MULTI-USER SCHEMES USING NONLINEAR TIME-VARYING MODULATION

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ABSTRACT

We propose to optimally reduce multiple access interference in multi-user systems using modulation schemes based on time-varying signals with nonlinear phase function. This minimization is achieved by deriving constraints on the frequency modulation parameters of these signals. The success of the new method over frequency-shift keying signaling is demonstrated in frequency-hopped code division multiple access (FH-CDMA) systems. In addition, we propose to estimate the transmitted symbol information at the receiver using time-varying processing techniques based on unitary versions of the optimal periodogram spectral estimator.

1. INTRODUCTION

Time-varying (TV) signal and system processing has recently been applied successfully in wireless communication systems. Time-frequency representations have been used, for example, for pulse shaping in multicarrier systems [1], for suppressing intentional wideband jamming interference [2], and for wireless channel modeling [3]. In addition, wireless systems have employed linear TV chirp signals for modulation [4–7].

Multiple access interference (MAI) can restrict the number of users that can access a channel at the same time. We have recently shown that a substantial reduction in MAI can be obtained when wideband modulation schemes, instead of narrowband ones, are used in frequency-hopped code division multiple access (FH-CDMA) communication systems. In particular, we have proposed the multi-chirp rate (MCR) modulation scheme for FH-CDMA systems; this scheme identifies each user with a unique linear chirp signal in order to limit interference between users [6,7]. The scheme ensures that the chirp signals make efficient use of the available hop bandwidth such that the resulting MAI is greatly reduced in comparison to the use of frequency-shift keying (FSK) signaling. This unique user representation, together with unique spreading sequences for each user, can successfully estimate the different users information at the receiver.

In this paper, we extend the MCR to investigate the use of TV signals with nonlinear phase in modulation schemes. In particular, we propose to assign each user with a fixed TV waveform signature that is unique for that user by choosing a different frequency modulation (FM) rate parameter for each user. These FM rate parameters are chosen optimally such that the MAI between users is minimized; to achieve this, we derive some simple conditions that the FM rate parameters must satisfy. We also propose matched TV signal transforms that can be used at the receiver to optimally estimate the FM rate information of each user when it is corrupted during transmission.

2. WIDEBAND TIME-VARYING MODULATION

2.1. Multiple Rate Time-Varying Modulation

TV signals are signals that, unlike sinusoids, have frequency content that changes with time. Examples of such signals include most real waveforms such as speech and the nonlinear phase FM (NFM) signals described as [8]:

$$x(t) = \sqrt{|\nu(t)|} e^{j2\pi c_0 \eta(t/t_r)}$$
(1)

where c_0 is the FM rate and $t_r > 0$ is a reference time. The phase of the NFM signal is given by $\eta(t/t_r)$ whose derivative corresponds to the linear or nonlinear instantaneous frequency (IF) $\nu(t) = \frac{1}{t_r} \frac{d}{dt} \eta(t/t_r)$. For example, a linear FM chirp is a TV signal as in (1) with $\eta(t/t_r) =$ $\operatorname{sgn}(t)|t/t_r|^2$ (with $\operatorname{sgn}(t) = u(t) - u(-t)$ where u(t) is the unit step) whereas a hyperbolic FM (HFM) signal has logarithmic phase function $\eta(t/t_r) = \ln(t/t_r), t > 0$.

We propose a modulation scheme that uses the signature waveform

$$s_k(t) = \sqrt{|\nu(t)|} \ e^{j2\pi I_k c_k \eta(t/t_r)} \ p_T(t)$$
(2)

to transmit the information of User k, k = 1, ..., K. Here, $p_T(t) = u(t - \epsilon) - u(t - T - \epsilon)$ where ϵ is a small number (usually zero) that depends on the domain of $\eta(t/t_r)$. The value of I_k depends on the transmitted bit. When bit 1 is transmitted, then the FM rate c_k is positive with $I_k = 1$; to transmit bit 0, the same FM rate c_k is used but made negative by setting $I_k = -1$.

The amplitude modulation in (1) depends on the IF of the NFM. Because of this amplitude modulation, any two infinite duration NFMs can be shown to be orthogonal [7]

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and thus important in wireless communication modulation schemes. The finite duration NFMs in (2) can be useful in multi-user modulation schemes provided we can show that they can optimally minimize MAI. We demonstrate next that we can achieve this by deriving conditions on the FM rate parameters of the NFMs. In particular, we need to derive an expression for the FM rates c_k and c_{k-1} such that the correlation $q_{k,k-1} = \int s_k(t) s_{k-1}^*(t) dt$ between the modulating signals of User k and User (k-1) is minimized. The correlation can be computed (with $t_r = 1$ in (2)) as

$$q_{k,k-1} = \int_{\epsilon}^{T+\epsilon} |\nu(t)| \, e^{j2\pi(c_k - c_{k-1})\eta(t)} \, dt$$

with magnitude

$$|q_{k,k-1}| = |\eta_T| \operatorname{sinc}((c_k - c_{k-1})\eta_T)$$

where $\eta_T = \eta(T+\epsilon) - \eta(\epsilon)$, and $\operatorname{sinc}(x) = \frac{\sin(\pi x)}{(\pi x)}$. In order to minimize the correlation, the FM rates c_k and c_{k-1} can be chosen such that $h_{k,k-1} = (c_k - c_{k-1})\eta_T = l$ for any integer l. We choose

$$\hat{c}_k = (K+1-k)/\eta_T$$
 (3)

as an optimum value for the rate of the FM signal in (2) so that the correlation between Users k and k - 1 is exactly $q_{k,k-1} = 0$. Note that if we choose the FM rates such that $h_{k,k-1} >> 1$, we have a less restrictive condition that can still result in reducing (but not minimizing) the correlation magnitude. In order to ensure that the modulating signals are not aliased, $\hat{c}_k \nu(t_a) \leq f_s/2$ where f_s is the sampling frequency and t_a results in the maximum IF value $\hat{c}_k \nu(t_a)$ which is limited by the signal bandwidth B. Note that this optimal value of c_k can easily be employed in real systems as can be shown when realistic values of the different parameters are chosen.

We call the new modulation scheme multi-NFM rate or MNR. In order to demonstrate it in the time-frequency (TF) plane, we consider HFM signals (multi-hyperbolic FM rate or MHR scheme) for K = 4 users in Figure 1(a). The signal is given in (2) as $s_k(t) = \frac{1}{\sqrt{t}} e^{j2\pi I_k c_k \ln(t/t_r)} p_T(t)$ with $\epsilon = pT_s = p/f_s$. Here, p is a small integer that is chosen based on the chirp rate. Each hyperbolic IF curve in the TF plane corresponds to the transmitted signature waveform of a different user. Note that for the same FM rate, a concave hyperbola represents the transmission of bit 1 by a user whereas a convex hyperbola represents the transmission of bit 0 by the same user.

2.2. MNR FH-CDMA Systems

The MCR scheme using linear FM chirps was successfully used in FH-CDMA systems to reduce MAI in [6, 7]. Note, however, that the optimum chirp rate conditions were not



Fig. 1. (a) Unique HFMs of different users in the TF plane. (b) Performance comparison of coherent MHR, MCR and FSK FH-CDMA over a Rayleigh fading channel, TB = 4.



Fig. 2. MNR FH-CDMA system block diagram.

used at the time and also a set of two chirps was assigned to each user to make efficient use of the available hop bandwidth. Here, we apply the new scheme using NFMs, including chirp signals, but with the optimum condition in (3) satisfied, in FH-CDMA systems. Specifically, the MNR FH-CDMA scheme assigns to each user a unique FM rate (that satisfies condition (3)) and a unique pseudo noise (PN) spreading sequence. When K users occupy the FH-CDMA system (with hop duration T and instantaneous hop bandwidth B), then User k, k = 1, ..., K, can use the NFM signal in (2) with unique FM rate c_k . For example, for the MCR scheme, (2) is used with $\eta(t) = (t/t_r)^2$ and $\epsilon =$ 0, whereas for the MHR scheme, (2) is used with $\eta(t) =$ $\ln(t/t_r)$ and $\epsilon = pT_s$. As we will demonstrate, both the MCR and MHR schemes that use FM rate parameters to represent different users outperform the FSK scheme that uses two sinusoids to represent the transmitted bit.

The new scheme is demonstrated in Figure 2. After propagation over a wireless channel, the PN sequence is first used at the receiver to identify the hops used for transmission by User k. The overall signal in each hop also contains the transmitted signals of other users transmitting in that hop based on their PN sequence. Since the transmitter has knowledge of the FM rates of each user, demodulation and matched filtering simply detects the transmitted bits of each user.

3. FM RATE ESTIMATION

3.1. Matched signal transform

The matched signal transform or MST is a linear transformation that is highly localized at the FM rate of TV signals with nonlinear phase provided the basis function of the transform is matched to this phase [8]. For a signal x(t), the MST is given by

$$\aleph_x^{(\eta)}(c) = \int_{t \in \wp} x(t) \sqrt{|\nu(t)|} \, e^{-j2\pi c \, \eta(t/t_r)} \, dt \tag{4}$$

where \wp has the values of t in the domain of $\eta(t/t_r)$. Known MSTs include the Mellin transform [9] that is matched to signals with hyperbolic IF, and the linear MST [7] that is matched to linear chirps. As the MST is a linear transformation, it is suitable for detecting multiple NFMs as it localizes them at their respective FM rates. Thus, the MST of $x(t) = \sqrt{|\nu(t)|} \sum_{l=1}^{L} e^{j2\pi c_l \eta(t/t_r)}$ is given by $\aleph_x^{(\eta)}(c) =$ $\sum_{l=1}^{L} \delta(c-c_l)$. For finite length signals as in (2), the MST yields a sinc function whose main lobe width decreases with increasing signal duration T [7]. The MST can be efficiently implemented using the fast Fourier transform since (4) can be expressed as the Fourier transform of a nonlinearly warped version of the signal [10]. The MST is also important when processing signals in dispersive mediums; if a signal x(t) passes through a dispersive system that nonlinearly changes the phase function of the signal by $c_0 \eta(t/t_r)$ to obtain $y(t) = x(t)e^{-j2\pi c_0 \eta(\frac{t}{t_r})}$, then the MST in (4) of y(t) is $\aleph_{y}^{(\eta)}(c) = \aleph_{x}^{(\eta)}(c-c_{0})$. This is an important result for detection and estimation applications in radar, sonar and communications.

3.2. MST-Based Estimator of User Identification

As the MST is highly localized for NFMs, we consider its use in estimating their FM rate parameters. This would be needed at the receiver of the MNR FH-CDMA receiver when the rate information is not available. For example, the information can be corrupted when transmitted over dispersive mediums such as shallow water; or small perturbations due to motion in the medium could cause a change in the FM rate. This additional signal processing step before despreading and detection is demonstrated in Figure 2.

We can show that the peak location of the MST squared magnitude is the maximum likelihood estimate (MLE) of the NFM rate. Specifically, if we consider N samples of the discrete time NFM $x[n; c_0] = x(nT_s)$ in (1) with unknown FM rate c_0 , then the received signal is $r[n] = x[n; c_0] + w[n]$ with zero-mean, additive white Gaussian noise (AWGN) w[n]. The MLE of c_0 is computed to be

$$\hat{c}_0 = \arg\max_{c_0} \frac{1}{N} \left| \sum_{n=0}^{N-1} r[n] x^*[n; c_0] \right|^2 = \arg\max_{c_0} \mathcal{P}_r^{(\eta)}(c_0)$$

where the MST periodogram (PMST) is given as $\mathcal{P}_r^{(\eta)}(c) = \frac{1}{N} |\aleph_r^{(\eta)}(c)|^2$. The PMST is a unitarily warped version of the classical periodogram $P_y(f) = \frac{1}{N} |\sum_{n=0}^{N-1} y[n] e^{-j2\pi fn}|^2$ [11]. Specifically, it can be shown that $\mathcal{P}_r^{(\eta)}(c) = P_y(c/t_r)$ where $y[n] = |\nu(\eta^{-1}(nTs))|^{-1/2} x(\eta^{-1}(nTs))$. Note that in practice, the variance of the PMST-based estimate can be shown to be high. Thus, to actually compute the PMST, we window and overlap the data and compute the average of the PMST of the resulting segments (similar, in concept, to the Welch periodogram [11]). For the MNR FH-CDMA system, due to the presence of MAI from other users in a hop, the PMST of the received signal in the hop results in multiple peaks. To allocate the corresponding estimated FM rates to the users, we find the maximum sum of the PMSTs of the signals from the particular hops that each user occupied as identified by the corresponding spreading sequence.

4. SIMULATION RESULTS

Using simulations, we demonstrate the improved performance of the MNR FH-CDMA scheme over FSK signaling when HFMs (MHR scheme) and chirps (MCR scheme) are used. The plots were obtained using 500 Monte Carlo simulations with 1,000 bits per user, and the performance is given as bit error rate (BER) versus signal-to-noise ratio (SNR) per bit for different time-bandwidth products (TB).

In Figure 3, we compare the performance of MHR, MCR and FSK in FH-CDMA systems over an AWGN channel for 5 and 10 users and TB = 4. In Figure 3(a), the MCR and MHR signaling use linear chirps and HFMs, respectively, that satisfy the FM rate condition given in (3). Thus, their performance is optimal in minimizing MAI and they both outperform the FSK signaling scheme. To emphasize the importance of choosing the optimal FM rates, in Figure 3(b) the FM rate conditions are not satisfied. Although the performance of MCR [6] and MHR is not as high as in (a), since wideband TV signals have an inherent resistance to MAI, they still perform better than FSK. Figure 1(b) shows similar results for a Rayleigh fading channel with the FM rate conditions satisfied. We also demonstrate the performance of the three schemes when TB = 1 for a fair comparison with the FSK. As we can see in Figure 4, the MCR and MHR still outperform the FSK scheme for an AWGN channel (Figure 4(a)) and a Rayleigh fading channel (Figure 4(b)).

We also provide simulations to demonstrate the performance obtained when the FM rates need to be estimated at the receiver. For an AWGN channel and 5 users in Figure 5(a), we used the MCR scheme with the corresponding linear MST for estimation. Even though for the FSK we do not perform any estimation, the MCR with chirp rate estimation performs better than the FSK one for SNR > 7dB in in Figure 5(a). In Figure 5(b), similar results are ob-



Fig. 3. Coherent MHR, MCR and FSK FH-CDMA in an AWGN channel with TB = 4 for 5 and 10 users. FM rates are chosen (a) to minimize MAI and (b) arbitrarily.



Fig. 4. Coherent MHR, MCR and FSK FH-CDMA with TB = 1 for 5 and 10 users. (a) AWGN channel, and (b) Rayleigh fading channel.

tained with varying number of users. We observe that the estimated MCR with 5 users performs better than the FSK with 3 users for SNR > 10 dB.

5. CONCLUSION

By deriving optimal conditions on the FM rates of wideband TV signals with nonlinear phase, we developed new modulation schemes that can reduce the MAI when compared to FSK modulation. This is achieved due to the inherent wideband nature of the signals as well as the fact that different users are represented by a fixed signal with different FM rate. We also derived estimators of the FM rates when they are corrupted at the receiver. These are MLE estimators that are based on warped versions of the periodogram spectral estimator. The high performance of the new modulation schemes was demonstrated for FH-CDMA systems.

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Fig. 5. Coherent MCR and FSK and estimated MCR (EMCR) FH-CDMA for an AWGN channel with (a) TB = 4, and (b) TB = 8 and varying users.

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