PERFORMANCE OF CDMA RECEIVERS UNDER IMPERFECT CHANNEL ESTIMATION

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ABSTRACT

In a CDMA system, imperfect channel estimation causes performance degradation of a minimum mean-square-error (MMSE) receiver. This paper studies effects of channel estimation error on the performance of direct matrix inversion (DMI) MMSE receiver and subspace MMSE receiver when channel parameters are estimated by a covariance-matching technique based on finite data samples. Receivers' output signal to interference plus noise ratios (SINRs) and bit-error-rates (BERs) are adopted for performance measures. Those performance indicators under such an imperfect condition are derived from a perturbation perspective and verified by simulation examples.

1. INTRODUCTION

In a CDMA system, multiuser interference (MUI) is a typical obstacle to be obviated. Minimum mean-square-error (MMSE) receivers can be designed to detect input signals while suppressing MUI. However, in a wireless communication environment, channel parameters are not known *a priori* and need to be estimated. Blind methods offer an increased effective data rate. Typical secondorder statistics based methods include subspace approaches [1], [2], minimum variance or minimum output energy (MOE) methods [3], [4], power of R (POR) technique [5], etc.

Most multiuser detection and channel estimation techniques are developed under perfect conditions first and then applied to practical scenarios to test performance degradation and robustness. Since various imperfectness may stem from many sources such as background noise, finite sample size, unknown channel order, synchronization error, user variation, etc., performance prediction under those conditions is necessary to better evaluate individual method. For example, sensitivity of multiuser detectors' performance to channel mismatch is analyzed in [6] when transmitted signals suffer from flat fading. Subspace based multipath channel estimation errors for a multirate CDMA system are derived for given finite number of observations [7]. In [8], perturbation to subspace decomposition of a matrix is studied when errors are introduced to the matrix by either noise or finite sample size.

In this paper, we analyze performance of direct matrix inversion (DMI) MMSE and subspace MMSE receivers [2] when channel is imperfectly estimated by a covariance-matching technique [9]. Each receiver's output signal to interference plus noise ratio (SINR) and bit-error-rate (BER) are adopted for performance measures. Imperfectness from finite sample size is treated as perturbation whose effect is particularly studied. To achieve our goal, statistics of sample covariance are provided in the light of [10]. Similar to [11], they can be regarded as general results and possibly applied to analysis of other methods. Then SINR and BER under such an imperfect condition are analyzed from a perturbation perspective and verified by extensive simulations.

Notations throughout the paper are defined as follows. Denote Hermitian - complex conjugate $(\cdot)^*$ transpose $(\cdot)^T$ by $(\cdot)^H$, integer ceiling by $\lceil \cdot \rceil$, vector 2-norm by $|| \cdot ||$ and matrix Frobenius norm by $|| \cdot ||_F$ [12], trace by $tr(\cdot)$, expectation by $E\{\cdot\}$, the *a*th column of a matrix U as u_a , $\mathbf{1}_a$ as a column vector of length *a* with all elements equal to one, I_a as an identity matrix of degree *a* whose *b*th column is denoted as $e_{a,b}$, the Kronecker product as " \otimes " [12], "*vec*" as a vectorized operation, matrix Hadamard product " \odot " to represent element wise multiplication, the Khatri-Rao product " \Box " to represent column-wise Kronecker product [12]: $U \Box W =$ $[u_1 \otimes w_1, u_2 \otimes w_2, \cdots]$. A diagonal or block diagonal matrix with main diagonal entries x_i is denoted as $diag\{x_1, x_2, \cdots\}$. We also denote a perturbation by preceding the corresponding quantity by δ , and the perturbed quantity with $\widehat{}$. For example, $\delta \mathbf{R} = \widehat{\mathbf{R}} - \mathbf{R}$.

2. DATA MODEL

Consider an uplink CDMA system with J users. User j is assigned periodic spreading codes $c_j(k)$ of length P to spread its information symbol $w_j(n)$ of zero-mean and unit variance. Let its chip sequence be transmitted through a discrete-time chip-rate channel $g_j(l)$. Then the received signal $y_j(n)$ at the chip-synchronized receiver has a form [3]

$$y_j(n) = \sum_{l=-\infty}^{\infty} w_j(l) h_j(n - d_j - lP), h_j(n) = \sum_{i=-\infty}^{\infty} g_j(i) c_j(n - i),$$
(1)

where d_j is the propagation delay of user j in chip periods. After considering all J users and zero-mean additive white Gaussian noise (AWGN) v(n) whose variance is denoted as σ_v^2 , the received signal becomes $y(n) = \sum_{j=1}^J y_j(n) + v(n)$. The discrete-time model can be easily formulated into a ma-

The discrete-time model can be easily formulated into a matrix/vector representation. For convenience, we assume a quasisynchronous system with $d_j \ll P$ and absorb propagation delay into the channel for each user. The maximum delay spread of all multipath channels is q chips. If we collect $\nu = MP$ chip-rate samples in a vector y_n at the receiver, then it is given by [3]

$$\boldsymbol{y}_{n} = \sum_{j=1}^{J} \sum_{m=-K}^{M-1} \boldsymbol{C}_{j,m} \boldsymbol{g}_{j} w_{j}(n+m) + \boldsymbol{v}_{n} = \boldsymbol{H} \boldsymbol{w}_{n} + \boldsymbol{v}_{n},$$
(2)

where $K = \lceil (q-1)/P \rceil$, g_j is the channel vector of length q, $C_{j,m}$ is the code filtering matrix of user j for symbol $w_j(n + 1)$

This work was supported in part by the U.S. National Science Foundation under Grant NSF-CCR 0207931.

m) which can be obtained from $C_{j,0}$ (corresponding to symbol $w_j(n)$) by shifting it up (if m < 0) or down (if m > 0) by |m|P rows, H is called a signature waveform matrix

$$\boldsymbol{H} = [\boldsymbol{H}_1, \cdots, \boldsymbol{H}_J], \quad \boldsymbol{H}_j = [\boldsymbol{C}_{j,-K}\boldsymbol{g}_j, \cdots, \boldsymbol{C}_{j,M-1}\boldsymbol{g}_j], \quad (3)$$

 w_n contains all L = J(M + K) inputs, v_n is the noise component. Without loss of generality, assume $w_1(n)$ is to be detected at time n. Its signature vector is $h_{K+1} = C_{1,0}g_1$. Structures of users' signature waveforms have been exploited to estimate all channel vectors blindly by a covariance-matching technique [9] under assumptions that all spreading codes and the maximum channel delay spread are known.

3. COVARIANCE-MATCHING CHANNEL ESTIMATION AND LINEAR DETECTION

Covariance-matching technique is based on the covariance of \boldsymbol{y}_n

$$R = \sum_{j=1}^{J} \sum_{m=-K}^{M-1} C_{j,m} G_j C_{j,m}^{H} + \sigma_v^2 I_\nu$$
(4)

where $G_j = g_j g_j^H$. Define $\alpha_j = tr(G_j) = ||g_j||^2$. It is observed that R is parameterized by G_j . If it is matched with its estimate from N data vectors

$$\widehat{\boldsymbol{R}} = \frac{1}{N} \sum_{n=1}^{N} \boldsymbol{y}_n \boldsymbol{y}_n^H, \qquad (5)$$

and the resulting error $||\mathbf{R} - \widehat{\mathbf{R}}||_F^2$ is minimized, G_j can be estimated. Define $\mathbf{r} = vec(\mathbf{R})$, $\widehat{\mathbf{r}} = vec(\widehat{\mathbf{R}})$, $\mathbf{x}_j = vec(G_j)$, and $\mathbf{x} = [\mathbf{x}_1^T, \dots, \mathbf{x}_J^T, \sigma_v^2]^T$. Noticing $||\mathbf{A}||_F^2 = ||vec(\mathbf{A})||^2$, the criterion can be described as follows [9]

$$\widehat{\boldsymbol{x}} = \arg\min||\boldsymbol{r} - \widehat{\boldsymbol{r}}||^2 = \arg\min||\boldsymbol{S}\boldsymbol{x} - \widehat{\boldsymbol{r}}||^2$$
 (6)

where

$$\boldsymbol{S} = [\boldsymbol{S}_1, \cdots, \boldsymbol{S}_J, vec(\boldsymbol{I}_{\nu})], \ \boldsymbol{S}_j = \sum_{m=-K}^{M-1} \boldsymbol{C}_{j,m}^* \otimes \boldsymbol{C}_{j,m}.$$
 (7)

Under some identifiability conditions [9], the solution to (6) becomes

$$\widehat{\boldsymbol{x}} = \boldsymbol{Q}\widehat{\boldsymbol{r}}, \quad \boldsymbol{Q} = (\boldsymbol{S}^{H}\boldsymbol{S})^{-1}\boldsymbol{S}^{H},$$
 (8)

Once x is estimated, x_j can be extracted. Then G_j is reconstructed by the reverse *vec* operation. These operations can be described by

$$\widehat{\boldsymbol{G}}_{j} = [\boldsymbol{A}_{j,1}\widehat{\boldsymbol{r}}, \cdots, \boldsymbol{A}_{j,q}\widehat{\boldsymbol{r}}]$$
(9)

where

$$\boldsymbol{A}_{j,i} = (\boldsymbol{e}_{q,i}^T \otimes \boldsymbol{I}_q) [\boldsymbol{e}_{J,j}^T \otimes \boldsymbol{I}_{q^2}, \boldsymbol{0}_{q^2 \times 1}]_{q^2 \times (Jq^2+1)} (\boldsymbol{S}^H \boldsymbol{S})^{-1} \boldsymbol{S}^H$$

for $i = 1, \dots, q, j = 1, \dots, J$. If singular value decomposition is performed on \boldsymbol{G}_j , then the maximum singular value is

 $\alpha_j = ||\mathbf{g}_j||^2$ and corresponding singular vector becomes the channel vector up to a scalar ambiguity. Therefore, once $\widehat{\mathbf{G}}_j$ is obtained, channel vector \mathbf{g}_j can be estimated by finding the maximum singular vector of $\widehat{\mathbf{G}}_j$ and scaling the vector by $\sqrt{\widehat{\alpha}_j}$ to adjust its norm.

The estimated channel vector \hat{g}_1 can be used for design of DMI MMSE and subspace MMSE receivers. The DMI MMSE receiver can be defined from direct inversion of R as

$$f_{mmse,dmi} = R^{-1} C_{1,0} g_1.$$
 (10)

The MMSE receiver can also be expressed in terms of the subspace components of \mathbf{R} . Let the eigenvalue decomposition of \mathbf{R} be

$$\boldsymbol{R} = \boldsymbol{U}_s \boldsymbol{\Lambda}_s \boldsymbol{U}_s^H + \boldsymbol{U}_n \boldsymbol{\Lambda}_n \boldsymbol{U}_n^H, \qquad (11)$$

where $\Lambda_s = diag\{\lambda_1^2, \ldots, \lambda_{\xi}^2\}$, $\Lambda_n = \sigma_v^2 I$, U_s and U_n represent the signal and noise subspaces respectively. Invoking the orthogonality between U_n and $C_{1,0}g_1$, the subspace MMSE receiver takes the following form [2]

$$\boldsymbol{f}_{mmse,sub} = \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} \boldsymbol{U}_s^H \boldsymbol{C}_{1,0} \boldsymbol{g}_1.$$
(12)

It can be observed that these receivers are coupled with channel vectors. Their performance will be investigated jointly with the channel estimator next.

4. PERFORMANCE STUDY

Performance loss is incurred to receivers when finite received data samples are processed as (5). Assume N is sufficiently large such that perturbation technique is applicable [8]. Performance of the channel estimator is required to evaluate each receiver's performance and is thus studied first. We will propose more compact results than those in [9] based on the property of cumulant.

4.1. Channel Estimation Performance

All perturbations are due to an estimation error for R or equivalently r. If \hat{r} has an estimation error $\delta r = \hat{r} - r$ due to finite N, then from (9), G_j is perturbed by δG_j as

$$\delta \boldsymbol{G}_j = [\boldsymbol{A}_{j,1} \delta \boldsymbol{r}, \cdots, \boldsymbol{A}_{j,q} \delta \boldsymbol{r}]. \tag{13}$$

Then the first-order perturbation in its maximum singular vector becomes [8]

$$\delta \boldsymbol{g}_{j} \approx \frac{1}{\alpha_{j}} \boldsymbol{\Pi}_{\boldsymbol{g}_{j}}^{\perp} \delta \boldsymbol{G}_{j} \boldsymbol{g}_{j}, \ \boldsymbol{\Pi}_{\boldsymbol{g}_{j}}^{\perp} = \boldsymbol{\Sigma}_{j} \boldsymbol{\Sigma}_{j}^{H}$$
(14)

where Σ_j is in size of $q \times (q-1)$ and spans a (q-1)-dimensional subspace orthogonal to g_j . Substituting (13) into (14), we obtain

$$\delta \boldsymbol{g}_{j} \approx \boldsymbol{\Gamma}_{j} \delta \boldsymbol{r}, \quad \boldsymbol{\Gamma}_{j} = \frac{1}{\alpha_{j}} \boldsymbol{\Pi} \boldsymbol{g}_{j} \sum_{i=1}^{q} g_{j}(i) \boldsymbol{A}_{j,i}.$$
 (15)

The auto-covariance of channel estimate thus becomes

$$\operatorname{Cov}(\delta \boldsymbol{g}_{j}, \delta \boldsymbol{g}_{j}) = E\{\delta \boldsymbol{g}_{j} \delta \boldsymbol{g}_{j}^{H}\} \approx \boldsymbol{\Gamma}_{j} \boldsymbol{\Phi}(\widehat{\boldsymbol{r}}) \boldsymbol{\Gamma}_{j}^{H}, \quad (16)$$

where $\Phi(\hat{r}) = E\{\delta r \delta r^H\}$ is the covariance of \hat{r} . It depends on data model (2), covariance estimation method (5) and up to the fourth-order statistics of channel inputs and noise. We will present general results in the light of [10] whose results for complex symmetric sources are extended to both a real system and a complex system here. The following properties of "*vec*", " \otimes " and " \Box " have found to be useful [12]

$$vec(ABC) = (C^T \otimes A)vec(B),$$
 (17)

$$(\boldsymbol{A} \otimes \boldsymbol{B})(\boldsymbol{C} \otimes \boldsymbol{D}) = (\boldsymbol{A}\boldsymbol{C}) \otimes (\boldsymbol{B}\boldsymbol{D}),$$
 (18)

$$(\boldsymbol{A} \otimes \boldsymbol{B})(\boldsymbol{C} \Box \boldsymbol{D}) = (\boldsymbol{A}\boldsymbol{C}) \Box (\boldsymbol{B}\boldsymbol{D}). \tag{19}$$

Although (5) does not require independence of different data vectors, they are assumed independent for convenience of analysis.

Proposition: If channel model follows (2) with ν outputs and L inputs with same distribution and fourth-order cumulant κ_{4w} , and data covariance is estimated from N independent data vectors as (5), then for a real system, $\Phi(\hat{r})$ is given by

$$N\boldsymbol{\Phi}(\hat{\boldsymbol{r}}) = \kappa_{4w}(\boldsymbol{H}\Box\boldsymbol{H})(\boldsymbol{H}\Box\boldsymbol{H})^{T} + \boldsymbol{R}\otimes\boldsymbol{R} + \boldsymbol{A}\odot\boldsymbol{A}^{T},$$
(20)

$$oldsymbol{A} = [(oldsymbol{I}_
u \otimes oldsymbol{1}_
u) oldsymbol{R} (oldsymbol{1}_
u^T \otimes oldsymbol{I}_
u)]$$

while for a complex system it has a form

$$N\boldsymbol{\Phi}(\widehat{\boldsymbol{r}}) = \kappa_{4w} (\boldsymbol{H}^* \Box \boldsymbol{H}) (\boldsymbol{H}^* \Box \boldsymbol{H})^H + \boldsymbol{R}^* \otimes \boldsymbol{R}.$$
(21)

Proof: It is omitted due to limited space. See [13] for details.

4.2. Performance of Receivers

Perturbation in channel estimation induced by finite data samples inevitably causes each receiver perturbed to be $\hat{f} = f + \delta f$, where the first-order perturbation in δf can be assumed to have zero mean due to zero-mean of δR . According to (2), perturbed SINR has the following form

$$\widehat{\text{SINR}} = \frac{\boldsymbol{f}^{H}\boldsymbol{R}_{1}\boldsymbol{f} + E\{\delta\boldsymbol{f}^{H}\boldsymbol{R}_{1}\delta\boldsymbol{f}\}}{\boldsymbol{f}^{H}\boldsymbol{R}_{int}\boldsymbol{f} + E\{\delta\boldsymbol{f}^{H}\boldsymbol{R}_{int}\delta\boldsymbol{f}\}}$$
(22)

where $\mathbf{R}_1 = \mathbf{C}_{1,0} \mathbf{g}_1 \mathbf{g}_1^H \mathbf{C}_{1,0}^H$ and $\mathbf{R}_{int} = \mathbf{R} - \mathbf{R}_1$. Then BER can be evaluated by assuming Gaussian interference [14]

$$\widehat{\text{BER}} \approx Q(\sqrt{\text{SINR}})$$
(23)

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{\frac{-t^2}{2}} dt$. According to (22), the SINR depends on both unperturbed terms (signal power, interference plus noise power) and corresponding perturbations. Perturbations follow a typical form of $\Psi(X) = E\{\delta f^H X \delta f\}$, where X can be replaced by R_1 or R_{int} . Since different receivers take different forms with correspondingly different δf , evaluation of $\Psi(X)$ will be discussed for each receiver respectively. For shorter notations, receivers' subscripts are dropped later and simply denoted by f. However, no confusion will be caused in the context.

4.2.1. DMI MMSE Receiver

Replacing \boldsymbol{R} in (10) by $\boldsymbol{R} + \delta \boldsymbol{R}$ and considering channel estimation error, we obtain the first-order perturbation

$$\delta \boldsymbol{f} \approx \boldsymbol{R}^{-1} \boldsymbol{C}_{1,0} \delta \boldsymbol{g}_1 - \boldsymbol{R}^{-1} \delta \boldsymbol{R} \boldsymbol{R}^{-1} \boldsymbol{C}_{1,0} \boldsymbol{g}_1.$$
(24)

Substituting (15) into (24), δf is related to perturbation in covariance estimation by

$$\delta \boldsymbol{f} \approx \boldsymbol{R}^{-1} \boldsymbol{C}_{1,0} \boldsymbol{\Gamma}_1 \delta \boldsymbol{r} - \boldsymbol{R}^{-1} \delta \boldsymbol{R} \boldsymbol{R}^{-1} \boldsymbol{C}_{1,0} \boldsymbol{g}_1.$$
(25)

Then perturbation of signal/noise power $\Psi(\mathbf{X})$ is given by ...

$$\Psi(\mathbf{X}) \approx tr(\underline{E\{\delta r \delta r^{H}\}} \Gamma_{1}^{H} C_{1,0}^{H} R^{-1} \mathbf{X} R^{-1} C_{1,0} \Gamma_{1}) + g_{1}^{H} C_{1,0}^{H} R^{-1} \underline{E\{\delta R R^{-1} \mathbf{X} R^{-1} \delta R\}} R^{-1} C_{1,0} g_{1} - g_{1}^{H} C_{1,0}^{H} R^{-1} \underline{E\{\delta R R^{-1} \mathbf{X} R^{-1} C_{1,0} \Gamma_{1} \delta r\}} - \underline{E\{\delta r^{H} \Gamma_{1}^{H} C_{1,0}^{H} R^{-1} \mathbf{X} R^{-1} \delta R\}} R^{-1} C_{1,0} g_{1}(26)$$

.. ..

Those underlined terms are required for evaluation of $\Psi(\mathbf{X})$. The first term can be easily obtained from Proposition. The second term has been derived in [7] in a general form $E\{\delta R Z \delta R\}$ where Z is an arbitrary deterministic matrix. The third underlined term $E\{\delta RZ\delta r\}$ can be easily related to a form $E\{\delta RZ\delta R\}$ after noticing $\delta r = vec(\delta R)$. Therefore, all of them can be evaluated from given system parameters.

4.2.2. Subspace MMSE Receiver

According to (12) and expanding $(\Lambda_s + \delta \Lambda_s)^{-1}$, we obtain

$$\delta \boldsymbol{f} \approx \delta \boldsymbol{U}_{s} \boldsymbol{\Lambda}_{s}^{-1} \boldsymbol{U}_{s}^{H} \boldsymbol{C}_{1,0} \boldsymbol{g}_{1} - \boldsymbol{U}_{s} \boldsymbol{\Lambda}_{s}^{-1} \delta \boldsymbol{\Lambda}_{s} \boldsymbol{\Lambda}_{s}^{-1} \boldsymbol{U}_{s}^{H} \boldsymbol{C}_{1,0} \boldsymbol{g}_{1}$$
$$+ \boldsymbol{U}_{s} \boldsymbol{\Lambda}_{s}^{-1} \delta \boldsymbol{U}_{s}^{H} \boldsymbol{C}_{1,0} \boldsymbol{g}_{1} + \boldsymbol{U}_{s} \boldsymbol{\Lambda}_{s}^{-1} \boldsymbol{U}_{s}^{H} \boldsymbol{C}_{1,0} \delta \boldsymbol{g}_{1}.$$
(27)

Perturbation $\delta \mathbf{R}$ or $\delta \mathbf{r}$ causes not only estimated channel vector perturbed, but also subspace components of R perturbed [8]

$$\delta \boldsymbol{U}_{s} \approx \boldsymbol{U}_{n} \boldsymbol{U}_{n}^{H} \delta \boldsymbol{R} \boldsymbol{U}_{s} \boldsymbol{\Omega}^{-1}, \ \delta \boldsymbol{U}_{n} \approx -\boldsymbol{U}_{s} \boldsymbol{\Omega}^{-1} \boldsymbol{U}_{s}^{H} \delta \boldsymbol{R} \boldsymbol{U}_{n},$$
$$\delta \boldsymbol{\Lambda}_{s} \approx \boldsymbol{U}_{s}^{H} \delta \boldsymbol{R} \boldsymbol{U}_{s}, \ \delta \boldsymbol{\Lambda}_{n} \approx \boldsymbol{U}_{n}^{H} \delta \boldsymbol{R} \boldsymbol{U}_{n},$$
(28)

where $\mathbf{\Omega} = \mathbf{\Lambda}_s - \sigma_v^2 \mathbf{I}$, and approximation is valid up to the first order of $\delta \mathbf{R}$. Since $\mathbf{U}_n^H \mathbf{C}_{1,0} \mathbf{g}_1 = \mathbf{0}$, substituting (28) in (27) and invoking (15), we obtain

$$\delta \boldsymbol{f} \approx \boldsymbol{B}_n \delta \boldsymbol{R} \boldsymbol{B}_{\gamma} \boldsymbol{C}_{1,0} \boldsymbol{g}_1 - \boldsymbol{B}_s \delta \boldsymbol{R} \boldsymbol{B}_s \boldsymbol{C}_{1,0} \boldsymbol{g}_1 + \boldsymbol{B}_s \boldsymbol{C}_{1,0} \boldsymbol{\Gamma}_1 \delta \boldsymbol{r}$$
(29)

where for convenience we have defined

$$\boldsymbol{B}_n \stackrel{\Delta}{=} \boldsymbol{U}_n \boldsymbol{U}_n^H, \ \boldsymbol{B}_s \stackrel{\Delta}{=} \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} \boldsymbol{U}_s^H, \ \boldsymbol{B}_\gamma \stackrel{\Delta}{=} \boldsymbol{U}_s (\boldsymbol{\Omega} \boldsymbol{\Lambda}_s)^{-1} \boldsymbol{U}_s^H.$$

Then $\Psi(\mathbf{X})$ can be expressed in terms of statistics of the covariance estimation error

$$\Psi(\mathbf{X}) \approx g_{1}^{H} C_{1,0}^{H} B_{\gamma} \underline{E}\{\delta R B_{n} X B_{n} \delta R\} B_{\gamma} C_{1,0} g_{1}$$

$$- g_{1}^{H} C_{1,0}^{H} B_{\gamma} \underline{E}\{\delta R B_{n} X B_{s} \delta R\} B_{s} C_{1,0} g_{1}$$

$$+ g_{1}^{H} C_{1,0}^{H} B_{\gamma} \underline{E}\{\delta R B_{n} X B_{s} C_{1,0} \Gamma_{1} \delta r\}$$

$$- g_{1}^{H} C_{1,0}^{H} B_{s} \underline{E}\{\delta R B_{s} X B_{n} \delta R\} B_{\gamma} C_{1,0} g_{1}$$

$$+ g_{1}^{H} C_{1,0}^{H} B_{s} \underline{E}\{\delta R B_{s} X B_{s} \delta R\} B_{s} C_{1,0} g_{1}$$

$$- g_{1}^{H} C_{1,0}^{H} B_{s} \underline{E}\{\delta R B_{s} X B_{s} \delta R\} B_{s} C_{1,0} g_{1}$$

$$- g_{1}^{H} C_{1,0}^{H} B_{s} \underline{E}\{\delta R B_{s} X B_{s} \delta R\} B_{s} C_{1,0} g_{1}$$

$$- g_{1}^{H} C_{1,0}^{H} B_{s} \underline{E}\{\delta R B_{s} X B_{s} \delta R\} B_{\gamma} C_{1,0} g_{1}$$

$$- \underline{E}\{\delta r^{H} \Gamma_{1}^{H} C_{1,0}^{H} B_{s} X B_{s} \delta R\} B_{s} C_{1,0} g_{1}$$

$$+ tr(\underline{E}\{\delta r \delta r^{H}\} \Gamma_{1}^{H} C_{1,0}^{H} B_{s} X B_{s} \delta R\} B_{s} C_{1,0} \Gamma_{1}). (30)$$

Each underlined term in (30) can be evaluated similarly as before.

5. SIMULATION EXAMPLES

Since channel estimation MSEs have been well studied in [9], we only show SINRs and BERs of each receiver averaged over 100 independent realizations. We assume J = 8, Gold sequences of P = 31, and Gaussian random channels of length 5 chips. Fig. 1 presents effect of N when SNR = 10dB. Dashed-dotted lines are based on ideal receivers ($N = \infty$), dashed lines represent analytical results obtained according to (22) or (23) and solid lines stand for experimental results. Analytical results agree well with experimental ones for relatively large N, say 1000. However, the DMI MMSE receiver has slower convergence than the subspace MMSE receiver because it is more sensitive to the estimation error in the data covariance due to matrix inversion. Effect of noise is tested when N = 500 with results presented in Fig. 2. Performance changes differently for different receivers. The DMI MMSE receiver (with finite N) does not improve with increased SNR monotonically, due to the same reason as before. Also the error between the analytical and experimental results turns out to be larger. The subspace MMSE receiver performs satisfactorily. It is close to the ideal MMSE receiver. The sensitivity to noise and

sample size can be further observed from Fig. 3. Dashed line in each subplot represents performance of the ideal MMSE receiver associated with $N = \infty$ while other six solid lines correspond to N = 50, 150, 300, 500, 1000, 2000 sequentially. Only marks N = 50 and N = 150 are made for clear representation. The SINR peaks for the DMI MMSE receiver are clearly observed for small to moderate N. They shift to high SNR regions as N increases. As $N \rightarrow \infty$, peak disappears and the solid line converges to the dashed one. However, the SINR of the subspace MMSE receiver almost shows no peak for all N. It converges to the ideal one much faster than that of the DMI MMSE receiver. Similar conclusions can be made for BER results.

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Fig. 1. Effect of sample size.



Fig. 2. Effect of noise.



Fig. 3. Effects of sample size and noise.