ROBUST BLIND MULTIUSER DETECTION BASED ON WORST-CASE MMSE PERFORMANCE OPTIMIZATION

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ABSTRACT

In this paper, we propose a new blind multiuser receiver which is robust against the effects of erroneously presumed desired user signature and short data length. Our approach is based on the explicit modeling of possible mismatches in the mean-square error cost function and worst-case performance optimization. We show that this approach leads to a multiuser receiver which uses the data covariance matrix with an *adaptive diagonal loading*. Simulation results show performance improvements achieved by our approach relative to existing techniques.

1. INTRODUCTION

Linear multiuser detection techniques can be classified into two categories: training-based (non-blind) and blind algorithms. The main drawback of training-based multiuser receivers is that they may have quite poor performances when the length of the training sequence is short and/or when the channel impulse response of the desired user varies rapidly.

As an alternative, one can employ blind multiuser detection techniques which are based entirely on the spreading code of the desired user and do not exploit any channel impulse response information [1]. However, the performance of most of these techniques may degrade substantially in scenarios with low signal-to-noise ratios (SNRs) and short data lengths.

An effective approach to overcome the aforementioned shortcomings of multiuser detection methods is to introduce robustness in the detection procedure. For example, robust multiuser receivers which are based on the *diagonally loaded* minimum output energy (MOE) approach have been presented in [2] and [3]. However, the main shortcoming of these receivers is that it is not clear how to obtain the optimal value of the diagonal loading factor. Motivated by this drawback of the diagonal loading technique, the authors of [4] and [5] have proposed multiuser detectors that explicitly model an arbitrary (but norm-bounded) uncertainties in the desired user signature and use *worst-case* performance optimization to improve the robustness of the MOE receiver. However, the approach of [4] is not suitable for on-line implementation, whereas the approach of [5] suffers from the fact that it optimizes the lower bound of the worst-case performance rather than the worst-case performance itself.

In this paper, we use the MMSE multiuser detection approach along with the idea of worst-case performance optimization to develop a new blind multiuser receiver which is robust against possible uncertainties in the mean-square error (MSE) cost function. We show that this approach is equivalent to the diagonal loading-based multiuser receiver with the *optimal* choice of the diagonal loading factor obtained based on the known level of uncertainty in the desired signal signature. A computationally efficient algorithm is proposed for our robust multiuser receiver. In contrast to the algorithm [4], our technique is suitable for on-line implementation.

2. DATA MODEL

Let us consider a K-user synchronous CDMA system with short spreading codes (for which the chip sequence period coincides with the symbol period T_s). We assume that the quasi-static channel FIR impulse response is much shorter than T_s, so that the effect of inter-symbol-interference (ISI) can be neglected [1], [2]. However, the duration of the channel impulse response can be comparable to the chip period T_c, so that there can be a substantial inter-chip-interference (ICI) [1], [3]. We also assume that user data symbols are zero-mean independent random variables which are equally likely drawn from the BPSK constellation.

Sampling the received data at $t = nT_s + pT_c$ for $p = 0, 1, \ldots, L-1$ and using the vector notation, we obtain the following familiar model [2], [4]

$$\mathbf{x}(n) = \sum_{k=1}^{K} A_k b_k(n)\mathbf{s}_k + \mathbf{v}(n) \tag{1}$$

where A_k is the received signal amplitude of the kth user, $b_k(n)$ is the nth data symbol of this user, $\mathbf{x}(n) = [x(nT_s), x(nT_s + T_c), \ldots, x(nT_s + (L-1)T_c)]^T$ is the received data vector, $\mathbf{s}_k = [s_k(0), s_k(T_c), \ldots, s_k((L-1)T_c)]^T$ is the signature vector of the kth user, $s_k(t) = \sum_{l=0}^{L-1} c_k(l) g_k(t - lT_c)$ is its normalized signature waveform ($\int_{T_s} |s_k(t)|^2 dt = 1$), $\mathbf{v}(n) = [v(nT_s), v(nT_s + T_c), \ldots, v(nT_s + (L-1)T_c)]^T$ is the noise vector, $g_k(t)$ is the chip waveform convolved with the kth user channel impulse response, $\mathbf{c}_k = [c_k(0), c_k(1), \ldots, c_k(L-1)]^T$ is the spreading code vector of this user, L is the spreading factor (i.e., $T_c = T_s/L$), $v(t)$ is the zero-mean additive random noise with the variance σ^2, and $(\cdot)^T$ stands for the transpose. In ICI-free scenarios, $g_k(t)$ spans only one chip, while in dispersive channels $g_k(t)$ can span several chips and this can cause ICI.

3. CONVENTIONAL MMSE RECEIVERS

The output of a linear multiuser receiver is given by [1]

$$y(n) = \mathbf{f}^H \mathbf{x}(n) \tag{2}$$

where $\mathbf{f} = [f_0, f_1, \ldots, f_{L-1}]^T$ is an $L \times 1$ complex vector of the receiver coefficients, and $(\cdot)^H$ stands for the Hermitian transpose.

In the MMSE approach, the receiver coefficient vector $\mathbf{f}$ is designed to minimize the mean-square error between the desired user symbol and the receiver output, so that

$$\mathbf{f}_{\rm opt} \quad = \quad \arg\min_{\mathbf{f}} \mathrm{E}\{|b_1(n) - \mathbf{f}^H \mathbf{x}(n)|^2\} \qquad (3)$$

where the first user is assumed to be the desired one. The optimal vector $\mathbf{f}_{\rm opt}$ is given by the classic Wiener formula $\mathbf{f}_{\rm opt} = \mathbf{R}^{-1}\mathbf{d}$ where $\mathbf{R} = \mathrm{E}\{\mathbf{x}(n)\mathbf{x}^H(n)\}$ is the data covariance matrix, $\mathbf{d} = \mathrm{E}\{\mathbf{x}(n)b_1^*(n)\}$ is the correlation vector, and $(\cdot)^*$ denotes the complex conjugate.

Note that under the assumptions made, it is easy to show that $\mathbf{d} = \beta\,\mathbf{s}_1$ where $\beta \triangleq A_1 \mathrm{E}\{|b_1(n)|^2\}$. As multiplying $\mathbf{f}_{\rm opt}$ by any positive constant does not affect the probability of error at the output of the symbol detector, we obtain another multiuser receiver,

$$\tilde{\mathbf{f}}_{\rm opt} = \mathbf{R}^{-1}\mathbf{s}_1 \qquad (4)$$

The "tilde" here stresses that although (4) is equivalent in the bit error rate (BER) performance to the MMSE receiver, the MSEs of these receivers are different because (4) does not minimize the MSE.

In practice, the exact knowledge of the desired user signature $\mathbf{s}_1$ is often unavailable. In this case, one can use $\mathbf{c}_1$ instead of $\mathbf{s}_1$ [2]-[4]. This corresponds to the blind receiver that can be written as

$$\mathbf{f}_{\rm blind} = \mathbf{R}^{-1}\mathbf{c}_1 \qquad (5)$$

Unfortunately, the receiver (5) is very sensitive to the difference between $\mathbf{c}_1$ and $\mathbf{s}_1$ [2]-[4].

In practice, the exact covariance matrix $\mathbf{R}$ is unavailable and is replaced by its sample estimate $\hat{\mathbf{R}} = \frac{1}{N}\sum_{n=1}^{N} \mathbf{x}(n)\mathbf{x}^H(n)$ where N is the number of data vectors in the observation period. Using $\hat{\mathbf{R}}$, the multiuser receivers (4) and (5) can be written as

$$\tilde{\mathbf{f}}_{\rm opt} = \hat{\mathbf{R}}^{-1}\mathbf{s}_1\,, \quad \mathbf{f}_{\rm blind} \quad = \quad \hat{\mathbf{R}}^{-1}\mathbf{c}_1 \qquad (6)$$

In scenarios with a short data length, the performance of the multiuser receivers (6) can degrade severely [3]. To provide robustness against short data length effects, it has been proposed in [2] and [3] to use the so-called *diagonal loading* technique whose essence is to replace $\hat{\mathbf{R}}$ by $\hat{\mathbf{R}} + \gamma\mathbf{I}$ where γ is the loading factor and $\mathbf{I}$ is the identity matrix, that is

$$\mathbf{f}_{\rm dl} = (\hat{\mathbf{R}} + \gamma\mathbf{I})^{-1}\mathbf{c}_1 \qquad (7)$$

Although the receiver (7) is known to (potentially) provide an improved robustness against short data length effects and signature mismatches, it is not clear how to choose the diagonal loading factor.

4. ROBUST MULTIUSER DETECTION

The MMSE receiver (3) assumes that $\mathbf{d}$ and $\mathbf{R}$ are exactly known. In practice, these values are known with certain errors. Let us consider the error $\mathbf{e} = \mathbf{s}_1 - \tilde{\mathbf{s}}_1$ between the *actual* desired user signature $\mathbf{s}_1$ and its *presumed* (e.g., estimated) value $\tilde{\mathbf{s}}_1$. Let the error vector $\mathbf{e}$ be norm-bounded by some known constant $\varepsilon > 0$, that is, let $\|\mathbf{e}\| \leq \varepsilon$, where $\|\cdot\|$ denotes the vector l_2 norm. Then, we obtain that $\mathbf{d} = \tilde{\mathbf{d}} + \beta\mathbf{e}$ where $\tilde{\mathbf{d}} \triangleq \beta\,\tilde{\mathbf{s}}_1$ is the presumed correlation vector.

Similarly, let us consider the error $\mathbf{E} = \mathbf{R} - \hat{\mathbf{R}}$ between the true data covariance matrix and its sample estimate. We assume that $\mathbf{E}$ is bounded in its Frobenius norm by some known constant $\gamma > 0$, i.e., $\|\mathbf{E}\|_{\mathcal{F}} \leq \gamma$.

To incorporate robustness against such norm-bounded errors $\mathbf{e}$ and $\mathbf{E}$, let us modify the MMSE optimization problem in (3) as

$$\min_{\mathbf{f}} \max_{\substack{\|\mathbf{E}\|_{\mathcal{F}} \leq \gamma \\ \|\mathbf{e}\| \leq \varepsilon}} \left\{ \mathbf{f}^H(\hat{\mathbf{R}} + \mathbf{E})\mathbf{f} - \mathbf{f}^H(\tilde{\mathbf{d}} + \beta\mathbf{e}) - (\tilde{\mathbf{d}} + \beta\mathbf{e})^H\mathbf{f} \right\} \qquad (8)$$

The problem statement in (8) guarantees that the MSE cost function is minimized for the worst-case scenario which corresponds to the largest value of the MSE over all possible norm-bounded errors in the desired user signature and data covariance matrix. Therefore, the proposed design should improve the MMSE receiver robustness via protecting its performance against worst-case errors.

The problem in (8) can be rewritten as

$$\min_{\mathbf{f}} \Big\{ \max_{\|\mathbf{E}\|_{\mathcal{F}} \leq \gamma} \{\mathbf{f}^H(\hat{\mathbf{R}} + \mathbf{E})\mathbf{f}\} + \max_{\|\mathbf{e}\| \leq \varepsilon} \{-\mathbf{f}^H(\tilde{\mathbf{d}} + \beta\mathbf{e}) - (\tilde{\mathbf{d}} + \beta\mathbf{e})^H\mathbf{f}\} \Big\} \qquad (9)$$

To simplify (9), the following two lemmas will be used.

Lemma 1. For any Hermitian $\mathbf{E}$ *and* $\hat{\mathbf{R}}$ *and any fixed* $\mathbf{f}$*,*

$$\max_{\|\mathbf{E}\|_{\mathcal{F}} \leq \gamma} \mathbf{f}^H(\hat{\mathbf{R}} + \mathbf{E})\mathbf{f} = \mathbf{f}^H(\hat{\mathbf{R}} + \gamma\mathbf{I})\mathbf{f} \qquad (10)$$

Lemma 2. For any fixed $\mathbf{f}$*,*

$$\max_{\|\mathbf{e}\| \leq \varepsilon} \{-\mathbf{f}^H(\tilde{\mathbf{d}} + \beta\mathbf{e}) - (\tilde{\mathbf{d}} + \beta\mathbf{e})^H\mathbf{f}\} = -\mathbf{f}^H\tilde{\mathbf{d}} - \tilde{\mathbf{d}}^H\mathbf{f} + 2\varepsilon\beta\,\|\mathbf{f}\| \qquad (11)$$

Using (10) and (11), we can transform (9) to

$$\min_{\mathbf{f}} \{\mathbf{f}^H(\hat{\mathbf{R}} + \gamma\mathbf{I})\mathbf{f} - \mathbf{f}^H\tilde{\mathbf{d}} - \tilde{\mathbf{d}}^H\mathbf{f} + 2\varepsilon\beta\|\mathbf{f}\|\} \qquad (12)$$

Differentiating the objective function in (12) with respect to $\mathbf{f}^H$ and equating it to zero, we obtain that the solution to (12) satisfies the equation

$$(\hat{\mathbf{R}} + \gamma\mathbf{I})\mathbf{f} + \varepsilon\beta\mathbf{f}/\|\mathbf{f}\| = \beta\,\tilde{\mathbf{s}}_1 \qquad (13)$$

To solve (13) directly, one needs to know β or, equivalently, A_1. In order to avoid this difficulty, let us rescale the vector $\mathbf{f}$ by the factor of β (note that, rescaling $\mathbf{f}$ by an arbitrary constant, we do not change the probability of error of any linear multiuser receiver). Taking into account that our ultimate goal is the probability of error performance and using, for the sake of simplicity, the same notation $\mathbf{f}$ for the rescaled vector, we rewrite (13) as

$$(\hat{\mathbf{R}} + \gamma\mathbf{I})\mathbf{f} + \varepsilon\mathbf{f}/\|\mathbf{f}\| = \tilde{\mathbf{s}}_1 \qquad (14)$$

Now, the knowledge of β (or A_1) is not required.

To solve (14), let us rewrite it as

$$\mathbf{f} = (\hat{\mathbf{R}} + (\gamma + \varepsilon/\|\mathbf{f}\|)\mathbf{I})^{-1}\tilde{\mathbf{s}}_1 \qquad (15)$$

We observe that the robust multiuser receiver (15) uses an adaptive diagonal loading factor $\gamma + \varepsilon/\|\mathbf{f}\|$ which depends on $\|\mathbf{f}\|$. This factor is optimally matched to known amounts of uncertainty in the desired user signature and data covariance matrix. From (15), it follows that, if $\|\mathbf{f}\|$ is available, then we can use (15) to calculate the coefficient vector of the proposed robust blind receiver. In what follows, we propose a simple method to determine $\|\mathbf{f}\|$.

Taking the norm of the both sides of (15), we have $\|\mathbf{f}\|^2 = \|(\hat{\mathbf{R}} + (\gamma + \varepsilon/\|\mathbf{f}\|)\mathbf{I})^{-1}\tilde{\mathbf{s}}_1\|^2$. Introducing $\tau \triangleq \|\mathbf{f}\| > 0$ we obtain that solving this equation is equivalent to finding a positive value of τ such that $\tau^2 = \|(\hat{\mathbf{R}} + (\gamma + \varepsilon/\tau)\mathbf{I})^{-1}\tilde{\mathbf{s}}_1\|^2$. Write the eigendecomposition of $\hat{\mathbf{R}}$ as $\hat{\mathbf{R}} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$ where $\mathbf{U}$ is the $L \times L$ unitary matrix whose columns are the eigenvectors of $\hat{\mathbf{R}}$ and $\mathbf{\Lambda}$ is the diagonal matrix of the real positive eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_L > 0$ of $\hat{\mathbf{R}}$. Then, we have

$$\|\mathbf{U}\mathbf{\Psi}^{-1}(\tau)\mathbf{U}^H\tilde{\mathbf{s}}_1\|^2 - \tau^2 = 0 \qquad (16)$$

where $\mathbf{\Psi}(\tau) \triangleq \mathbf{\Lambda} + (\gamma + \varepsilon/\tau)\mathbf{I}$. Introducing the $L \times 1$ vector $\check{\mathbf{s}} = [\check{s}_1, \ldots, \check{s}_L]^T \triangleq \mathbf{U}^H\tilde{\mathbf{s}}_1$, we can express the r.h.s. of (16) as

$$\|\mathbf{U}\mathbf{\Psi}^{-1}(\tau)\mathbf{U}^H\tilde{\mathbf{s}}_1\|^2 - \tau^2 = \left[\sum_{i=1}^{L}\left(\frac{|\check{s}_i|}{\varepsilon + \tau(\lambda_i + \gamma)}\right)^2 - 1\right]\tau^2 \qquad (17)$$

Using (17) and noting that $\tau > 0$, we obtain that solving (16) is equivalent to finding a positive value for τ such that

$$f(\tau) \triangleq \sum_{i=1}^{L}\left(\frac{|\check{s}_i|}{\varepsilon + \tau(\lambda_i + \gamma)}\right)^2 - 1 = 0 \qquad (18)$$

The following lemma states the necessary and sufficient conditions under which (18) has a unique positive solution.

Lemma 3. Equation (18) *has a unique real-valued and positive solution if and only if* $\|\tilde{\mathbf{s}}_1\| > \varepsilon$.

An intuitively appealing interpretation of Lemma 3 is that our approach is applicable only if the maximum of the norm of the error $\mathbf{e}$ does not exceed the norm of the presumed desired user signature itself. We assume that this condition is always satisfied.

Using (18), we can upper-bound the function $f(\tau)$ as

$$f(\tau) < \frac{\|\tilde{\mathbf{s}}_1\|^2}{(\varepsilon + \tau(\lambda_L + \gamma))^2} - 1 \triangleq f_{\mathrm{up}}(\tau) \qquad (19)$$

Noting that $f(\tau)$ and $f_{\mathrm{up}}(\tau)$ are both decreasing functions for positive values of τ and that, according to Lemma 3, the root τ of $f(\tau)$ is positive, we obtain from (19) that this root is always smaller than the root $\tau_{\mathrm{up}} = \dfrac{\|\tilde{\mathbf{s}}_1\| - \varepsilon}{\lambda_L + \gamma}$ of $f_{\mathrm{up}}(\tau)$. Hence, the value of τ belongs to the interval $(0, \tau_{\mathrm{up}})$. With this condition, the problem of finding τ becomes standard using the algorithm of [6] which consists of a binary search followed by Newton-Raphson iterations. The binary search technique is used to obtain a proper initial point for the subsequent Newton-Raphson procedure. As shown in [6], this algorithm converges to a ζ-neighborhood of τ in $\mathcal{O}(\log\log(\tau_{\mathrm{up}}/\zeta))$ iterations. The algorithm to obtain $\|\mathbf{f}\|$ can be summarized as follows:

1. Use binary search to find $\tau_0 \in (0, \tau_{\mathrm{up}})$ such that $f(\tau_0) > 0$ and $f(\frac{13}{12}\tau_0) < 0$ (see [6] for details).
2. Set $l = 1$ and select a small positive value of ξ which will be used in the algorithm stopping criterion.
3. Obtain τ_l as $\tau_l = \tau_{l-1} - f(\tau_{l-1})/f'(\tau_{l-1})$ where $f'(\tau_{l-1})$ is the derivative of $f(\tau)$ at $\tau = \tau_{l-1}$.
4. If $|f(\tau_l)| < \xi$, go to the next step. Otherwise, repeat steps 2 and 3.
5. Determine $\|\mathbf{f}\|$ as $\tau = \tau_l$.

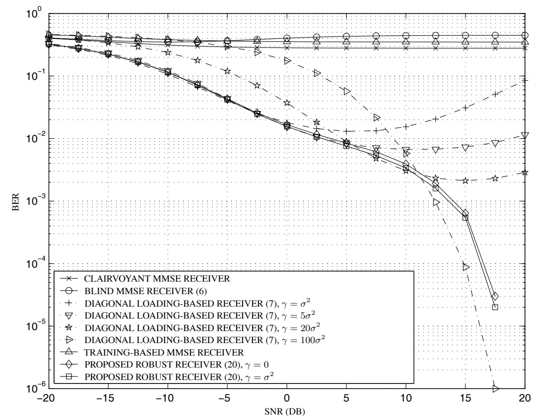

Figure 1: BERs versus SNR.

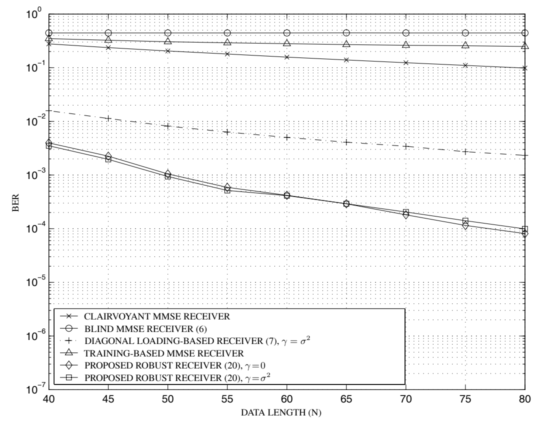

Figure 2: BERs versus N.

As the procedure developed above enables us to find the value of $\|\mathbf{f}\|$, it can be directly used to compute the coefficient vector of our multiuser receiver. Using this fact, the proposed multiuser detection algorithm can be summarized as follows:

1. Compute the sample covariance matrix $\hat{\mathbf{R}}$ and find its eigendecomposition.
2. Compute $\check{\mathbf{s}} = \mathbf{U}^H\tilde{\mathbf{s}}_1$ and find the value of $\tau = \|\mathbf{f}\|$ using the Newton-Raphson procedure.
3. Compute the receiver coefficient vector as $\mathbf{f}_{\mathrm{rob}} = (\hat{\mathbf{R}} + (\gamma + \varepsilon/\tau)\mathbf{I})^{-1}\tilde{\mathbf{s}}_1$.

If the channel is unknown, $\mathbf{c}_1$ can be used as a presumed desired user signature [2]-[4]. In this case, the receiver in the last step can be rewritten as

$$\mathbf{f}_{\mathrm{rob}} = (\hat{\mathbf{R}} + (\gamma + \varepsilon/\tau)\mathbf{I})^{-1}\mathbf{c}_1 \qquad (20)$$

5. SIMULATION RESULTS

We consider a 7-user synchronous CDMA system. The BPSK modulation scheme is used and binary Gold codes of the length $L = 31$ are employed as user spreading codes. The interfering

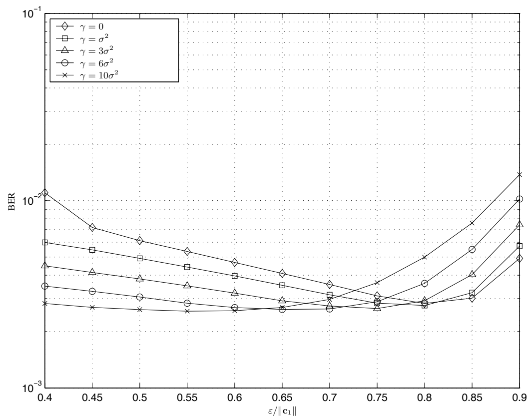

Figure 3: BERs of the proposed receiver versus $\varepsilon/\|\mathbf{c}_1\|$.

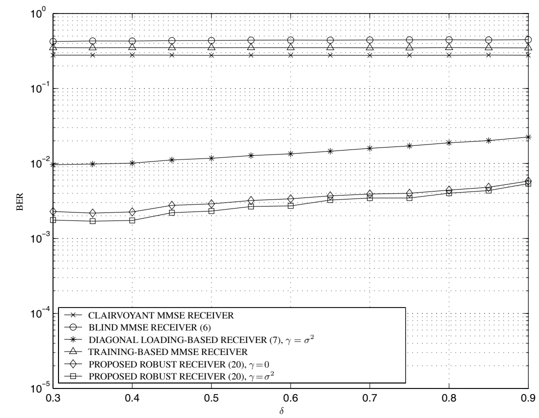

Figure 4: BERs versus δ.

users are assumed to have the interference-to-noise-ratio (INR) equal to 20 dB.

The performances of the following multiuser receivers are compared in terms of the BER at the output of the symbol detector: the *clairvoyant* Wiener receiver which corresponds to the ideal case when the desired user signature $\mathbf{s}_1$ is known exactly (this algorithm is considered for comparison reasons only); the blind multiuser receiver (6); the diagonal loading-based blind multiuser receiver (7) with different *ad hoc* values of γ; the training-based MMSE multiuser receiver; and the proposed blind receiver (20).

In the training-based receiver, 30 samples are used to estimate $\mathbf{d}$. A total of 1000 runs is used to obtain each point of the BER curves.

To model the effect of multipath channel, each of the user spreading codes is distorted by an additive random Gaussian vector drawn uniformly from the interval $[-\delta, \delta]$. For each user, such a random vector is added to the spreading code vector to simulate the effect of ICI [4]. The upper bound for the norm of the error vector $\mathbf{e}$ is equal to $\delta\sqrt{L}$, and hence, $\varepsilon = \delta\sqrt{L}$ has been chosen.

Figure 1 shows the BERs of the multiuser receivers tested versus the SNR of the desired user. In this figure, $N = 40$ data vectors are used to obtain $\hat{\mathbf{R}}$, and $\delta = 0.7$ is chosen. Note that this choice of δ implies that the amount of ICI per chip is up to 70%. As it can be seen from Figure 1, our robust multiuser receiver provides the best performance tradeoff over all SNR values. It can be observed that the clairvoyant multiuser receiver shows a poor performance which is due to the short data length effect. Furthermore, the BER of the diagonal loading-based multiuser receiver does not decrease monotonically when the SNR increases. Note that our robust multiuser receiver (20) uses an adaptive diagonal loading factor whose value varies with the SNR and is optimally matched to the uncertainties in the presumed desired user signature and data covariance matrix. This explains why the performance of (20) is good over a wide range of SNR.

Figure 2 shows the BERs of the multiuser receivers tested versus the data length N. In this figure, SNR = 10 dB, $\delta = 0.7$ and $\varepsilon = \delta\sqrt{L}$ are chosen. We can observe that the proposed multiuser receiver has substantially faster convergence rate as compared to the other multiuser detection techniques.

To study the effect of selection of the parameters ε and γ, the BER of our robust multiuser receiver is shown in Fig. 3 versus $\varepsilon/\|\mathbf{c}_1\| = \varepsilon/\sqrt{L}$ for different values of γ. Here, SNR = 10 dB, $\delta = 0.7$ and $N = 40$. As we see from Fig. 3, when γ is comparable to the noise power σ^2, the performance of the proposed method is less sensitive to ε compared to the case when $\gamma = 0$ or when γ is much larger than σ^2.

To study the effect of an improper choice of δ, the BERs are shown versus δ in Fig. 4 for SNR = 10 dB, $N = 40$, and $\varepsilon/\sqrt{L} = 0.7$. Such a choice of ε implies that we assume that $\delta = 0.7$ while the actual value of δ varies between 0.3 and 0.9. As can be seen from Fig. 4, overestimating δ can even improve the performance of the proposed multiuser receiver while underestimating δ does not affect the performance significantly.

6. CONCLUSIONS

A new blind multiuser receiver has been proposed which is robust against the effects of erroneous presumed desired user signature and short data length. Our approach is based on the explicit modeling of possible mismatches in the mean-square error cost function and worst-case performance optimization.

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