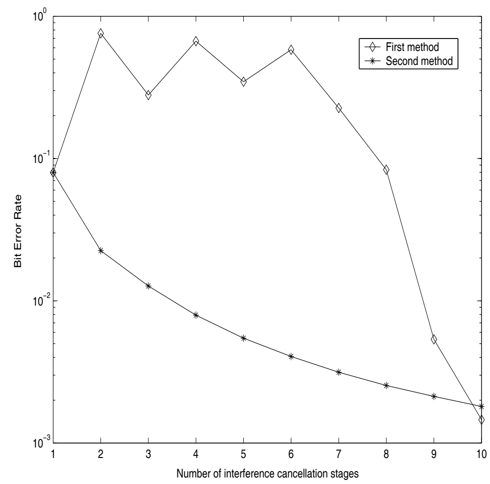

Fig. 3. Comparison of performance improvement of PPIC receiver by applying two methods of PCF optimization for $N = 15$, $N = 31$, and $SNR = 10dB$.

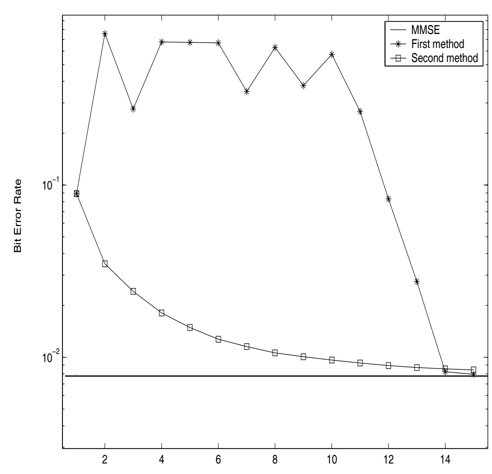

Fig. 4. Convergence of the performance of PPIC receiver to the performance of MMSE receiver using two methods of PCF optimization for $N = 15$, $N = 31$, and $SNR = 7dB$.