NEAR-FAR RESISTANCE OF CDMA COMMUNICATION SYSTEMS WITH SPATIAL DIVERSITY

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ABSTRACT

In this paper, the advantages of the spatial diversity are investigated with respect to two metrics, i.e., near-far resistance and rank of the channel matrix. The impact of spatial diversity on the rank of the channel matrix and its significance is first studied. Then the near-far resistance of the minimum mean square error (MMSE) detector is derived for wireless code division multiple access (CDMA) communication systems with spatial diversity. The derived near-far resistance under different transmit-receive antenna configurations is analyzed and compared. It is shown that near-far resistance is enhanced when spatial diversity is introduced.

1. INTRODUCTION

Throughout the history of wireless communications, spatial antenna diversity has been important in improving the radio link between wireless users. In this paper, the advantages of the spatial diversity are investigated according to the near-far resistance and the rank of the channel matrix. It was recently shown that the full column rank condition of the channel matrix could be violated in the single antenna CDMA systems [1]. In this paper, we show that the full column rank assumption could also be violated in CDMA systems with receiver antenna arrays but with a much smaller probability compared with single antenna systems.

On the other hand, the near-far resistance is by all means one of the most important performance measures for a CDMA detector. In this paper the near-far resistance of the MMSE detector is also derived for wireless CDMA communication systems with spatial diversity. The near-far resistance under different transmit-receive antenna settings is also analyzed and compared. It is shown that near-far resistance is enhanced when spatial diversity is introduced.

2. SIGNAL MODEL

Consider a CDMA system with J active users and a P-element antenna array in the receiver. The jth user's spreading code is denoted by $\mathbf{c}_j = [c_j(0), \cdots, c_j(L_c - 1)]^T$. Then, the jth user's transmitted signal at the chip rate in a baseband discrete-time model representation is given by $s_j(k) = \sum_n b_j(n)\, c_j(k - n\,L_c)$, where $b_j(n)$ is the jth user's nth symbol at the symbol rate $1/T_s$, $c_j(k)$ and $s_j(k)$ are at the chip rate $1/T_c$. In the presence of a linear multipath channel where the receiver collects one sample per chip, the received discrete-time sampled signal from user j on the lth antenna is $x_j^l(k) = \sum_m \sum_i b_j(i)\, c_j(m - i\,L_c)\, g_j^l(k - m - d_j^l)$, where $g_j^l(\cdot)$ is the effective channel impulse response between the jth user and the lth receiver antenna which is sampled at the chip interval, and d_j^l is the transmission delay (mod L_c) of user j seen at the lth antenna in chip periods. It is straightforward to show that the above equation is equivalent to $x_j^l(k) = \sum_i b_j(i) h_j^l(k - i\,L_c - d_j^l)$, where $h_j^l(k) \triangleq \sum_{i=0}^{L_c-1} c_j(i) g_j^l(k-i)$. The total received signal at the lth antenna at the chip rate is the superposition of contributions of all users as $x^l(k) = \sum_{j=1}^{J} x_j^l(k)$. Stack up L_c samples of $x^l(k)$ into $\mathbf{x}^l(n) \triangleq \left[x^l(n\,L_c), \cdots, x^l(n\,L_c + L_c - 1)\right]^T$ to obtain, at the symbol rate, the signal model

$$\mathbf{x}^l(n) = \sum_{j=1}^{J} \sum_{i=0}^{L_j-1} b_j(n-i) \begin{bmatrix} h_j^l(i\,L_c - d_j^l) \\ \vdots \\ h_j^l(i\,L_c + L_c - 1 - d_j^l) \end{bmatrix}$$
$$= \sum_{j=1}^{J} \sum_{i=0}^{L_j-1} b_j(n-i)\mathbf{h}_j^l(i) = \sum_{i=0}^{L_h-1} \mathbf{H}^l(i)\bar{\mathbf{b}}(n-i) \quad (1)$$

where L_j is the length of the jth user's channel impulse response and is related to the length of $g_j^l(k)$ and the delay d_j^l. $L_h \triangleq \max_{1\le j\le J} L_j$, $\mathbf{h}_j^l(i) \triangleq [h_j^l(i\,L_c - d_j^l), \cdots, h_j^l(i\,L_c + L_c - 1 - d_j^l)]^T$, $\mathbf{H}^l(i) \triangleq \left[\mathbf{h}_1^l(i), \cdots, \mathbf{h}_J^l(i)\right]$, $\bar{\mathbf{b}}(i) \triangleq [b_1(i), \cdots, b_J(i)]^T$. Let $\mathbf{x}^H(n) = \left[\ \mathbf{x}^{1H}(n) \quad \cdots \quad \mathbf{x}^{PH}(n)\ \right]$. By stacking up M successive $\mathbf{x}(n)$ vectors of the received data, the discrete time signal model for the dispersive CDMA channel observed in additive white Gaussian noise can be represented as follows

$$\mathcal{X}(n) = \mathcal{H}\mathbf{b}(n) + \mathbf{v}(n) \quad (2)$$

where $\mathcal{X}(n) \triangleq \left[\mathbf{x}^H(n), \cdots, \mathbf{x}^H(n+M-1)\right]$,

$$\mathcal{H} = \begin{bmatrix} \mathbf{H}(L_h-1) & \cdots & \mathbf{H}(0) & \cdots & \mathbf{0} \\ \vdots & \ddots & & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{H}(L_h-1) & \cdots & \mathbf{H}(0) \end{bmatrix}, \ \mathbf{H}(i) =$$

$[\mathbf{H}^{1H}(i), \cdots, \mathbf{H}^{PH}(i)]^H$, $\mathbf{b}^H(n) = [\bar{\mathbf{b}}^H(n - L_h + 1), \cdots, \bar{\mathbf{b}}^H(n + M - 1)]$ and $\mathbf{v}(n)$ the noise vector and defined in a manner similar to $\mathcal{X}(n)$. The dimension of the matrix $\mathcal{H}$ is $PML_c \times J(M +$

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$L_h - 1)$. Since the channel effect on received energy can always be incorporated into a diagonal amplitude matrix $\mathbf{A}$, without loss of generality, we assume the columns of the channel matrix $\mathcal{H}$ are all *normalized* in the following sections. Then (2) can be rewritten as

$$\mathcal{X}(n) = \mathcal{H}\mathbf{A}\mathbf{b}(n) + \mathbf{v}(n) \tag{3}$$

3. RANK OF CHANNEL MATRIX FOR SINGLE AND MULTIPLE RECEIVER ANTENNA

The full column rank assumption of the channel matrix $\mathcal{H}$ is essential in many second order statistics (SOS) based blind identification and equalization algorithms such as the subspace method [6] and the linear prediction method [7]. Therefore, it is interesting to investigate how the receive antenna arrays may affect the rank of the channel matrix. It has recently been shown that the full column rank assumption of $\mathcal{H}$ could be violated in single antenna CDMA systems [1]. There is a trivial case and other non-trivial cases considered in [1]. The trivial case is, if some user channels have short delay spread $L_j < L_h$, then $\mathbf{H}(l)$ contains zero columns for $l > L_j$. However, since these zero columns have no impact on the received signal vector $\chi_M(n)$ (one can always delete those zero columns to form a narrower channel matrix) [1], in the following, we assume that there is no zero column in $\mathcal{H}$ and consider only the non-trivial cases. We have the following proposition.

Proposition 1 *Let $\mathcal{H}_s$ represent the channel matrix in a CDMA system with a single receiver antenna, i.e., $P = 1$. Assume that the probability for columns of $\mathcal{H}_s$ to be linearly dependent is p_s. The probability p_m for columns of the channel matrix $\mathcal{H}$ to be linearly dependent for CDMA systems with multiple receiver antennas is $p_m = p_1 p_s^P$, where p_1 is the probability for having the same linear dependence in columns of $\mathcal{H}_s$ for all antennas across the array.*

Proof: The channel matrix $\mathcal{H}$ can be divided into $M + L_h - 1$ block columns from left to right, each block having J_c columns. Due to the Toeplitz structure of the channel matrix, it is straightforward to show that linear dependence can only occur within each block of J columns of $\mathcal{H}$, and can not occur among different blocks due to the existence of the zero rows. It has been shown in [1] that in single receiver antenna CDMA systems, the channel matrix $\mathcal{H}_s$ can be rank deficient, which is a special case of $\mathcal{H}$ in (3) when $P = 1$. Without loss of generality, we assume that the probability for columns of $\mathcal{H}_s$ to be linearly dependent is p_s for single receiver antenna CDMA systems ($P = 1$). By carefully examining the structure of $\mathbf{H}(l)$, in CDMA with receiver antenna arrays ($P > 1$), the columns of $\mathcal{H}$ to be linearly dependent requires satisfaction of both of the following two conditions. 1) on each lth antenna element, the columns $\xi_j^l \triangleq \left[\mathbf{0}^T, \mathbf{h}_j^{lT}(0), \cdots, \mathbf{h}_j^{lT}(L_h - 1), \mathbf{0}^T\right]^T$ (existence of the zero entries and the number of $\mathbf{h}_i^l(n)$ vectors may vary regarding different blocks of columns under consideration), $j = 1, \cdots, J$, should be linearly dependent (with probability p_s); and 2) the same linear relationship should be satisfied among different antenna elements (denote this probability to be p_1). E.g., if $\sum_{j=1}^{J} k_j\, \xi_j^1 = \mathbf{0}$, then $\sum_{j=1}^{J} k_j\, \xi_j^l = \mathbf{0}$, $\forall l \neq 1$, where k_j are constants. Therefore, in CDMA systems with receiver antenna arrays, the probability for the columns in $\mathcal{H}$ to be linearly dependent is $p_m = p_1 p_s^P$. It is obvious that $p_m = p_1 p_s^P \leq p_s$. The equality holds in single antenna CDMA systems (i.e., $P = 1$ and $p_1 = 1$). In most cases, $p_m << p_s$. □

This Proposition implies that, for CDMA with receiver antenna arrays, the probability for $\mathcal{H}$ to be rank deficient is much lower than that of in single receiver antenna CDMA systems. Furthermore, one can see that this probability decreases exponentially when the number of antenna elements increases. This phenomena is not surprising since more spatial diversity is exploited when antenna elements increase. Finally, it is straightforward to see that, when the number of columns in the channel matrix (i.e., system load) increases, the probability that the channel matrix is rank deficient also increases in both CDMA systems with single receiver antenna and with receiver antenna arrays (both systems have the same number of columns in their channel matrix). These results indicate that it is much safer to use the SOS based blind detection and equalization algorithms on receivers with multiple antennas than a single antenna.

4. NEAR-FAR RESISTANCE OF MMSE DETECTOR IN CDMA WITH RECEIVER ANTENNA ARRAY

Without loss of generality, we assume that the dth symbol in $\mathbf{b}(n)$ is the desired transmitted symbol of the desired user and simply denote it by $b_d(n)$. $\mathbf{H}_d$ is the dth column in $\mathcal{H}$ corresponding to the desired transmitted symbol $b_d(n)$. Let $\mathcal{I}$ denote the subspace spanned by interference channel vectors $\mathbf{H}_i$, $i \neq d$, where $\mathbf{H}_i$ denotes the ith column in $\mathcal{H}$. It is easy to show that $\mathcal{C}(\tilde{\mathcal{H}}) = \mathcal{I}$, where $\mathcal{C}(\cdot)$ represents the column subspace and $\tilde{\mathcal{H}}$ is the matrix obtained by deleting the dth column $\mathbf{H}_d$ from $\mathcal{H}$. We have the following proposition.

Proposition 2 *The near-far resistance of the MMSE detector is $\bar{\eta}_d = \frac{1}{(\mathcal{H}^H\mathcal{H})^{+}_{(d,d)}}$, where "+" represents Moore-Penrose pseudoinverse, and the subscript (d, d) denotes choosing the element at the dth row and the dth column.*

Brief Proof: Denote $\mathbf{M} \triangleq \mathcal{H}^H\mathcal{H}$, $\mathbf{R}_d \triangleq \tilde{\mathcal{H}}^H\tilde{\mathcal{H}}$ and $\mathbf{r}_d \triangleq \tilde{\mathcal{H}}^H\mathbf{H}_d$. Note that $\mathbf{M}$ is singular if $\mathcal{H}$ is rank deficient. The definition of the near-far resistance of the MMSE detector is presented in [2] based on a geometric point of view. That is the near-far resistance of the MMSE detector is equal to square norm of the projection of $\mathbf{H}_d$ onto the orthogonal complement of the space $\mathcal{I}$ (Note that $\mathbf{H}_d$ and the columns in $\mathcal{I}$ should be all normalized [2].). The above statement can be written in the mathematical expression as

$$\begin{aligned}
\bar{\eta}_d &= \left\| \left(\mathbf{I} - \tilde{\mathcal{H}} \left(\tilde{\mathcal{H}}^H \tilde{\mathcal{H}} \right)^{-} \tilde{\mathcal{H}}^H \right) \mathbf{H}_d \right\|^2 \\
&= \mathbf{H}_d^H \left(\mathbf{I} - \tilde{\mathcal{H}} \left(\tilde{\mathcal{H}}^H \tilde{\mathcal{H}} \right)^{-} \tilde{\mathcal{H}}^H \right) \mathbf{H}_d \\
&= 1 - \mathbf{H}_d^H\, \tilde{\mathcal{H}} \left(\tilde{\mathcal{H}}^H \tilde{\mathcal{H}} \right)^{-} \tilde{\mathcal{H}}^H\, \mathbf{H}_d = 1 - \mathbf{r}_d^H\, \mathbf{R}_d^{-}\, \mathbf{r}_d
\end{aligned} \tag{4}$$

where $(\cdot)^-$ denotes the generalized inverse. In (4) we have used the facts that the projection matrix $\mathbf{I} - \tilde{\mathcal{H}} \left(\tilde{\mathcal{H}}^H \tilde{\mathcal{H}} \right)^{-} \tilde{\mathcal{H}}^H$ is idempotent and $\mathbf{H}_d$ is normalized.

Since in literature the expression of the near-far resistance is usually expressed in the form of the crosscorrelation matrix of the signature vector [3] which in our case is $\mathbf{M}$, in order to facilitate comparison with existing work, we need to express (4) in terms of $\mathbf{M}$. By adopting the properties of elementary matrix and the generalized inverse, we can show

$$\left(\mathcal{H}^H\,\mathcal{H}\right)^{-}_{(d,d)} = \frac{1}{1 - \mathbf{r}_d^H\, \mathbf{R}_d^{-}\, \mathbf{r}_d} \tag{5}$$

Refer to [4] for a proof due to lack of space here. Then, based on (4) and (5), we obtain the near-far resistance of the MMSE detector

$$\bar{\eta}_d = \frac{1}{(\mathcal{H}^H\mathcal{H})^{-}_{(d,d)}} \tag{6}$$

Generally speaking, the generalized inverse is not unique. However, it is shown in [5] (pp. 166), the projection matrix $\mathbf{I} - \tilde{\mathcal{H}}(\tilde{\mathcal{H}}^H \tilde{\mathcal{H}})^- \tilde{\mathcal{H}}^H$ is invariant to the choice of the generalized inverse $(\tilde{\mathcal{H}}^H \tilde{\mathcal{H}})^-$. This means that (4) and thus the near-far resistance in (6) is unique. Therefore, we can choose the widely used Moore-Penrose pseudoinverse (one special case of the generalized inverse), which is unique, to replace the generalized inverse in (6). Then Proposition 2 is proved. Note that when $\mathcal{H}$ is of full column rank, matrix $\mathcal{H}^H\mathcal{H}$ is non-singular. It is straightforward to show that the above derivations are still valid and the Moore-Penrose generalized inverse becomes the conventional inverse. Note that (6) is a generalization of the corresponding result in [3] to multipath channels, singular crosscorrelation matrix $\mathcal{H}^H\mathcal{H}$, and receiver antenna array scenario. □

5. NEAR-FAR RESISTANCE COMPARISON AMONG TRANSMIT/RECEIVE ANTENNA SETTINGS

We now use the results derived in the last section to demonstrate the advantages of spatial diversity on near-far resistance. One of the most promising techniques to achieve higher data rates is space-time (ST) coding which adopts not only multiple receive antennas but also multiple antennas at the transmitter. A simple space-time block coding (STBC) scheme developed by Alamouti [8] has been adopted in several wireless standards such as wideband CDMA (W-CDMA) and CDMA-2000. In this section, we will compare the near-far resistance of the MMSE detector under different antenna configurations. We focus on four different scenarios. 1) one transmitter antenna, one receiver antenna (uncoded); 2) one transmitter antenna, two receiver antennas (uncoded); 3) two transmitter antennas, one receiver antennas (ST coded); 4) two transmitter antennas, two receiver antennas (ST coded). It is assumed that the Alamouti STBC is employed in systems with two transmitter antennas.

The signal model for scenarios 3 and 4 is derived in [9] which turns out to have the same form as (3). From the derivation of Proposition 2, it is straightforward to find that the near-far resistance of the MMSE detector of scenarios 3 and 4 should also have the same form as (6) by substituting $\mathcal{H}$ with the corresponding channel matrix for scenarios 3 and 4 (Actually, the results of Proposition 2 is valid for any CDMA system as long as its signal model is of the same form as (3)).

In order to facilitate fair comparison among different scenarios, we make the following assumption. $\mathbf{AS1}$: the processing gain L_c, the system load J, the smoothing factor M, the distributions of multipath delay spread and asynchronous user delay are the same under different scenarios. Since the dimension of the channel matrix will prove to be useful for our following derivations, we now specify those parameters. Let $\mathcal{H}_i,\ i = 1, \cdots, 4$, denote the channel matrix corresponding to the above four scenarios. Under $\mathbf{AS1}$, the dimensions of the channel matrix under scenarios 1 through 4 are $MN \times J(M+L_h^1-1)$, $2MN \times J(M+L_h^2-1)$, $2MN \times 2J(M+\lceil\frac{L_h^3}{2}\rceil)$ and $4MN \times 2J(M+\lceil\frac{L_h^4}{2}\rceil)$, respectively [7][9], where L_h^i (non-negative integer) is related to the maximum multipath delay spread and the maximum asynchronous user delay of the ith scenario, and is defined as in (2.11)[1] of [7] and in (132) of [9], respectively.

Furthermore, since the channels and asynchronous transmission delays are random in nature, it is more meaningful to compare the statistical average of the near-far resistance rather than a particular random realization. To this end, we need an additional assumption. $\mathbf{AS2}$: Under the ith scenario, assume $\mathbf{h}_d^i$ (the vector in $\mathcal{H}_i$ which is corresponding to the desired transmitted symbol of the desired user) is a random vector with a probability density function $\mathcal{N}_c(\mathbf{0}_{row(\mathcal{H}_i)}, \frac{1}{row(\mathcal{H}_i)}\mathbf{I}_{row(\mathcal{H}_i)})$ and is statistically independent of the interference subspace $\mathcal{I}$ (Since it won't affect the derivation, here $\mathcal{I}$ is a general expression which includes all 4 scenarios.), where $\mathcal{N}_c$ represents the complex normal distribution, $row(\mathcal{H}_i)$ denotes the number of rows in $\mathcal{H}_i$, $\mathbf{0}_{row(\mathcal{H}_i)}$ represents the $row(\mathcal{H}_i) \times 1$ zero vector, and $\mathbf{I}_{row(\mathcal{H}_i)}$ represents the $row(\mathcal{H}_i) \times row(\mathcal{H}_i)$ identity matrix. The fact that the variance is $1/row(\mathcal{H}_i)$ is because $\mathbf{h}_d^i$ is normalized.

In Proposition 1, we have shown that the probability for rank of the channel matrix under spatial diversity to be deficient is very low. Therefore, in the following we assume $\mathbf{AS3}$: the channel matrix $\mathcal{H}_i,\ i = 1, \cdots, 4$ is of full column rank. We have the following proposition.

Proposition 3 *Denote $\bar{\bar{\eta}}_d^i,\ i = 1, \cdots, 4$, as the expectation of the near-far resistance of the MMSE detector under the ith scenario. Then under $\mathbf{AS1}$, $\mathbf{AS2}$ and $\mathbf{AS3}$ for each and every above scenarios, we have 1) $\bar{\bar{\eta}}_d^3 < \bar{\bar{\eta}}_d^2$; 2) If $L_h^i \geq 2,\ i = 3, 4$, then $\bar{\bar{\eta}}_d^1 < \bar{\bar{\eta}}_d^3$, $\bar{\bar{\eta}}_d^2 < \bar{\bar{\eta}}_d^4$; otherwise $\bar{\bar{\eta}}_d^1 \approx \bar{\bar{\eta}}_d^3$, $\bar{\bar{\eta}}_d^2 \approx \bar{\bar{\eta}}_d^4$. Based on 1 and 2, we have $\bar{\bar{\eta}}_d^1 \leq \bar{\bar{\eta}}_d^3 < \bar{\bar{\eta}}_d^2 \leq \bar{\bar{\eta}}_d^4$.*

Proof: Under $\mathbf{AS1}$, starting from (4) and adopting the similar derivations in [3], the conditional expectation of $\bar{\eta}_d^i$, conditioning on the interference subspace $\mathcal{I}$, is then given by

$$\begin{aligned}
E[\bar{\eta}_d^i\,|\mathcal{I}] &= 1 - E[\mathbf{r}_d^{iH}(\mathbf{R}_d^i)^{-1}\mathbf{r}_d^i|\mathcal{I}] = 1 - E[tr\{\mathbf{r}_d^{iH}(\mathbf{R}_d^i)^{-1}\mathbf{r}_d^i\}|\mathcal{I}] \\
&= 1 - E[tr\{\mathbf{r}_d^i\mathbf{r}_d^{iH}(\mathbf{R}_d^i)^{-1}\}\,|\mathcal{I}] \\
&= 1 - E[tr\{\tilde{\mathcal{H}}_i^H\mathbf{h}_d^i\mathbf{h}_d^{iH}\tilde{\mathcal{H}}_i(\mathbf{R}_d^i)^{-1}\}\,|\mathcal{I}] \\
&= 1 - E[tr\{\mathbf{h}_d^i\mathbf{h}_d^{iH}\tilde{\mathcal{H}}_i(\mathbf{R}_d^i)^{-1}\tilde{\mathcal{H}}_i^H\}\,|\mathcal{I}] \\
&= 1 - tr\{E[\mathbf{h}_d^i\mathbf{h}_d^{iH}\,|\mathcal{I}]\tilde{\mathcal{H}}_i(\mathbf{R}_d^i)^{-1}\tilde{\mathcal{H}}_i^H\} \\
&= 1 - \frac{1}{row(\mathcal{H}_i)}tr\{\tilde{\mathcal{H}}_i^H\tilde{\mathcal{H}}_i(\mathbf{R}_d^i)^{-1}\} \\
&= 1 - \frac{1}{row(\mathcal{H}_i)}tr\{\mathbf{R}_d^i(\mathbf{R}_d^i)^{-1}\} = 1 - \frac{col(\mathcal{H}_i) - 1}{row(\mathcal{H}_i)} \qquad (7)
\end{aligned}$$

where $tr(\cdot)$ represents the trace of a matrix, $\tilde{\mathcal{H}}_i$, $\mathbf{r}_d^i$ and $\mathbf{R}_d^i$ are defined similarly as in section 4 for the ith scenario. The sixth equality is based on the property of conditional expectation and the seventh equality is based on the fact that $E[\mathbf{h}_d^i\mathbf{h}_d^{iH}\,|\mathcal{I}] = E[\mathbf{h}_d^i\mathbf{h}_d^{iH}] = \frac{1}{row(\mathcal{H}_i)}\mathbf{I}$ due to $\mathbf{AS2}$. Note (7) is an extension from the similar result shown in [3] under AWGN channel case for DS-CDMA. However, in order to obtain (7) in the asynchronous multipath channel case, in $\mathbf{AS2}$ we assume a much more restrictive assumption ($\mathbf{h}_d^i$ and vectors in $\mathcal{I}$ are the combination of multipath channel and spreading code while in [3] the statistical independence assumption is only between different spreading codes). Based on (7) and $\mathbf{AS1}$, it is straightforward to show that

$$E[\bar{\eta}_d^1\,|\mathcal{I}] = 1 - \frac{J(M + L_h^1 - 1) - 1}{ML_c} \qquad (8)$$

$$E[\bar{\eta}_d^2\,|\mathcal{I}] = 1 - \frac{J(M + L_h^2 - 1) - 1}{2ML_c} \qquad (9)$$

$$E[\bar{\eta}_d^3\,|\mathcal{I}] = 1 - \frac{2J(M + \lceil\frac{L_h^3}{2}\rceil) - 1}{2ML_c} \qquad (10)$$

$$E[\bar{\eta}_d^4\,|\mathcal{I}] = 1 - \frac{2J(M + \lceil\frac{L_h^4}{2}\rceil) - 1}{4ML_c} \qquad (11)$$

[1] There is a notation error (i.e., the floor operation should be replaced by the ceiling operation) in (2.11) of [7].

Under $\mathbf{AS1}$, J, M, L_c, the maximum multipath delay spread and the maximum asynchronous user delay are the same under different scenarios for each realization of $\mathcal{I}$ (these parameters in general may be different from one realization of $\mathcal{I}$ to another). By carefully examining (2.11) and (9), (2.17) and (12) of [7] and [9] respectively, it is straightforward to show that $L_h^1 = L_h^2 = L_h^3 + 1 = L_h^4 + 1$ for each realization of $\mathcal{I}$. Since $J(M + L_h^2 - 1) = JM + JL_h^3 < 2JM + 2J\lceil \frac{L_h^3}{2} \rceil$ for each realization of $\mathcal{I}$ where we have used the fact that $\gamma \leq 2\left\lceil \frac{\gamma}{2} \right\rceil$ when γ is non-negative integer, by comparing (9) and (10), we have $E[\bar{\eta}_d^3 \,|\mathcal{I}] < E[\bar{\eta}_d^2 \,|\mathcal{I}]$ for each realization of $\mathcal{I}$.

When $L_h^i \geq 2$, $i = 3, 4$, by comparing (8) and (10), (9) and (11), we obtain $E[\bar{\eta}_d^1 \,|\mathcal{I}] < E[\bar{\eta}_d^3 \,|\mathcal{I}]$ and $E[\bar{\eta}_d^2 \,|\mathcal{I}] < E[\bar{\eta}_d^4 \,|\mathcal{I}]$ for each realization of $\mathcal{I}$, where we have used the fact that $\left\lceil \frac{\gamma}{2} \right\rceil - 1/(2J) < \gamma - 1/J$ when $\gamma \geq 2$ is an integer. When $L_h^i = 0, 1$, $i = 3, 4$, $E[\bar{\eta}_d^1 \,|\mathcal{I}] - E[\bar{\eta}_d^3 \,|\mathcal{I}] = 1/(2ML_c)$ and $E[\bar{\eta}_d^2 \,|\mathcal{I}] - E[\bar{\eta}_d^4 \,|\mathcal{I}] = 1/(4ML_c)$ for each realization of $\mathcal{I}$. Since $1/(2ML_c)$ and $1/(4ML_c)$ are negligible for values of M and N in practical systems, $E[\bar{\eta}_d^1 \,|\mathcal{I}] \approx E[\bar{\eta}_d^3 \,|\mathcal{I}]$ and $E[\bar{\eta}_d^2 \,|\mathcal{I}] \approx E[\bar{\eta}_d^4 \,|\mathcal{I}]$ for each realization of $\mathcal{I}$.

Since $\bar{\bar{\eta}}_d^i = E_{\mathcal{I}}[E[\bar{\eta}_d^i \,|\mathcal{I}]]$, $i = 1, \cdots, 4$, the claims in Proposition 3 are proven. □

Remarks: 1) Transmitter and receiver diversity does have benefits on near-far resistance compared with systems without those diversity (i.e., $\bar{\bar{\eta}}_d^1 \leq \bar{\bar{\eta}}_d^3$, $\bar{\bar{\eta}}_d^1 < \bar{\bar{\eta}}_d^2$). Furthermore, receiver diversity has more impact on the near-far resistance (i.e., $\bar{\bar{\eta}}_d^3 < \bar{\bar{\eta}}_d^2$). 2) Although Proposition 3 are derived based on Alamouti STBC for up to two-transmit/receive antenna configurations, these results can be extended to any STBC for any transmit/receive antenna configurations as long as the signal model has the same form as in (3).

6. SIMULATIONS

In this section, simulations are conducted to verify the theoretical findings. Gold sequence of length L_c is employed as user spreading sequences. The multipath channels for each user and each transmit-receive pair have N_p paths with a total delay spread (including the user transmission delay) of two symbol durations, and all multipaths have mutually independent delays uniformly distributed over two symbol intervals. All N_p multipath amplitudes are mutually independent, complex Gaussain with zero-mean and unit variance. The spreading sequences and multipath channels for each user are randomly generated in each of the Monte Carlo run (i.e., they are different in different runs).

Example 1: Rank of the channel matrix

In this simulation, we investigate how spatial diversity may affect the probability for channel matrix to be column rank deficient. $L_c = 31$ and $N_p = 12$. System load varies from 5 users to 20 users. The smoothing factor is 5. p_s and p_m are calculated after averaging over 100000 Monte Carlo runs. The user channels, spreading codes, asynchronous delays were randomly generated in each of the 100000 Monte Carlo runs. Zero columns are excluded from calculating p_s and p_m. The simulation results are presented in Table 1. From the table, it can be seen that the probability for $\mathcal{H}$ to be rank deficient decreases rapidly when the number of receiver antennae increases. Furthermore, the probability for $\mathcal{H}$ to be rank deficient increases when the system load increases. It is worth to point out that, from Table 1, we can find that most of the SOS blind identification and equalization algorithms such as the subspace method may fail with high probability under single receiver antenna CDMA systems. However, those SOS algorithms are much safer to use in CDMA systems with receiver antenna arrays even the number of receiver antenna is 2.

	J=5	J=10	J=15	J=20
P=1	0.0107	0.0528	0.1428	0.3002
P=2	0.00003	0.00021	0.00087	0.0014
P=4	$< 10^{-5}$	$< 10^{-5}$	$< 10^{-5}$	$< 10^{-5}$

Table 1. Probability for channel matrix (after deleting all zero columns) to be column rank deficient under different number of receiver antennas and system load

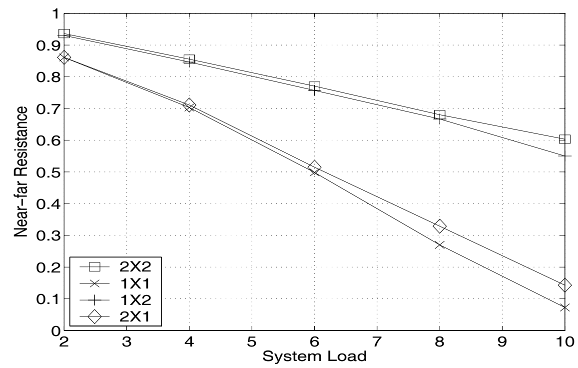

Fig. 1. Near-far Resistance Comparison Among Different transmit/receive antenna Settings.

Example 2: Near-far resistance comparison among different antenna configurations

In this simulation, we compare the theoretical near-far resistance of MMSE detector among different space-time settings. $L_c = 15$ and $N_p = 6$. The smoothing factor $M = 3$. All near-far resistances are calculated by averaging 2000 Monte Carlo runs. The result is presented in Fig. 1. It can be seen from Fig. 1 that the simulation result has a good agreement with the theoretical findings in Proposition 3.

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