# BLOCK CMA-BASED BLIND AND GROUP-BLIND MULTIUSER DETECTORS

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## ABSTRACT

We derive a new iterative multiuser detection algorithm based on a generalized sidelobe canceller which uses the constant modulus property of digitally modulated signals together with code knowledge given to a receiver in CDMA system. The proposed algorithm can be seen as an extension of least square constant modulus algorithm in a sense that it simultaneously minimizes the constant modulus (CM) and the least square (LS) cost functions iteratively. The proposed detector performs as well as the ideal MMSE detector in the high SNR region, in addition to resolving the interference capture problem existing in most constant modulus algorithm (CMA).

## 1. INTRODUCTION

The performance of CDMA system is mainly limited by multiple access interference. This has motivated considerable research on multi user detection (MUD) algorithm design in CDMA system. Among these are algorithms which can be used when receivers do not have full knowledge of all codes in a system. This encompasses blind MUD, where the receiver has knowledge of only its own code, [4] and group-blind MUD [7], where the receiver knows a subset of codes.

The principles in beamforming in array signal processing has many similarities with linear detection in CDMA system. In the area of beamforming, weight vectors are calculated by either block iterative algorithms or by symbol base adaptive algorithms. Good examples of block iterative algorithms are LS-CMA [1], ILSP and ILSE [2]. The LCCM algorithm [6] using Generalized Sidelobe Canceller (GSC) to deal with a constraint problem is an example of adaptive algorithms. Among the above mentioned algorithms, LS-CMA and LCCM utilize the constant modulus property of digitally modulated signals, which is a powerful tool in blind equalization and blind source separation. Analytical constant modulus algorithm was developed for the blind beamforming of CM signal in [3], and most algorithms using CM cost function suffer from the interference capture problem, i.e., it locks onto interference instead of the desired signal.

In this paper, we develop a block iterative CMA-based multiuser detection algorithm. The motivation is that we want to take fully advantage of the information known to a receiver: 1. most digitally modulated communication signals have a CM property; 2. some of the codes are known to a receiver in CDMA system; the latter can be put as a constraint in a cost function. The proposed detection algorithm is similar to the LS-CMA [1] in the area of adaptive beamforming, but uses efficiently the code knowledge available at a receiver in CDMA system.

## 2. SYSTEM MODEL

We consider a synchronous CDMA system with $K$ users. The received signal is

$$r(t) = \sum_{k=1}^{K} b_k \underbrace{A_k \sum_{l=1}^{L} \alpha_{l,k} s_k\left(t - iT\right)}_{\bar{s}_k(t-iT)} + v(t), \tag{1}$$

where $A_k$ and $b_k$ are the amplitude and the transmitted symbol of the $k$-th user, respectively; $s_k(t) = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} s_{j,k} \cdot\psi(t - jT_c)$ is the spreading waveform of the $k$-th user, with $\{s_{j,k}\}_{j=0}^{N-1}$ the signature sequence of the $k$-th user and $\psi(t)$ the chip waveform; $\bar{s}_k(t)$ is a composite signature waveform representing the code convolved with the channel impulse response between the $k$-th user's transmitter and the receiver, $g_k(t) = \sum_{l=1}^{L} \alpha_{l,k}\delta\left(t - \tau_{l,k}\right)$, where $L$ is the total number of paths in the channel; $N$, $T$, $T_c$ denote spreading factor, symbol interval and chip interval, respectively; $\boldsymbol{v}[n]$ is additive white Gaussian noise. At the receiver, the received signal $r(t)$ is chip-matched filtered and sampled at the chip-rate. Thus, the corresponding output signal can be modeled in vector form as

$$\boldsymbol{r}[n] = \sum_{k=1}^{K} \bar{\boldsymbol{s}}_k b_k[n] + \boldsymbol{v}[n] = \bar{\boldsymbol{S}}\boldsymbol{b}[n] + \boldsymbol{v}[n], \tag{2}$$

where

$$\bar{\boldsymbol{S}} = [\bar{\boldsymbol{s}}_1 \cdots \bar{\boldsymbol{s}}_K], \quad \boldsymbol{b}[n] = [b_1[n] \cdots b_K[n]]^T. \tag{3}$$

We construct the received signal matrix by stacking $M$ received symbols in columns

$$\underbrace{\left[\ \boldsymbol{r}[i] \quad \cdots \quad \boldsymbol{r}[i+M-1]\ \right]}_{\boldsymbol{R}} = \tag{4}$$
$$\bar{\boldsymbol{S}} \underbrace{\left[\ \boldsymbol{b}[i] \quad \cdots \quad \boldsymbol{b}[i+M-1]\ \right]}_{\boldsymbol{B}} + \boldsymbol{V},$$

where $\boldsymbol{V} = [\boldsymbol{v}[i] \cdots \boldsymbol{v}[i+M-1]]$.

## 3. ALGORITHM DEVELOPMENT

In this Section, we will develop a new algorithm for blind multiuser detection based on the synchronous CDMA system model in Section 2 for simplicity of notation. The results, however, can be easily extended to an asynchronous model, and simulation results for both a synchronous and an asynchronous CDMA systems will be presented in Section 4. We first assume that signals have constant modula. Let us define a constant modulus signal set as

$$CM \stackrel{\triangle}{=} \{\ \boldsymbol{b}\ \mid\ \|(\boldsymbol{b})_i\| = 1, \quad all\ i\}. \tag{5}$$

The CM property of digitally modulated signals can be exploited by minimizing the following nonlinear CM cost function

$$J_{CM} = \sum_{i=0}^{M-1} E\left\{(1 - |\boldsymbol{w}^H \boldsymbol{r}[i]|)^2\right\}. \tag{6}$$

The minimization of (6) is achieved instantly in terms of projection of signals onto the constant modulus signal set, $CM$, and we consequently define the CM projection as

$$\boldsymbol{P}_{CM}(\boldsymbol{b}) = \left[\frac{(\boldsymbol{b})_1}{\|(\boldsymbol{b})_1\|}, \cdots, \frac{(\boldsymbol{b})_n}{\|(\boldsymbol{b})_n\|}\right]. \tag{7}$$

Denote by $\boldsymbol{w}^{(k)}$ a current weight vector at the $k$-th iteration. Then, the $(1 \times M)$ vector of temporal samples at the $k$-th iteration are generated as

$$\boldsymbol{y}^{(k)} = \boldsymbol{w}^{(k)H} \boldsymbol{R}. \tag{8}$$

The LS-CMA makes intermediate decisions by projecting the temporal sample vector $\boldsymbol{y}^{(k)}$ onto the constant modulus set

$$\hat{\boldsymbol{b}}^{(k)} = \boldsymbol{P}_{CM}(\boldsymbol{y}^{(k)}) = \left[\frac{y^{(k)}[0]}{\|y^{(k)}[0]\|}, \cdots, \frac{y^{(k)}[M-1]}{\|y^{(k)}[M-1]\|}\right]. \tag{9}$$

By using these intermediate decisions and by minimizing the following LS criterion a new weight vector is formed

$$J_{LS} = \left\| \boldsymbol{w}^{(k+1)H} \boldsymbol{R} - \hat{\boldsymbol{b}}^{(k)} \right\|^2. \tag{10}$$

It is used as the new weight vector at the $(k+1)$-th iteration

$$\boldsymbol{w}^{(k+1)} = (\boldsymbol{R}\boldsymbol{R}^H)^{-1} \boldsymbol{R} \hat{\boldsymbol{b}}^{(k)H} = \boldsymbol{C}_{rr}^{-1} \boldsymbol{C}_{rb}, \tag{11}$$

where the auto- and cross-correlation terms $\boldsymbol{C}_{rr}$ and $\boldsymbol{C}_{rb}$ are defined as

$$\boldsymbol{C}_{rr} = \frac{1}{M} \sum_{n=0}^{M-1} \boldsymbol{r}[n] \boldsymbol{r}[n]^H \tag{12}$$

$$\boldsymbol{C}_{rb} = \frac{1}{M} \sum_{n=0}^{M-1} \boldsymbol{r}[n] \hat{b}[n]^*. \tag{13}$$

In order to take advantage of the available code knowledge at the receiver, we put a constraint on (10)

$$\boldsymbol{C}^H \boldsymbol{w} = \boldsymbol{f}, \tag{14}$$

where $\boldsymbol{C}$ is a constraint matrix(known code matrix in CDMA system), and $\boldsymbol{f}$ is a response vector. Then the LS problem of minimizing (10) becomes a constrained LS problem. To solve this problem effectively and iteratively, we decompose the weight vector into two components at each iteration

$$\boldsymbol{w}^{(k)} = \boldsymbol{w}_f - \boldsymbol{B} \boldsymbol{w}_a^{(k)} \tag{15}$$

where $\boldsymbol{w}_f$ is the fixed weight vector, $\boldsymbol{w}_a$ is the adaptive weight vector (iterative weight vector in this paper) and $\boldsymbol{B}$ is the pre-designed blocking matrix. Both $\boldsymbol{w}_f$ and $\boldsymbol{B}$ are completely determined by the constraint equation, and their values are fixed while processing. Only $\boldsymbol{w}_a$ changes on each iteration. The two orthogonal components $\boldsymbol{w}_f$ and $\boldsymbol{B}\boldsymbol{w}_a$ lie in the range and null space of the constraint matrix, respectively. The fixed weight vector $\boldsymbol{w}_f$ is a minimum norm solution to the constraint (14)

$$\boldsymbol{w}_f = \boldsymbol{C}(\boldsymbol{C}^H \boldsymbol{C})^{-1} \boldsymbol{f}. \tag{16}$$

Using the decomposed weight vector (15), the LS cost function becomes

$$J_{LS} = \left\| \hat{\boldsymbol{b}}^{(k)} - \left( \boldsymbol{w}_f^H - (\boldsymbol{B} \boldsymbol{w}_a^{(k+1)})^H \right) \boldsymbol{R} \right\|^2, \tag{17}$$

and a minimizing solution is

$$\boldsymbol{w}_a^{(k+1)} = \left( \boldsymbol{B}^H \boldsymbol{R} \boldsymbol{R}^H \boldsymbol{B} \right)^{-1} \boldsymbol{B}^H \boldsymbol{R} \boldsymbol{d}^{(k)H} \tag{18}$$

$$= \left( \boldsymbol{B}^H \boldsymbol{C}_{rr} \boldsymbol{B} \right)^{-1} \boldsymbol{B}^H \boldsymbol{C}_{rd}^{(k)}, \tag{19}$$

where $\boldsymbol{d}^{(k)} = \boldsymbol{w}_f^H \boldsymbol{R} - \hat{\boldsymbol{b}}^{(k)} = \left[d^{(k)}[0], \cdots, d^{(k)}[M-1]\right]$. The correlation matrix $\boldsymbol{C}_{rr}$ is found in (12) and $\boldsymbol{C}_{rd}^{(k)}$ is

$$\boldsymbol{C}_{rd}^{(k)} = \frac{1}{M} \sum_{n=0}^{M-1} \boldsymbol{r}[n] d^{(k)}[n]^*. \quad (20)$$

Finally using the result of (19) we can obtain the weight vector at the $(k+1)$-th iteration as $\boldsymbol{w}^{(k+1)} = \boldsymbol{w}_f - \boldsymbol{B}\boldsymbol{w}_a^{(k+1)}$. This method that converts constrained problem into an unconstrained one is known as Generalized Sidelobe Canceller (GSC) in the area of adaptive signal processing. We summarize the proposed algorithm in Table 1.

| |
|---|
| [Initial step] |
| Compute $\boldsymbol{w}_f$, $\boldsymbol{B}$, $\boldsymbol{C}_{rr}$, and $\boldsymbol{T} \triangleq \boldsymbol{B}(\boldsymbol{B}^H \boldsymbol{C}_{rr} \boldsymbol{B})^{-1} \boldsymbol{B}^H$. |
| Set $\boldsymbol{w}_a^{(0)} = \boldsymbol{0}$. |
| [At the $k$-th iteration, $k$=0,1,2,$\cdots$] |
| Compute $\boldsymbol{y}^{(k)} = \boldsymbol{w}^{(k)H}\boldsymbol{R}$, $\hat{\boldsymbol{b}}^{(k)} = \boldsymbol{P}_{CM}(\boldsymbol{y}^{(k)})$, and $\boldsymbol{d}^{(k)} = \boldsymbol{w}_f^H \boldsymbol{R} - \hat{\boldsymbol{b}}^{(k)}$. |
| Compute $\boldsymbol{C}_{rd}^{(k)} = \frac{1}{M}\sum_{n=0}^{M-1} \boldsymbol{r}[n] d^{(k)}[n]^*$. |
| Compute $\boldsymbol{w}^{(k+1)} = \boldsymbol{w}_f - \boldsymbol{T}\boldsymbol{C}_{rd}^{(k)}$. |
| Continue until $\lvert \boldsymbol{w}^{(k+1)} - \boldsymbol{w}^{(k)} \rvert \le \varepsilon$. |

**Table 1**. Algorithm Summary

### 3.1. Blind Case

In a CDMA downlink, a receiver typically has the knowledge of its own signature sequence. Assuming user 1 is the desired user, we here put the following constraint on the LS cost function: $\bar{\boldsymbol{s}}_1^H \boldsymbol{w} = 1$. Then, we get a fixed weight vector

$$\boldsymbol{w}_f = \frac{\bar{\boldsymbol{s}}_1}{\bar{\boldsymbol{s}}_1^H \bar{\boldsymbol{s}}_1}. \quad (21)$$

As can easily be seen, the fixed weight vector is the normalized matched filter vector of the desired user. Then, according to (15) and (19) the weight vector at the $(k+1)$-th iteration is

$$\boldsymbol{w}^{(k+1)} = \frac{\bar{\boldsymbol{s}}_1}{\bar{\boldsymbol{s}}_1^H \bar{\boldsymbol{s}}_1} - \boldsymbol{B}\left(\boldsymbol{B}^H \boldsymbol{C}_{rr} \boldsymbol{B}\right)^{-1} \boldsymbol{B}^H \boldsymbol{C}_{rd}^{(k)}, \quad (22)$$

where notice that only $\boldsymbol{C}_{rd}^{(k)}$ needs to be updated every iteration. We call this case *blind LS-CMA*.

### 3.2. Group-blind Case

We consider the case where some of the codes are known and some are unknown. The CDMA uplink as well as the downlink of certain systems, such as UMTS TDD, are good examples. For this case, the constraint is

$$\breve{\bar{\boldsymbol{S}}}^H \boldsymbol{w} = \boldsymbol{1}_k \quad (23)$$

where $\breve{\bar{\boldsymbol{S}}}$ is the known code matrix and $\boldsymbol{1}_k$ is a vector containing zero elements except the $k$-th element, which is one. Then, the fixed weight vector is

$$\boldsymbol{w}_f = \breve{\bar{\boldsymbol{S}}}(\breve{\bar{\boldsymbol{S}}}^H \breve{\bar{\boldsymbol{S}}})^{-1} \boldsymbol{1}_k. \quad (24)$$

The fixed weight vector is the decorrelating detector assuming that there exist only $\breve{K}$ users with known codes. Then, the weight vector at the $(k+1)$-th iteration is

$$\boldsymbol{w}^{(k+1)} = \breve{\bar{\boldsymbol{S}}}(\breve{\bar{\boldsymbol{S}}}^H \breve{\bar{\boldsymbol{S}}})^{-1} \boldsymbol{1}_k - \boldsymbol{B}\left(\boldsymbol{B}^H \boldsymbol{C}_{rr} \boldsymbol{B}\right)^{-1} \boldsymbol{B}^H \boldsymbol{C}_{rd}^{(k)}, \quad (25)$$

where only $\boldsymbol{C}_{rd}^{(k)}$ is updated every iteration. The more codes we know the less dimensions of the weight vector we need to iterate. We call this case *group-blind LS-CMA*. Fig.1 shows the general block diagram for blind and group-blind LS-CMA.

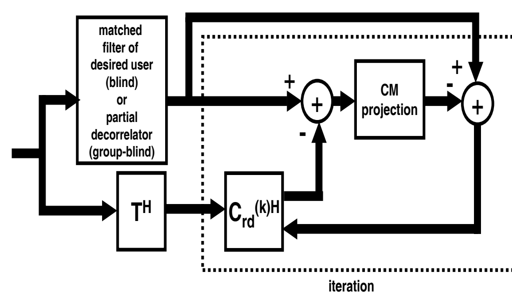


**Fig. 1**. Block Figure

## 4. SIMULATION RESULTS

In this Section we provide a few simulation results of the proposed blind and group-blind LS-CMA from the signal to interference plus noise ratio (SINR) viewpoint. The systems we consider here are a synchronous with negligible ISI and an asynchronous multipath system with ISI. The processing gain N is 11 with randomly generated spreading sequences. The randomly generated channels which have length between 1 and 15 are assumed to be known at the receiver. The number of users in the simulation is 7, with 5 known in-cell users and 2 unknown intercell users with strong power. The desired user is assumed to be the first in-cell user. The transmitted signals are QPSK modulated, which has CM property. For an ultimate performance we use the ideal MMSE detector with all codes known. To terminate the iteration we set $\varepsilon$ as 1e-3.

Fig.2 and Fig.3 illustrate the output SINR of the proposed algorithm as well as the original blind and group-blind algorithms [4, 7] and the ideal MMSE detector. Both blind and group-blind LS-CMA outperform the original blind/group-blind algorithms, and approach the ideal MMSE detector in the high SNR region.

In order to show how the proposed detector increase the SINR on each iteration and how the number of symbols effect the performance, we show the output SINR on each iteration with respect to the number of symbols $M$ in Fig.4 and Fig.5. For both blind and group-blind cases, a couple of iteration is enough for the proposed algorithms to converge.

## 5. CONCLUSIONS

We have presented a block iterative CMA-based multiuser detection algorithm. The proposed algorithm can be used in a blind or a group-blind setting. Simulation results showed that these detectors offer substantial performance improvement over the original blind and group-blind detectors and approach the ideal MMSE detector in the high SNR region. Further, the proposed algorithm does not suffer from the interference capture problem existing in most CMA algorithms.

## 6. REFERENCES


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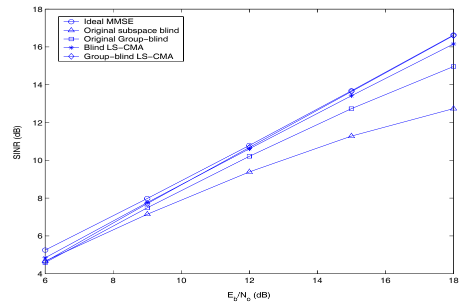


**Fig. 2**. Output SINR for a synchronous CDMA system

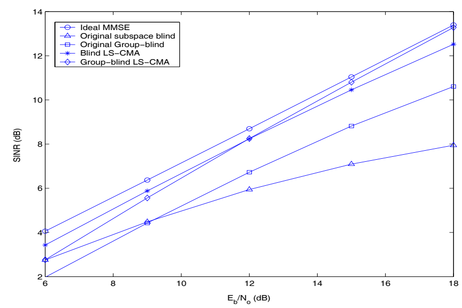


**Fig. 3**. Output SINR for an asynchronous CDMA system

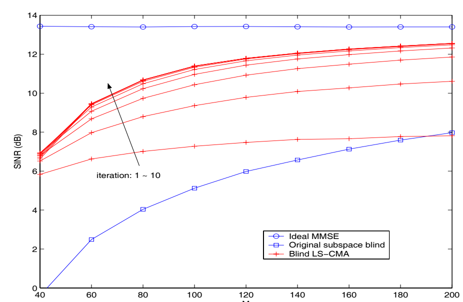


**Fig. 4**. SINR for blind algorithm with respect to number of symbols (M) for an asynchronous CDMA system

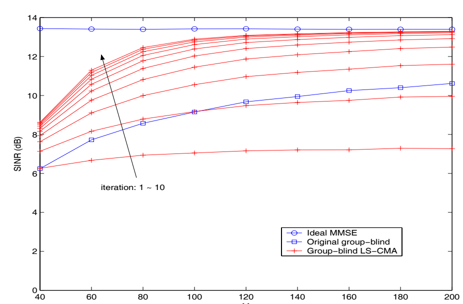


**Fig. 5**. SINR for group-blind algorithm with respect to number of symbols (M) for an asynchronous CDMA system