INTERFERENCE SUPPRESSION FOR GPS COARSE/ACQUISITION SIGNALS USING ANTENNA ARRAY

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ABSTRACT

This paper considers the problem of interference suppression in GPS coarse/acquisition (C/A) signals. Particularly, an anti-jam receiver is developed utilizing the unique repetitive feature of the GPS C/A-signal. The proposed receiver does not require the knowledge of the transmitted GPS symbols or the satellite positions. It utilizes the repetition of the Gold code within each navigation symbol to simultaneously suppress the interference of all satellites.

1. INTRODUCTION

GPS employs direct-sequence spread-spectrum (DS-SS) signaling. For each satellite, two different pseudorandom codes, a coarse/acquisition (C/A) code and a precision (P) code, are used to modulate the navigation information, which are binary phase shift key (BPSK) symbols transmitted at a data rate of 50 bps [1]. The C/A-code is a Gold code with a chip rate of 1.023 Mchips/sec (or code period $P = 1023$) and repeats every millisecond, i.e., there are twenty replicas of the code within each symbol. This property allows a different approach for interference suppression in GPS than that used in typical CDMA communication systems.

The spread-spectrum (SS) scheme, which underlies the GPS signal structure, provides a certain degree of protection against interference. However, when the interferer power becomes much stronger than the signal power, the spreading gain alone is insufficient to yield any meaningful information. For the C/A-signal, the GPS receiver is vulnerable to strong interferers whose power exceeds the approximately 30 dB gain offered via the spreading/despreading process.

In GPS, interference can be combated in the time, space, or frequency domain, or in a domain of joint variables, e.g., time-frequency [2], or space-time [3]. Time-frequency signal representations equip the receiver with the ability to detect the time-frequency signature of the nonstationary interferer and remove it through synthesis or subspace projection methods [4]. Space-time processing relies on antenna arrays to provide the receiver with spatial and temporal selectivity. It is noted, however, that the existing techniques for GPS interference suppression do not fully utilize the GPS signal structure, namely the replication of the GPS C/A-code.

This paper proposes a novel interference suppression technique in GPS using the repetitive structure of the GPS C/A-signals. Due to the repetition of the spreading code, the GPS C/A-signal exhibits strong self-coherence between chip samples that are separated by integer multiples of the spreading gain. Using this unique feature, an anti-jam receiver based on the spectral self-coherence restoral (SCORE) algorithm [5] can be constructed to suppress interferers that are either uncorrelated with or of different self-coherent properties from that of the GPS signal. The proposed GPS receiver does not require the knowledge of the navigation data or satellite locations to perform interference suppression.

2. PROPOSED GPS RECEIVER

The proposed GPS receiver with an M-element array is displayed in Figure 1. The structure of the received noise-free GPS signal is depicted in Figure 2, where the BPSK modulated GPS navigation symbols are spread by a Gold code with spreading gain of $P = 1023$ and chip-rate sampled. The spreading code is repeated 20 times within each symbol. A *data block* and a *reference block* are formed at the receiver, each containing N consecutive samples. The distance between the respective samples in the data and reference blocks is set equal to jP chips, where $1 \leq j < 20$. Due to the repetition of the spreading code, the nth sample in the data block has the same value as the corresponding nth sample in the reference block, providing that the two samples belong to the same symbol.

The data block is processed by a beamformer $\mathbf{w}$, whereas an auxiliary beamformer $\mathbf{f}$ provides a reference signal by processing samples in the reference block. An error signal $e(t)$ is formed from the beamformer output and the reference signal. For the proposed receiver, the weight vector $\mathbf{w}$ is adaptively updated according to the cross-SCORE algorithm, while $\mathbf{f}$ is renewed using the least-squares algorithm.

The signal reaching the GPS receiver is the aggregate of the GPS signals of satellites currently in the field of view, signal multipaths, additive white Gaussian noise (AWGN), and broadband/narrowband interference. Thus, the signal received at the GPS receiver, after the frequency synchronization with

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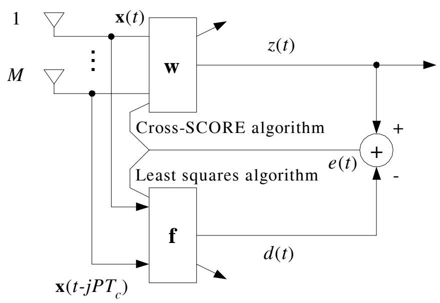


Fig. 1. Block diagram of the proposed GPS receiver.

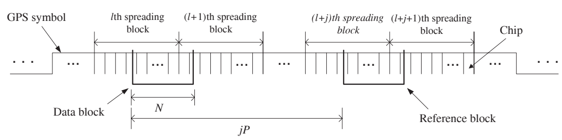


Fig. 2. Noise-free GPS signal structure.

the carrier, can be expressed as

$$\mathbf{x}(n) = \sum_{q=0}^{Q} s_q(nT_s - \tau_q)\mathbf{a}_q e^{j\phi_q} + \sum_{k=1}^{K} B_k u_k(n)\mathbf{d}_k + \mathbf{v}(n), \tag{1}$$

where $s_q(n)$ denotes the GPS signal, T_s is the Nyquist sampling interval, Q is the number of multipath components, with subscript 0 designated to the direct-path signal, $s_q(n)$, τ_q, and ϕ_q are the signal sample, time-delay, and phase-shift of the qth multipath component, respectively, K is the number of interferers, $u_k(n)$ is the waveform of the kth interferer with amplitude B_k, $\mathbf{a}_q$ and $\mathbf{d}_k$ are, respectively, $M \times 1$ spatial signatures of the qth satellite multipath and the kth interferer, and $\mathbf{v}(n)$ consists of noise samples. Let $\mathbf{s}(n) := s_0(nT_s - \tau_0)\mathbf{a}_0 e^{j\phi_0}$ denote the data vector across the array due to the direct-path signal. Then, equation (1) can be rewritten as

$$\mathbf{x}(n) = \mathbf{s}(n) + \tilde{\mathbf{s}}(n) + \mathbf{u}(n) + \mathbf{v}(n),$$

where $\tilde{\mathbf{s}}(n) := \sum_{q=1}^{Q} s_q(nT_s - \tau_q)\mathbf{a}_q e^{j\phi_q}$ represents the signal multipath and $\mathbf{u}(n) := \sum_{k=1}^{K} B_k u_k(n)\mathbf{d}_k$. The counterpart of $\mathbf{x}(n)$ in the reference block within the same symbol can be written as

$$\begin{aligned}\mathbf{x}(n - jP) &= \sum_{q=0}^{Q} s_q(nT_s - \tau_q)\mathbf{a}_q e^{j\phi_q} \\ &\quad + \sum_{k=1}^{K} B_k u_k(n - jP)\mathbf{d}_k + \mathbf{v}(n - jP) \\ &= \mathbf{s}(n) + \tilde{\mathbf{s}}(n) + \mathbf{u}(n - jP) + \mathbf{v}(n - jP),\end{aligned}$$

where we have assumed that, when considered within the same symbol, $s_q(nT_s - \tau_q) = s_q(nT_s - \tau_q - jP)$, $q = 0, \cdots, Q$.

The beamformer output and the reference signal are given by $z(n) := \mathbf{w}^H \mathbf{x}(n)$ and $d(n) := \mathbf{f}^H \mathbf{x}(n - jP)$, respectively. We define the following covariances:

$$\begin{aligned}R_{zd} &:= E\{z(n)d^H(n)\} = \mathbf{w}^H E\{\mathbf{x}(n)\mathbf{x}^H(n - jP)\}\mathbf{f}, \\ R_{zz} &:= E\{z(n)z^H(n)\} = \mathbf{w}^H E\{\mathbf{x}(n)\mathbf{x}^H(n)\}\mathbf{w}, \\ R_{dd} &:= E\{d(n)d^H(n)\} = \mathbf{f}^H E\{\mathbf{x}(n - jP)\mathbf{x}^H(n - jP)\}\mathbf{f}.\end{aligned}$$

Under the assumption that the GPS signal, interference, and noise are independent, we have

$$\mathbf{R}_{xx} := E\{\mathbf{x}(n)\mathbf{x}^H(n)\} = \mathbf{R}_s + \mathbf{R}_u + \mathbf{R}_v. \tag{2}$$

The three terms in (2) denote, respectively, the covariance matrices of the GPS, including both the direct and multipath signals, interference, and noise: $\mathbf{R}_s := E\{[\mathbf{s}(n) + \tilde{\mathbf{s}}(n)][\mathbf{s}(n) + \tilde{\mathbf{s}}(n)]^H\}$, $\mathbf{R}_u := E\{\mathbf{u}(n)\mathbf{u}^H(n)\}$, and $\mathbf{R}_v := E\{\mathbf{v}(n)\mathbf{v}^H(n)\}$. Providing that the only signals are correlated when delayed jP samples are those of the GPS, the cross-correlation matrix between the corresponding data vectors in the data and reference blocks simplifies to

$$\mathbf{R}_{xx}^{(P)} := E\{\mathbf{x}(n)\mathbf{x}^H(n - jP)\} = \mathbf{R}_s.$$

Let $e(n) := d(n) - z(n)$ be the error between the output of the beamformer and the reference signal. For a fixed beamformer $\mathbf{w}$, the error $e(n)$ is minimized in the least-squares sense when $\mathbf{f}$ is given by $\mathbf{f}_{\text{LS}} = \mathbf{R}_{xx}^{-1}\mathbf{r}_{xz}$, where $\mathbf{r}_{xz} := E\{\mathbf{x}(n - jP)z^H(n - i)\} = \mathbf{R}_{xx}^{(P)H}\mathbf{w}$.

The beamformer $\mathbf{w}$ is obtained by maximizing the cross-correlation between $z(n)$ and $d(n)$. Define the following cost function [5]:

$$\begin{aligned}\mathcal{C}(\mathbf{w}, \mathbf{f}_{\text{LS}}) &:= \frac{|R_{zd}|^2}{R_{zz}R_{dd}} = \frac{|\mathbf{w}^H \mathbf{R}_{xx}^{(P)}\mathbf{f}|^2}{[\mathbf{w}^H \mathbf{R}_{xx}\mathbf{w}][\mathbf{f}^H \mathbf{R}_{xx}\mathbf{f}]} \\ &= \frac{\mathbf{w}^H \mathbf{R}_{xx}^{(P)} \mathbf{R}_{xx}^{-1} \mathbf{R}_{xx}^{(P)H}\mathbf{w}}{\mathbf{w}^H \mathbf{R}_{xx}\mathbf{w}}.\end{aligned} \tag{3}$$

Then, the weight vector $\mathbf{w}$ that maximizes $\mathcal{C}(\mathbf{w}, \mathbf{f}_{\text{LS}})$ is readily shown to be the eigenvector corresponding to the largest eigenvalue of the generalized eigenvalue problem:

$$\mathbf{R}_{xx}\mathbf{w} = \lambda_{\text{max}} \mathbf{R}_{xx}^{(P)} \mathbf{R}_{xx}^{-1} \mathbf{R}_{xx}^{(P)H}\mathbf{w}, \tag{4}$$

where λ_{max} is the largest eigenvalue.

In practice, $\mathbf{R}_{xx}$ and $\mathbf{R}_{xx}^{(P)}$ in (4) are unknown and have to be replaced by their sample estimates. Define the $M \times N$ data and reference matrices as $\mathbf{X}_N := [\mathbf{x}(n) \ \cdots \ \mathbf{x}(n - (N-1))]$ and $\mathbf{X}_{N\text{ref}} := [\mathbf{x}(n - jP) \ \cdots \ \mathbf{x}(n - (N-1) - jP)]$, where N is the block length and $N \leq P$. The sample covariance matrices are then given by $\hat{\mathbf{R}}_{xx} = \mathbf{X}_N\mathbf{X}_N^H/N$ and $\hat{\mathbf{R}}_{xx}^{(P)} = \mathbf{X}_N\mathbf{X}_{N\text{ref}}^H/N$. The beamformer $\mathbf{w}$ is obtained by solving the eigenvalue problem (4) using the sample estimates given above.

3. COVARIANCE MATRIX ESTIMATIONS

The key assumption made for the proposed GPS receiver is that both the data and reference samples, $\mathbf{x}(n)$ and $\mathbf{x}(n - jP)$,

$1 \leq j < 20$, belong to the same navigation symbol. However, since the data samples used for covariance matrix estimations are selected randomly, and interference suppression is performed prior to any symbol synchronization process, there is no guarantee that the data and reference samples belong to the same symbol. Questions arise as how will the receiver perform when the above assumption fails, i.e., the data and reference samples lie in two adjacent symbols?

To answer this question, we relax the condition imposed in Section 2, and develop the general expression of the covariance matrices. Define the following events: $\mathcal{A}_1$: $\mathbf{x}(n)$ & $\mathbf{x}(n-jP)$ are within the same symbol; $\mathcal{A}_{21}$: $\mathbf{x}(n)$ & $\mathbf{x}(n-jP)$ are in two adjacent symbols with the same sign; $\mathcal{A}_{22}$: $\mathbf{x}(n)$ & $\mathbf{x}(n-jP)$ are in two adjacent symbols with opposite signs. With random selection of time n, and using the repetitive property of the C/A-code, it is straightforward to show that the corresponding probabilities of the above events are $Prob\{\mathcal{A}_1\} = \frac{T-jP}{T} = 1 - \frac{jP}{T}$, $Prob\{\mathcal{A}_{21}\} = \frac{jP}{2T}$, and $Prob\{\mathcal{A}_{22}\} = \frac{jP}{2T}$, respectively, where $T = 20P$ is the total number of samples in one symbol. The exact expression $\mathbf{R}_{xx}^{(P)}$ can be written in terms of the above probabilities and conditional expectations as

$$\begin{aligned}\mathbf{R}_{xx}^{(P)} =& E\{\mathbf{x}(n)\mathbf{x}^H(n-jP)|\mathcal{A}_1\}Prob\{\mathcal{A}_1\} \\ &+ E\{\mathbf{x}(n)\mathbf{x}^H(n-jP)|\mathcal{A}_{21}\}Prob\{\mathcal{A}_{21}\} \\ &+ E\{\mathbf{x}(n)\mathbf{x}^H(n-jP)|\mathcal{A}_{22}\}Prob\{\mathcal{A}_{22}\} \\ =& \left(1 - \frac{jP}{T}\right)\mathbf{R}_s,\end{aligned} \tag{5}$$

which shows that $\mathbf{R}_{xx}^{(P)}$ depends on the distance between the data and reference samples jP. The maximum value of $\mathbf{R}_{xx}^{(P)}$ is achieved when $j = 1$, representing the closest repetition between the data and reference blocks.

In practice, however, sample estimates replace the exact values in equations (5). It can be readily shown that if $\mathbf{X}_N$ and $\mathbf{X}_{N\mathrm{ref}}$ are jP samples apart, $1 \leq j < 20$, the probability of the two blocks belonging to the same symbol or, equivalently, in two adjacent symbols with the same sign, is $1 - \frac{jP+N}{2T}$. The probability that $\mathbf{X}_N$ and $\mathbf{X}_{N\mathrm{ref}}$ are in two adjacent symbols with opposite signs is $\frac{jP-N}{2T}$. Using the above probabilities, the expected values of $\hat{\mathbf{R}}_{xx}$ and $\hat{\mathbf{R}}_{xx}^{(P)}$ are, respectively, $\bar{\hat{\mathbf{R}}}_{xx} = \mathbf{R}_s + \mathbf{R}_u + \mathbf{R}_v$ and $\bar{\hat{\mathbf{R}}}_{xx}^{(P)} = \left(1 - \frac{jP}{T}\right)\mathbf{R}_s$, which indicate that $\hat{\mathbf{R}}_{xx}$ and $\hat{\mathbf{R}}_{xx}^{(P)}$ are unbiased estimates of $\mathbf{R}_{xx}$ and $\mathbf{R}_{xx}^{(P)}$, respectively.

The covariance matrix estimations obtained above use only one data block and one reference block. It does not utilize the 20 replicas/symbol property. Consider the case shown in Figure 3, where the gth data block(respectively, the $(g-1)$th reference block) is split between two adjacent symbols with opposite signs. Using this data (reference) block and its associated reference (data) block will yield poor time-average estimation, due to term cancellation. This problem can be remedied if multiple data and reference blocks are used. With up to 19 data blocks (respectively, reference blocks), only one block could be at symbol transition, whereas each of the other remaining blocks belongs to one symbol. Using G data and reference blocks, we have $\hat{\mathbf{R}}_{xxG} = \frac{1}{G}\sum_{g=1}^{G}\mathbf{X}_N(g)\mathbf{X}_N^H(g)/N$ and $\hat{\mathbf{R}}_{xxG}^{(P)} = \frac{1}{G}\sum_{g=1}^{G}\mathbf{X}_N(g)\mathbf{X}_{N\mathrm{ref}}^H(g)/N$. A maximum of only two of the G terms in the above equation may suffer from symbol transition, whereas the rest of the terms will be coherently combined.

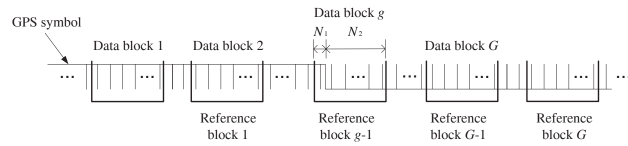


Fig. 3. Multiple data and reference blocks.

To analyze the receiver performance, we derive the mean and variance of the covariance matrix estimation. To simplify the derivation, we rewrite the received signal vector as $\mathbf{x}(n) = s(n)\mathbf{a}(\theta) + \mathbf{v}(n)$, where $\mathbf{v}(n)$ contains samples of interference and noise, with zero mean and variance σ_v^2. Therefore, $\mathbf{R}_s = E\{s(n)\mathbf{a}(\theta)\mathbf{a}^H(\theta)s^H(n)\} = \mathbf{a}(\theta)\mathbf{a}^H(\theta)$, where it is assumed $E\{s(n)s^H(n)\} = 1$. When G data and reference blocks are used, we define the following events: $\mathcal{F}_1$: the first data block or the last reference block is split, $\mathcal{F}_2$: one of the other $G-1$ data/reference blocks is split and $G_1 < G$ data blocks (respectively, $G_1 - 1$ reference blocks) are in one symbol, $\mathcal{F}_3$: the data blocks and reference blocks are within the same symbol, and $\mathcal{F}_4$: no split block and the data and reference blocks are in two adjacent symbols. The corresponding probabilities are: $Prob\{\mathcal{F}_1\} = \frac{N}{T}$, $Prob\{\mathcal{F}_2\} = \frac{N}{T}$, $Prob\{\mathcal{F}_3\} = 1 - \frac{N+GjP}{T}$, and $Prob\{\mathcal{F}_4\} = \frac{G(jP-N)}{T}$. From the above probability values, it is straightforward to show that the expected value of $\hat{\mathbf{R}}_{xx}^{(P)}$ is given by

$$\begin{aligned}\bar{\hat{\mathbf{R}}}_{xxG}^{(P)} &= \bar{\hat{\mathbf{R}}}_{xxG|\mathcal{F}_1}^{(P)} + \bar{\hat{\mathbf{R}}}_{xxG|\mathcal{F}_2}^{(P)} + \bar{\hat{\mathbf{R}}}_{xxG|\mathcal{F}_3}^{(P)} + \bar{\hat{\mathbf{R}}}_{xxG|\mathcal{F}_4}^{(P)} \\ &= \left(1 - \frac{jP}{T}\right)\mathbf{R}_s,\end{aligned} \tag{6}$$

which is equivalent to the the expected value given in (5).

The variance of $\hat{\mathbf{R}}_{xx}^{(P)}$ is given by

$$\mathrm{var}\left[\hat{\mathbf{R}}_{xx}^{(P)}\right] = E\left\{\hat{\mathbf{R}}_{xx}^{(P)}\hat{\mathbf{R}}_{xx}^{H(P)}\right\} - E^2\left\{\hat{\mathbf{R}}_{xx}^{(P)}\right\}.$$

Since the covariance estimate is unbiased, the variance of $\hat{\mathbf{R}}_{xx}^{(P)}$ is determined by $E\left\{\hat{\mathbf{R}}_{xx}^{(P)}\hat{\mathbf{R}}_{xx}^{H(P)}\right\}$. It is shown in [6] that using G data and reference blocks,

$$\begin{aligned}&E\left\{\hat{\mathbf{R}}_{xxG}^{(P)}\hat{\mathbf{R}}_{xxG}^{H(P)}\right\} \\ &= \left(1 - 2\frac{jP}{T} + 2\frac{jP}{GT}\right)\left[\frac{M(N+\sigma_v^2)}{N}\mathbf{R}_a + \frac{M(1+\sigma_v^2)}{N}\mathbf{R}_v\right],\end{aligned}$$

which shows the value of using a higher value of G data and reference blocks in time-averaging.

4. SIMULATIONS

We now examine the performance of the proposed GPS receiver. We use a linear uniform array consisting of $M = 7$ sensors with half-wavelength spacing. At the receiver, $N = 800$ samples are collected in both the data and reference blocks.

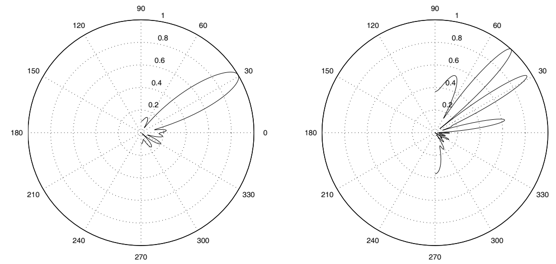
First, we consider the case when there is no interference and multipath. The SNR is -30 dB. We consider covariance matrix estimations using one data block and one reference block, both are taken within the same symbol. The performance of the proposed GPS receiver is shown in Figure 4(a), where the antenna pattern is formed towards the satellite located at $\theta = 30°$.

It is known that in GPS, at least four satellites are needed simultaneously in order to calculate the three-dimensional position and time. To demonstrate the receiver performance in the presence of multiple satellites, we select the first four from the satellite constellation. Since the proposed receiver relies on the special structure of the GPS signals to suppress interference and all satellite emitted signals share the same repetitive properties of the C/A-codes, it is expected that the receiver will pass the signals from all satellites with high gains. In the simulation, the satellites are located at $\theta_1 = 10°$, $\theta_2 = 30°$, $\theta_3 = 50°$, and $\theta_4 = 70°$, and SNR $= -30$ dB. In this particular case, we use $M = 9$ sensors and $G = 7$ data and reference blocks for covariance matrix estimations. The result is shown in Figure 4(b), where four clear beams are generated towards the four satellites.

We next study the performance of the GPS receiver in the presence of strong jammers. If the jammers have explicit bearings, we can generate the received GPS signals according to (1), but replacing the spatial signature $\mathbf{d}_k$ by the respective steering vector. The antenna pattern is shown in Figure 5, where the satellite is located at $10°$ and the direction-of-arrivals (DOA's) of the three jammers are $30°$, $50°$, and $70°$. It can be seen from Figure 5 that the receiver is able to generate deep nulls at the jammer locations.

5. CONCLUSIONS

In this paper, we presented a GPS receiver that is based on the inherent self-coherence of the GPS C/A-signal. By using this self-coherence feature, an anti-jam GPS receiver is constructed to mitigate a wide class of narrowband and broadband interferers. The proposed receiver does not require any knowledge of the transmitted signals and the locations of the satellites. Simulation results have shown that the proposed receiver is capable of suppressing strong jammers while preserving GPS signals.



(a) (b)

Fig. 4. Beam pattern with SNR $= -30$ dB (a) One satellite and $G = 1$; (b) Four satellites and $G = 7$.

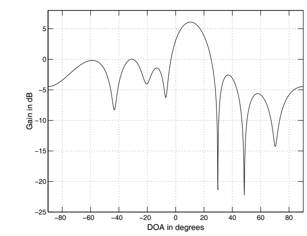


Fig. 5. Antenna gains of the proposed scheme with SINR $= -33$ dB and JSR $= 30$ dB, and $G = 3$.

6. REFERENCES

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