A New Efficient Chirp Modulation Technique for Multi-User Access Communications Systems

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***Abstract*— In this paper, we present a novel chirp modulation technique that completely suppresses the interference between the users in a multi-access communications system. The interference suppression is obtained by using an orthogonal set of chirp signals in the proposed modulation scheme. Simulations results, based on Monte-Carlo realizations, show that the new technique can easily handle a larger number of users, compared to existing techniques, without a degradation in performance.**

I. Introduction

Over the last few decades, a class of modulation techniques, called broadband modulation class, has been developed. This class of modulation, in contrast to the narrowband modulation class, is characterized by its wide frequency spectra. The use of a wide frequency band, in the signal transmission, gives broadband techniques many advantages such as combating interference and/or allowing multiple signals occupying the same bandwidth to be transmitted simultaneously without interfering with one another [1], [2]. With the fast growth of broadband communications systems hardware development, more and more of these broadband techniques are being implemented in military as well as in commercial applications.

Chirp modulation, as a broadband technique [3], has been applied successfully in many engineering problems [4], [5]. In [6], we have shown that this modulation technique has an inherent capability to mitigate the effects of channel Doppler shifts and fading due to a moving receiver. In this paper, we consider the use of chirp modulation in the context of multi-user access communications applications.

The use of chirp modulation as a multi-user access tool was reported in [7] where the author proposed to assign a pair of chirps, one with a positive and one with a negative chirp rate, to each user, and a particular user is identified by its specific chirp rates and bandwidth. This technique was later extended in [8] where the chirps were chosen in a way to have the same power as well as the same bandwidth. We note that the performances of the techniques in [7], [8] decrease when the number of users increases. This limitation stems from the fact that when there is an increase in the number of users, in a fixed communications resource, the interference between the users increases, yielding a degradation in performance.

To address the limitation stated above, an improved technique was proposed in [9]. This technique assigns a particular chirp rate as well as a particular initial phase to each user. The algorithm in [9] uses a chirp rate to select a user, and the corresponding initial phase to minimize the cross-correlation with the other users. Although the technique was shown to outperform those in [7], [8], it still suffers from the presence of interference between the users. This, in turn, reduces its overall channel capacity.

In this paper, we present a new chirp modulation technique that completely suppresses the interference between the users. The interference suppression is due to an appropriate design of the chirp signals used in the modulation scheme. We will show, using various Monte-Carlo simulations, that the proposed technique can easily handle a larger number of users, compared to that in [9], without sacrificing the performance.

The paper is organized as follows. In Section 2, we present the communications system considered and its related concepts. In Section 3, we outline the procedure to design the chirps used in the proposed modulation technique. In Section 4, we provide some simulations to prove the efficiency and superiority of the new proposed technique. Section 5 concludes the paper.

II. Basic System Concept

A block diagram of the communications system considered in this study is shown in Figure 1.

At the transmitter, for an arbitrary user $n = 0, 1, \ldots, M-1$ (where M is the total number of users), one of the two data source symbols -1 or +1 modulates a particular chirp signal $c_n(t)$. Thus, the general expression of the baseband transmitted signal, for user n, can be written as

$$s_n(t) = b_n(t) \cdot c_n(t) \tag{1}$$

where $b_n(t) \in \{\pm 1,\ 0 \le t \le T\}$ is the binary information sequence, T is the symbol period and $c_n(t)$ is a chirp signal associated with user n. In the sequel, we will give the details on how to design the chirp signal.

At the receiver, assuming perfect synchronization, the general expression of the baseband received signal can be written as

$$r(t) = \sqrt{E_s}\sum_{n=1}^{M} s_n(t) + w(t) \tag{2}$$

where E_s denotes the transmitted signal energy per symbol (per user) and $w(t)$ is an additive interference, assumed here to be white, complex, Gaussian noise.

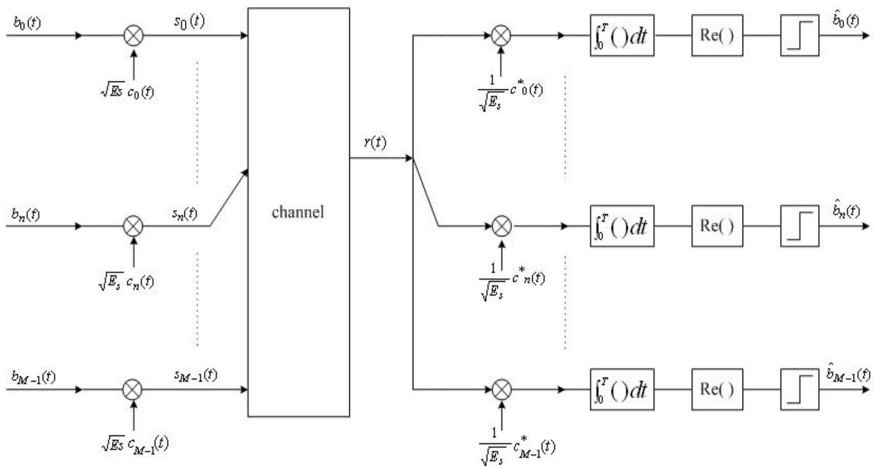

Fig. 1. A block diagram of the considered communications system.

By multiplying the received signal by an array of locally generated replica of the chirp signals, we obtain, for the decision variable u_n at branch n, the following expression

$$\begin{aligned} u_n &= \frac{1}{\sqrt{E_s}} \operatorname{Re}\left[\int_0^T r(t)\cdot c_n^*(t)\, dt\right] \\ &= b_n(t) + \operatorname{Re}\left[\sum_{\substack{m=0\\ m\neq n}}^{M-1} b_m(t)\cdot \rho_{n,m}\right] \\ &+ \operatorname{Re}\left[\frac{1}{\sqrt{E_s}}\int_0^T w(t)\cdot c_n^*(t)\, dt\right] \end{aligned} \tag{3}$$

where

$$\rho_{n,m} = \int_0^T c_n(t)\cdot c_m^*(t)\, dt \tag{4}$$

denotes the cross-correlation coefficient between two different chirps $c_n(t)$ and $c_m(t)$. An estimate of the transmitted symbol (-1 or +1) may, subsequently, be obtained by a hard decision thresholding.

It is clear from Equation (3) that, in addition to the channel noise effect (last term), the system suffers from a multiple access interference (MAI) (second term) which can further degrade the system performance. If the effect of the noise can be easily overcome by increasing the signal-to-noise ratio, the MAI effect can only be mitigated by reducing the cross-correlation coefficient $\rho_{n,m}$. The latter task can be accomplished using different approaches.

In [9], where a similar tranmitter-receiver system but with a different chirp modulation was used, the authors proposed to reduce the MAI by optimizing the value of the initial phase introduced in the chirp model. This optimization was carried out by minimizing the maximum absolute cross-correlation coefficient given by

$$\rho_{n,m;\max} = \max_{\substack{n,\, m\, =\, 0,1,\,...\,M-1\\ m\neq n}} |\rho_{n,m}|. \tag{5}$$

When the number of users increases, the optimization procedure suggested in [9] becomes difficult to implement and its performance drastically deteriorates. Here, we propose a simpler and more efficient alternative method to suppress the cross-correlation coefficient.

III. Chirp Signals Design

We see, from Equation (3), that in order to suppress the MAI, the cross-correlation coefficient $\rho_{n,m}$ should be made equal to zero. One way to accomplish this goal is to choose the chirp signals to be orthogonal.

Now, let us observe that the kernel, $K_\alpha(t,u)$, of the special linear canonical transform, called the fractional Fourier transform (FrFT), given by [11]

$$K_\alpha(t,u) = A_\alpha \cdot \exp\left\{j\pi(t^2+u^2)\cot\alpha - j2\pi\, t\, u\, \csc\alpha\right\} \tag{6}$$

with $A_\alpha = \frac{e^{-j\pi \operatorname{sign}(\sin\alpha)/4 + j\alpha/2}}{\sqrt{|\sin\alpha|}}$ and $0 < |\alpha| < \pi$, can be shown to verify the following expression

$$\int_0^T K_a(t,t_n)\cdot K_a^*(t,t_m)\, dt = \begin{cases} |A_\alpha|^2\, T, & t_n = t_m \\ \mathcal{C}\, \operatorname{sinc}[(t_n-t_m)\cdot\csc\alpha\cdot T], & t_n \neq t_m \end{cases} \tag{7}$$

where $\mathcal{C} = |A_\alpha|^2\, T\exp\{-j\pi(t_n^2-t_m^2)\cot\alpha\}$, $\operatorname{sinc}(x)$ is the usual sinc function defined as $\operatorname{sinc}(x) = \sin(\pi x)/(\pi x)$, and t_n and t_m are arbitrary values of the variable u. It is not difficult to check that we have $\int_0^T K_a(t,t_n)\cdot K_a^*(t,t_m)\; dt = 0$ for $(t_n - t_m) = k\cdot\sin\alpha/T$ where k is an arbitrary integer.

The above result implies that the infinite set of normalized chirps given by $\frac{1}{|A_\alpha|\sqrt{T}}\; K_a^*(t, n\sin\alpha/T)$, where n is an integer, forms a set of orthonormal chirps. Thus, by selecting a subset of M chirps from this orthonormal set and use each one of them to modulate a different user in the modulation model given by Equation (1), we ensure full suppression of the MAI. That is, by choosing the modulating signal as

$$c_n(t) = \frac{1}{|A_\alpha|\sqrt{T}}\, K_a^*\,(t, n\ \sin\alpha/T) \quad n = 0, 1, \ldots, M-1, \tag{8}$$

we guarantee that $\rho(n, m) = 0$ for $n \neq m$ in Equation (4).

The instantaneous frequency (IF) [12] of the selected chirp, for user n, is easily found to be equal to

$$f_n(t) = -\cot\alpha \cdot t + n/T, \quad n = 0, 1, \ldots, M-1. \tag{9}$$

The above result indicates that the selected chirp signals have the same chirp rate $-\cot\alpha$. It also indicates that when $\alpha = \pi/2$, each user will have a complex sinusoid, with a constant frequency $f_n(t) = n/T,\ n = 0, 1, \ldots, M-1$, as its modulating signal (or subcarrier). That is, for $\alpha = \pi/2$, the proposed technique reduces to the classical orthogonal frequency division multiplexing (OFDM) technique.

Note that the freedom in the choice of the parameter α allows us to have a flexible system design that meets the actual communications system requirements such as the desired number of users M who share the communications resources and the bandwidth occupied by each user.

As an illustration, in Figure 2 (top plot), we display the normalized IFs of 4 users sharing a system whose time-bandwidth product equals $TB = 32$. In this example, we chose the IFs to be increasing with time (i.e.,$-\cot\alpha > 0$). For a comparison purpose, we also display in the same figure (bottom plot) the IFs of 4 users sharing a system with $TB = 32$ when the technique in [9] is considered in the modulation scheme.

IV. Simulation and Results

In this section, we present some Monte-Carlo simulations to evaluate the performance of the proposed technique. A comparison with the performance of the technique in [9] is also presented here.

In the experiment a binary data sequence (-1 and 1), is generated and modulated according to the scheme proposed in the block diagram of Figure 1. The chirp modulating signals are generated using Equation (8) (for our proposed technique) and expressions [9, (2)] (for the technique in [9]), respectively. Here, we consider the same channel as that considered in [9], namely, an additive white Gaussian noise channel.

The first quantity used in the comparison is the maximum absolute sum of cross-correlation coefficient defined as [9]

$$\rho_{sum,max} = \max_{n=0,1,\ldots M-1} \sum_{\substack{m\,=\,0 \\ m\,\neq\,n}}^{M-1} |\rho_{n,m}| \tag{10}$$

where $\rho_{n,m}$ is the chirps cross-correlation quantity defined in Equation (4). This quantity may be regarded as a measure of multiple interference in the system [9]. The higher is the value of $\rho_{sum,max}$, the more severe the interference among the users, sharing the communications system, will be [9]. In Table I, we display the experimental values of $\rho_{sum,max}$ obtained for various values of the number of users M and the system time-bandwidth product TB.

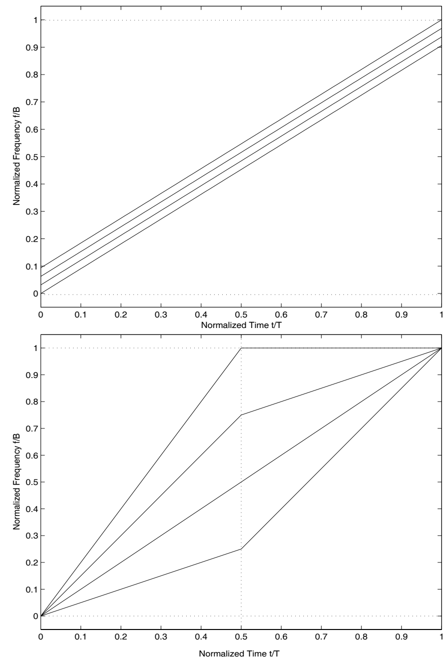

Fig. 2. Normalized IFs of modulating chirps of the proposed technique (top plot) and that proposed in [9]. Both systems are assumed to have the same time-bandwidth product $TB = 32$ and the same number of users $M = 4$.

M	TB	Proposed method		Method in [9]	
		$\rho_{sum,max}$	B_n/B	$\rho_{sum,max}$	B_n/B
4	32	$2.5 \cdot 10^{-15}$	0.91	0.28	1
8	64	$1.5 \cdot 10^{-14}$	0.90	0.74	1
8	128	$3.4 \cdot 10^{-14}$	0.95	0.52	1

TABLE I

Cross-correlation coefficient for various values of number of users, M, and system time-bandwidth products, TB.

As expected, we can see from the table that the proposed technique outperforms the other technique in terms of interference suppression between the users ($\rho_{sum,max} \sim 0$). The strong cancellation of the MAI should yield a better system performance, as shown below. In the same table, we also provide the ratios of the individual users' bandwidths, B_n, over the total communications system bandwidth B.

Also, we studied the MAI as a function of the number of users when the system time-bandwidth is kept fixed. The results of this analysis, for $TB = 128$, are shown in Table II. Once again, we see that the proposed method yileds a better interference suppression, compared to the technique in [9]. This suggests that the new method can accomodate more users in the same allocated channel without suffering from users interference. Note that the bandwidth occupied by the individual users becomes smaller with increasing number of users. This is expected as the algorithm tries to maintain the orthogonality between the users and not the frequency band they occupy.

M	TB	Proposed method		Method in [9]	
		$\rho_{sum,max}$	B_n/B	$\rho_{sum,max}$	B_n/B
2	128	$3.3 \cdot 10^{-16}$	0.99	$4 \cdot 10^{-04}$	1
4	128	$4.0 \cdot 10^{-15}$	0.98	0.13	1
8	128	$3.4 \cdot 10^{-14}$	0.95	0.52	1
16	128	$4.2 \cdot 10^{-14}$	0.88	1.19	1
32	128	$7.5 \cdot 10^{-14}$	0.76	2.68	1
64	128	$1.2 \cdot 10^{-13}$	0.51	5.77	1

TABLE II

CROSS-CORRELATION COEFFICIENT FOR VARIOUS VALUES OF NUMBER OF USERS AND A FIXED TIME-BANDWIDTH PRODUCT.

Next, we evaluated the bit-error-rates (BERs) of the two techniques, in the presence of additive white Gaussian noise. The considered values of M and TB are those given in Table I. In this experiment, the signal-to-noise ratio (SNR) is varied from -40 dB to 0 dB with a 5 dB increment. A total number of $100,000$ blocks (each block consisting of M users) is used, in the Monte-Carlo simulations, for each SNR value. The results of the simulations are shown in Figure 3. The continuous curves correspond to the proposed technique and the dashed curves correspond to the technique in [9]. For the three cases considered, we observe that the new technique has a better performance (a smaller BER value for a fixed SNR value).

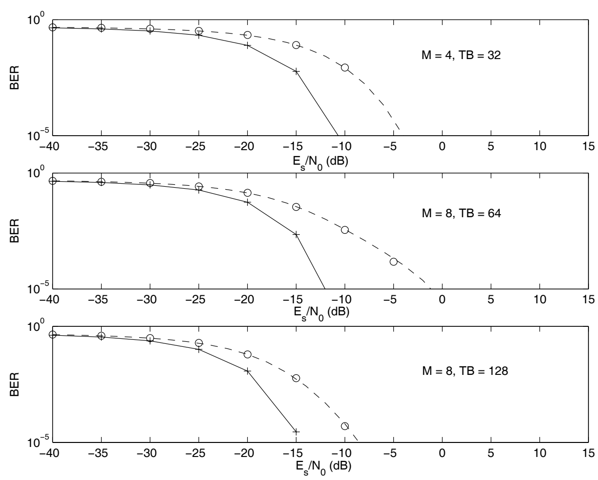

Fig. 3. BERs of the proposed technique (continuous curve) and that in [9] (dashed curve) for various values of TB and M.

Here again, we studied the system performance as a function of the number of users when the system time-bandwidth is kept fixed. For that, we evaluated the BERs of both techniques for the values $M = 2,\ 4,\ 8,\ 16,\ 32,\ 64$, $TB = 128$ and SNR$=-15$ dB. The results of this analysis are shown in Figure 4. The continuous curves correspond to the proposed technique and the dashed curves correspond to the other technique. As we can see, the proposed technique not only has a smaller BER but its value does not change with the number of users. This means that it totally removes the MAI, in conformity with the theoretical analysis of the previous sections. This is not the case with the technique in [9] whose BER increases with increasing number of users.

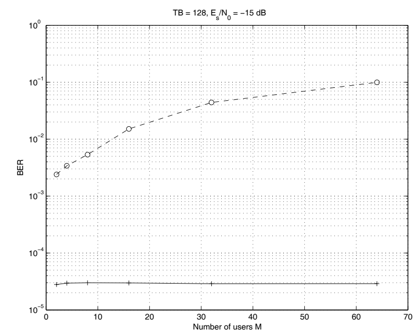

Fig. 4. BERs, as a function of M, of the proposed technique (continuous curve) and that in [9] (dashed curve) for $TB = 128$ and SNR=-15 dB.

V. CONCLUSION

In this paper, we introduced a new chirp modulation technique for multi-access communications systems. The technique uses a set of orthogonal chirps to modulate the tramsmitted data. Simulations results, based on Monte-Carlo realizations, have shown that this technique is very efficient in terms of multi-access interference suppression. Compared to existing techniques, the new one can allow a higher number of users, in a given communications system, without a degradation in performance.

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