TEMPORAL DIVERSITY ASSISTED BLIND CHANNEL ESTIMATION FOR DOWNLINK LONG-CODE CDMA SYSTEMS

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ABSTRACT

In CDMA downlink, subspace channel estimation can not be applied directly since long spreading codes destroy the rank condition of the received data covariance matrix. However, after introducing temporal diversity in the transmitted chip sequence for all users, subspace technique can be applied to estimate channel. Detection of the desired user's signal is then performed by a channel equalizer cascaded with the desired user's code despreader. Analytical channel estimation mean-square-error is derived based on perturbation theory. Experimental comparison with our previously proposed spatial diversity method shows superior performance of the proposed scheme.

1. INTRODUCTION

Direct sequence (DS) code division multiple access (CDMA) technology has become an appealing solution to support emerging multirate multiuser communications. Despite various advantages, adopted long spreading codes inevitably destroy cyclostationarity of CDMA signals, making many of the existing channel estimation and detection approaches for short code CDMA systems not directly applicable. Study on channel estimation and detection techniques for long code CDMA systems has received considerable attention in recent years. Given transmitted pilot symbols of all users, least squares (LS) fitting or iterative maximum likelihood (ML) approaches are proposed for symbol detection in [1], [2] in the absence of channel state information (ISI). Semi-blind channel estimation solutions via subspace based data projection for downlink are also derived [3]. A blind uplink channel estimation method using correlation matching techniques is proposed in [4]. Blind downlink channel estimation methods have been reported by applying subspace techniques [5], [6]. Under the assumption of known CSI, symbol-level and chip-level adaptive MMSE interference suppression and channel equalization schemes have also appeared [7], [8].

In CDMA downlink, user specific short Hadamard codes combined with the base station's long codes are used to spread users' signal spectrum, resulting in orthogonal transmitted chip sequences for different users. However, this orthogonality is destroyed at the receiver due to multipath propagation. Therefore, channel equalization is necessary to restore such orthogonality [9] before code despreading is applied to detect the desired user's information sequence. Most existing channel equalization methods assume perfect CSI, while only focusing on signal detection and performance evaluation. Although [5] considers blind channel estimation, it requires spreading codes of other users which may not be accessible by a particular mobile user. [6] employs spatial diversity to create multiple subchannels and thus improve the rank condition of the channel matrix, rendering the use of subspace technique for direct channel estimation. However, spatial diversity introduces more channel parameters to be estimated, resulting in degraded performance.

In this paper, we propose to deploy temporal diversity to assist blind channel estimation. By retransmitting partial chip sequence, or more generally, precoding the chip sequence to introduce temporal diversity in the transmitted data, rank condition on composite channel matrix including precoding effect is improved. As a result, noise subspace is created in the covariance of the received data vector. Subspace technique is then applied for channel estimation. The covariance and mean-square-error (MSE) of the channel estimate are derived in closed forms and verified by simulation examples. Comparison with our previously proposed spatial diversity assisted channel estimation method is also performed by simulation.

2. CONVENTIONAL CDMA DOWNLINK WITH LONG CODES

Consider a base station communicating with J mobile stations in a CDMA system. The jth user's aperiodic spreading codes $c_{j,n}(k)$ ($k = 0, \ldots, P-1$), which are the combination of its Hadamard codes and base station's long codes, is used to spread bit $w_j(n)$. Let the chip sequence be transmitted through a common FIR channel with unknown coefficients $g(n)$. Suppose the channel is casual with order q (i.e., $g(n) = 0$ for $n < 0$ or $n > q$). Then the chip-rate discrete-time signal is a superposition of signals from

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J users corrupted by noise [5]

$$\begin{aligned} y(n) &= \sum_{j=1}^{J}\sum_{l=0}^{q} g(l)s_j(n-l) + v(n), \\ s_j(n) &= \sum_{k=-\infty}^{\infty} w_j(k)c_{j,k}(n-kP) \end{aligned} \tag{1}$$

where $v(n)$ is additive white Gaussian noise (AWGN) with variance σ_v^2. If we collect P chip rate samples in a big vector as $\bar{\boldsymbol{y}}_n \triangleq [y(nP), \cdots, y(nP+P-1)]^T$, then

$$\bar{\boldsymbol{y}}_n = \bar{\mathcal{G}}[\boldsymbol{b}_{n-1}^T(P-q+1:P),\ \boldsymbol{b}_n^T]^T \tag{2}$$

where $\bar{\mathcal{G}}$ is a $P \times (P+q)$ Toeplitz matrix whose first row is given by $[g(0), \ldots, g(q), \mathbf{0}]$, $\boldsymbol{b}_n \triangleq \sum_{j=1}^{J} \boldsymbol{c}_{j,n} w_j(n)$ is a sum chip sequence associated with the nth symbol from all users, $\boldsymbol{c}_{j,n} = [c_{j,n}(0), \cdots, c_{j,n}(P-1)]^T$, and $\boldsymbol{b}_{n-1}(P-q+1:P)$ contains the last q elements of $\boldsymbol{b}_{n-1}$.

Clearly, direct subspace channel estimation based on (2) is impossible, since $\bar{\mathcal{G}}$ has no null column space. In [6], we have shown that employing spatial diversity with appropriate number of subchannels makes $\bar{\mathcal{G}}$ a block Toeplitz and tall matrix. Consequently, subspace approach can be applied for channel estimation. In this paper, we will solve this problem from a different perspective, which modifies the conventional CDMA scheme to include temporal diversity in the transmitted chip sequence, as shown next.

3. TEMPORAL DIVERSITY ASSISTED TRANSMISSION SCHEME

Let's consider the following communication scenario which mimics OFDM modulation [10]. Each user, after transmitting its nth symbol's spread chip sequence, will retransmit the first ν chips but in reverse order. Then in the nth symbol period, the transmitted sum chip sequence from all users becomes $\check{\boldsymbol{b}}_n \triangleq [\boldsymbol{b}_n,\ \boldsymbol{b}_n(\nu-1:0)]$. After multipath propagation, the new received data vector spanning $P+\nu$ chips can be shown to be

$$\begin{aligned} \check{\boldsymbol{y}}_n &\triangleq [y(n(P+\nu)), \cdots, y(n(P+\nu)+(P+\nu)-1)]^T \\ &= \check{\mathcal{G}}[\check{\boldsymbol{b}}_{n-1}^T(P+\nu-q+1:P+\nu),\ \check{\boldsymbol{b}}_n^T]^T + \check{\boldsymbol{v}}_n \end{aligned} \tag{3}$$

where $\check{\mathcal{G}}$ with dimension of $(P+\nu) \times (P+\nu+q)$ is still a Toeplitz matrix structured as $\bar{\mathcal{G}}$, and $\check{\boldsymbol{v}}_n$ is AWGN. If we focus on the partial data vector free of intersymbol interference (ISI), then we have

$$\boldsymbol{y}_n \triangleq \check{\boldsymbol{y}}_n(q+1:P+\nu) = \boldsymbol{\Gamma}\mathcal{G}\check{\boldsymbol{b}}_n + \boldsymbol{v}_n \tag{4}$$

where $\boldsymbol{\Gamma} \triangleq [\mathbf{0}_{(P+\nu-q)\times q},\ \boldsymbol{I}_{P+\nu-q}]$ is a selection matrix to discard the first q elements of $\check{\boldsymbol{y}}_n$, $\mathcal{G} \triangleq \check{\mathcal{G}}(:, q+1:P+\nu+q)$ contains the last $P+\nu$ columns of $\check{\mathcal{G}}$. By properly choosing ν, existence of $\boldsymbol{y}_n$ can be guaranteed irrespective of channel order q. Considering that the last ν elements of $\check{\boldsymbol{b}}_n$ are the repetition of the first ν chips of $\check{\boldsymbol{b}}_n$, one can rewrite $\boldsymbol{y}_n$ as the following,

$$\boldsymbol{y}_n = \boldsymbol{\Gamma}\check{\mathcal{G}}\boldsymbol{b}_n + \boldsymbol{v}_n \tag{5}$$

where $\check{\mathcal{G}}$ is obtained by removing the last ν columns of $\mathcal{G}$ and add them respectively into its first ν columns in reverse order. These column operations can be expressed by $\check{\mathcal{G}} = \mathcal{G}\boldsymbol{\Psi}$. Thus (5) becomes

$$\boldsymbol{y}_n = \boldsymbol{\Gamma}\mathcal{G}\boldsymbol{\Psi}\boldsymbol{b}_n + \boldsymbol{v}_n \tag{6}$$

where $\boldsymbol{\Psi}$ now can be regarded as a precoding matrix. After this restructure, $\boldsymbol{\Gamma}\check{\mathcal{G}}$ becomes a tall matrix with dimension $(P+\nu-q) \times P$ if we choose $\nu > q$, and it has full column rank under some channel condition. In this way, signal subspace and noise subspace are created in data covariance matrix $\boldsymbol{R} \triangleq E\{\boldsymbol{y}_n\boldsymbol{y}_n^H\}$.

In a more general way, we can apply an arbitrary precoding matrix $\boldsymbol{\Psi}$ of dimension $P_1 \times P$ to the conventional CDMA spread sequence to introduce temporal diversity in the transmitted chips. The data model for the general case still follows (6). Taking $P_1 > P+q$, the input/output transformation matrix $\boldsymbol{\Gamma}\mathcal{G}\boldsymbol{\Psi}$ is a tall matrix with dimension of $(P_1-q) \times P$. Clearly, (6) is a special case with $P_1 = P+\nu$. $\boldsymbol{\Psi} = [\boldsymbol{I}_P,\ [\mathcal{I}_{P_1-P},\ \mathbf{0}]^T]^T$ where $\mathcal{I}$ denotes anti-diagonal identity matrix. Next we turn to channel estimation based on model (6).

4. SUBSPACE CHANNEL ESTIMATION AND MULTIUSER DETECTION

4.1. Subspace Channel Estimation

It is observed that channel parameters are embedded in $\mathcal{G}$. For convenience, define a channel vector $\boldsymbol{g} = [g(0), \ldots, g(q)]^T$ and $\boldsymbol{H} = \boldsymbol{\Gamma}\mathcal{G}\boldsymbol{\Psi}$ in (6). Since $\mathcal{G}$ is a Toeplitz matrix with first column $[(\mathcal{I}_{q+1}\boldsymbol{g})^T, \mathbf{0}]^T$, we can express it by $\boldsymbol{B}(\boldsymbol{I}_{P_1} \otimes (\mathcal{I}_{q+1}\boldsymbol{g}))$ where $\boldsymbol{B} = [\boldsymbol{J}^0\boldsymbol{A}, \ldots, \boldsymbol{J}^{P_1-1}\boldsymbol{A}]$, $\boldsymbol{J}$ is a shifting matrix with all elements of the first sub-diagonal below the main diagonal to be ones, $\boldsymbol{A} = [\boldsymbol{I}_{q+1}, \mathbf{0}]$, $\otimes$ stands for Kronecker product. After expressing $\boldsymbol{\Psi}$ and $\boldsymbol{H}$ into columns as $[\boldsymbol{\xi}_0, \ldots, \boldsymbol{\xi}_{P_1-1}]$ and $[\boldsymbol{h}_0, \ldots, \boldsymbol{h}_{P_1-1}]$ respectively, and applying properties of $\otimes$, one can verify that $\boldsymbol{h}_i = \boldsymbol{A}_i\boldsymbol{g}$ where $\boldsymbol{A}_i = \boldsymbol{\Gamma}\boldsymbol{B}(\boldsymbol{\xi}_i \otimes \boldsymbol{I}_{P_1})\mathcal{I}_{q+1}$. Consequently, the covariance matrix is obtained as

$$\boldsymbol{R} = \rho\boldsymbol{H}\boldsymbol{H}^H + \sigma_v^2\boldsymbol{I} = \rho\sum_{i=0}^{P_1-1}\boldsymbol{A}_i\boldsymbol{g}\boldsymbol{g}^H\boldsymbol{A}_i^H + \sigma_v^2\boldsymbol{I} \tag{7}$$

where $\rho = J\sigma_c^2\sigma_w^2$, $\sigma_c^2 = E\{|c_{j,n}(i)|^2\}$, σ_w^2 is the symbol power. Since $\boldsymbol{A}_i\boldsymbol{g}$ for $i = 0, \ldots, P_1-1$ spans the signal subspace, $\boldsymbol{U}_n^H\boldsymbol{A}_i\boldsymbol{g} = 0$ for $i = 0, \ldots, P_1-1$ where $\boldsymbol{U}_n$ denotes the noise subspace of $\boldsymbol{R}$. As a result, the channel vector is the null vector of the matrix $\boldsymbol{X} \triangleq \sum_i \boldsymbol{A}_i^H\boldsymbol{U}_n\boldsymbol{U}_n^H\boldsymbol{A}_i$.

We thus propose to minimize it and obtain the corresponding channel estimation method

$$\boldsymbol{g} = \arg \min_{||\boldsymbol{\alpha}||=1} \boldsymbol{\alpha}^H \boldsymbol{X} \boldsymbol{\alpha}. \tag{8}$$

Eq. (8) suggests that $\boldsymbol{g}$ is the eigenvector of $\boldsymbol{X}$ corresponding to its minimum eigenvalue.

4.2. Multiuser Detection

Once the common channel vector is estimated by (8), zero forcing (ZF) and MMSE equalizers can be constructed respectively to detect the sum chip sequence $\boldsymbol{b}_n$ in (6). Then the desired user's codes are used to despread the sum signals, yielding the symbol estimate at each time instant. Suppose user 1 is the desired user. The estimated symbol by ZF detection is given by

$$\widehat{w}_{zf,1}(n) = \boldsymbol{c}_{1,n}^H \boldsymbol{H}^\dagger \boldsymbol{y}_n$$

where † denotes pseudoinverse. Correspondingly, the MMSE detection yields

$$\widehat{w}_{mmse,1}(n) = \boldsymbol{c}_{1,n}^H \boldsymbol{H}^H \boldsymbol{R}^{-1} \boldsymbol{y}_n.$$

5. CHANNEL ESTIMATION MEAN-SQUARE-ERROR

When estimated from finite data by sample average $\widehat{\boldsymbol{R}} = \frac{1}{N}\sum_{i=1}^N \boldsymbol{y}_n \boldsymbol{y}_n^H$, the covariance matrix is perturbed from its ideal value, resulting in a perturbation in the channel estimation. We next derive the covariance of the channel estimation error in a similar way to [6]. Let's denote the perturbation by preceding the corresponding quantity by δ, and the perturbed quantity with $\tilde{}$, i.e., $\delta \boldsymbol{g} = \widetilde{\boldsymbol{g}} - \boldsymbol{g}$. Since $\boldsymbol{R}$ is perturbed by $\delta \boldsymbol{R}$, its null space $\boldsymbol{U}_n$ is perturbed by $\delta \boldsymbol{U}_n \approx -\frac{1}{\rho}(\boldsymbol{H}\boldsymbol{H}^H)^\dagger \delta \boldsymbol{R} \boldsymbol{U}_n$ [11], which results in a perturbation to $\boldsymbol{X}$

$$\delta \boldsymbol{X} \approx \sum_{i=0}^{P_1-1} \boldsymbol{A}_i^H (\boldsymbol{U}_n \delta \boldsymbol{U}_n^H + \delta \boldsymbol{U}_n \boldsymbol{U}_n^H) \boldsymbol{A}_i. \tag{9}$$

Due to $\delta \boldsymbol{X}$, $\boldsymbol{g}$ obtained from (8) is perturbed with perturbation $\delta \boldsymbol{g}$ [11]

$$\delta \boldsymbol{g} \approx -\boldsymbol{X}^\dagger \delta \boldsymbol{X} \boldsymbol{g}. \tag{10}$$

After substituting (9) in (10), applying $\delta \boldsymbol{U}_n$ and noticing that $\boldsymbol{U}_n^H \boldsymbol{A}_i \boldsymbol{g} = \boldsymbol{0}$, we obtain the perturbation of channel estimate

$$\delta \boldsymbol{g} \approx \frac{1}{\rho} \sum_{i=-q}^{P-1} \boldsymbol{T}_i \boldsymbol{U}_n^H \delta \boldsymbol{R} \boldsymbol{t}_i \tag{11}$$

where $\boldsymbol{T}_i$ and $\boldsymbol{t}_i$ are deterministic quantities

$$\boldsymbol{T}_i = \boldsymbol{X}^\dagger \boldsymbol{A}_i^H \boldsymbol{U}_n, \quad \boldsymbol{t}_i = (\boldsymbol{H}\boldsymbol{H}^H)^\dagger \boldsymbol{A}_i \boldsymbol{g}.$$

Therefore, the covariance of $\delta \boldsymbol{g}$ becomes

$$\mathrm{Cov}_g \approx \frac{1}{\rho^2} \sum_{i,j} \boldsymbol{T}_i \boldsymbol{U}_n^H E\{\delta \boldsymbol{R} \boldsymbol{t}_i \boldsymbol{t}_j^H \delta \boldsymbol{R}\} \boldsymbol{U}_n \boldsymbol{T}_j^H, \tag{12}$$

and the mean-square-error is equal to the trace of Cov_g. Both quantities depend on the term $E\{\delta \boldsymbol{R} \boldsymbol{t}_i \boldsymbol{t}_j^H \delta \boldsymbol{R}\}$. Applying the result for $E\{\delta \boldsymbol{R} \boldsymbol{t}_i \boldsymbol{t}_j^H \delta \boldsymbol{R}\}$ in [12] and following similar steps as [6], one can verify that (12) reduces to

$$\mathrm{Cov}_g \approx \frac{\sigma_v^2}{N\rho^2} \sum_{i,j} (\boldsymbol{t}_j^H \boldsymbol{R} \boldsymbol{t}_i) \boldsymbol{T}_i \boldsymbol{T}_j^H. \tag{13}$$

Rewriting $\boldsymbol{R} = \rho \boldsymbol{H}\boldsymbol{H}^H + \sigma_v^2 \boldsymbol{U}_s \boldsymbol{U}_s^H + \sigma_v^2 \boldsymbol{U}_n \boldsymbol{U}_n^H$, with $\boldsymbol{U}_s$ as the signal subspace of $\boldsymbol{R}$, we obtain $(\boldsymbol{t}_j^H \boldsymbol{R} \boldsymbol{t}_i) = \rho\delta(i - j) + O(\sigma_v^2)$. Here with a little abuse of the notation, $\delta(.)$ denotes delta function. Using this result, and omitting the σ_v^4 term in (13), we have

$$\mathrm{Cov}_g \approx \frac{\sigma_v^2}{N\rho} \sum_i \boldsymbol{T}_i \boldsymbol{T}_i^H = \frac{\sigma_v^2}{N\rho} \boldsymbol{X}^\dagger. \tag{14}$$

Therefore, channel estimation error is proportional to the noise power and inversely proportional to data length and total transmission power (ρ).

The above analysis is based on the assumption that channel is identifiable from $\boldsymbol{X}$. Channel identifiability condition will be addressed in our future work. Moreover according to (14), the precoding matrix $\boldsymbol{\Psi}$ will affect both $\boldsymbol{A}_i$ and $\boldsymbol{U}_n$ in $\boldsymbol{X}$, and thus affect the channel MSE. The optimal $\boldsymbol{\Psi}$ yielding minimum MSE will also be investigated in our future work.

6. SIMULATION EXAMPLES

In this section, we compare the performance of the proposed channel estimator with the spatial diversity aided channel estimator [6], and also verify our analysis. Both the particular precoding matrix $\boldsymbol{\Psi} = [\boldsymbol{I}_P,\ [\mathcal{I}_{P_1-P},\ \boldsymbol{0}]^T]^T$ and binary random $\boldsymbol{\Psi}$ are considered for the proposed method. They are termed as 'TDM1' and 'TDM2' respectively. The approach in [6] is termed as 'SDM'. The transmitted sequences are drawn from a binary constellation $[-1, 1]$. Each user's short codes are Hadamard codes, which are then multiplied by base station's binary random codes. The system parameters are $P = 16$, $J = 10$ for all methods, $P_1 = 32$ for TDM1 and TDM2, and $M = 2$ for SDM. Moreover, the trace of $\boldsymbol{\Psi}^H \boldsymbol{\Psi}$ is constrained to be P such that the average transmission power remains constant either with or without precoding. The downlink channel is randomly generated with Gaussian distribution, and the channel for TDM1 and TDM2 is assumed to be the same as the first subchannel of SDM. All simulation results are based on 100 realizations.

Fig. 1 illustrates MSE in the presence of 15dB SNR. The channel is assumed to have order 3, and is fixed for all realizations once randomly generated. It is observed for both TDM1 and TDM2 that experimental MSE curves converge to their analytical ones from $N = 500$, verifying our MSE analysis. On the other hand, both TDM1 and TDM2 outperform SDM. To further show the applicability of the proposed scheme, we compare TDM2 and SDM in a more general situation, where channel with order $q = 6$ and the binary matrix $\mathbf{\Psi}$ are both randomly generated for each realization. 500 symbols are used for channel estimation. BER performance is then calculated based on an independent record of 5000 data symbols. The average MSE is plotted in Fig. 2 (a). As expected, TDM2 outperforms SDM, since the former has much fewer channel parameters to estimate. On the other hand, experimental MSE of TDM2 converges to its analytical value at high SNR. Fig. 2 (b) illustrates BER performance of both MMSE and ZF receivers for TDM2 and SDM. All receivers have similar performance at low SNR, while the receivers for TDM2 demonstrate much better BER performance than those of SDM from moderate to high SNR.

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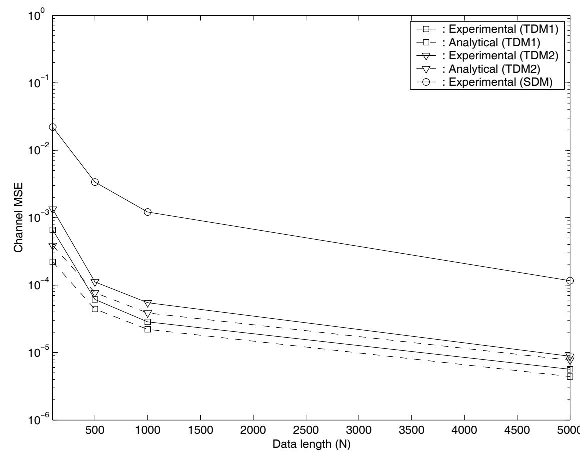

Fig. 1. Channel MSE v.s. N, P=16, q=2, SNR=$15dB$.

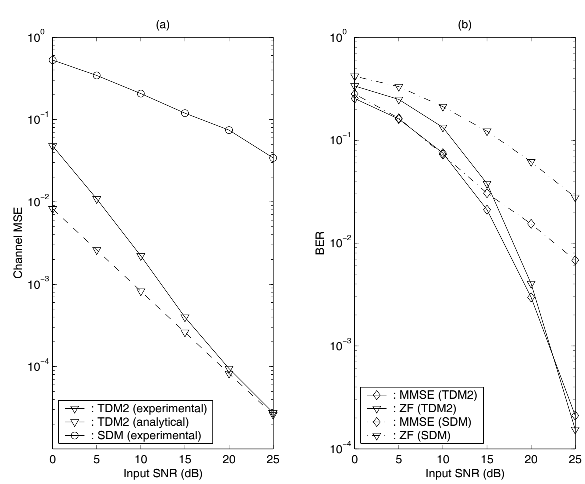

Fig. 2. Effect of SNR, P=16, q=6, N=500.