Iterative MAP Multiuser Detection for Constant Modulus Constellations in Synchronous CDMA

Shannon D. Blunt
Radar Division
US Naval Research Laboratory

K.C. Ho
Department of Electrical and Computer Engineering
University of Missouri-Columbia

ABSTRACT

This paper generalizes previous work on BPSK [1] to multiuser detection for constant modulus constellations in synchronous CDMA communication systems based on maximum *a posteriori* (MAP) estimation. The known finite alphabet of discrete transmitted symbols is approximated as a stochastic parameter with Gaussian distributions centered at the true values of the symbol constellation. This allows for the development of a MAP estimator that takes *a priori* knowledge of the symbol constellation into account in order to improve estimation accuracy. We examine the performance of the proposed algorithm for BPSK, QPSK, and 8PSK and compare the results with other well-known multiuser detection techniques.

1. INTRODUCTION

In a code-division multiple access (CDMA) communication system, the transmitted symbols from different users simultaneously occupy the same frequency band and hence act as multiple access interference (MAI) to one another. Separation of the user signals at the base-station receiver is accomplished by exploiting each user's known length *N* signature code as a means of discrimination. This paper extends previous work for the synchronous reception of BPSK modulated user signals to general constant modulus user signals.

Each user's binary data are encoded into one of *M* predetermined complex symbols. The complex symbols are denoted as $g_{i,k}(n)$ where $i \in [1,2,\ldots,M]$ indicates the distinct symbol for each of the $k = 1,2,\cdots,K$ users and n is the symbol index. For the k^{th} user at time n, the symbol $g_{i,k}(n)$ is multiplied by the signature code $\mathbf{s}_k$ so that the transmitted baseband discrete time signal is $g_{i,k}(n)\mathbf{s}_k$. Without loss of generality, we will henceforth suppress the symbol index, n.

For *K* synchronously received CDMA signals in additive white Gaussian noise (AWGN) without channel distortion, the received signal sampled at the chip-rate over a symbol period is

$$\mathbf{r} = \sum_{k=1}^{K} \lambda_k d_k \mathbf{s}_k + \mathbf{z} = \mathbf{S\Lambda d} + \mathbf{z} \tag{1}$$

where the vector $\mathbf{d} = [d_1 \; d_2 \; \cdots \; d_K]^T$ is the collection of all *K* users' complex received symbols, $\mathbf{\Lambda} = diag\{\lambda_1, \ldots, \lambda_K\}$ is a diagonal matrix of the respective received signal amplitudes (assumed real-valued and known to within some reasonable estimation error), $\mathbf{S}$ is the $N \times K$ CDMA code matrix with columns $\mathbf{s}_1, \ldots, \mathbf{s}_K$, and $\mathbf{z}$ is an $N \times 1$ complex AWGN vector with noise power σ_z^2. In order to perform continuous domain estimation, the finite set of discrete symbols $g_{i,k}$ at the transmitter becomes the continuous random variable d_k at the receiver. Without loss of generality, after normalization by the largest user's receiver signal amplitude, λ_k is assumed to be between 0 and 1. Recovering the users' data at the receiver therefore requires separating the *K* user's CDMA signals and then selecting the discrete symbol estimate $\hat{g}_{i,k}$ that most closely matches the continuous symbol estimate $\hat{d}_k$.

The conventional approach, otherwise known as the matched filter, estimates received symbols as

$$\hat{\mathbf{d}}_{MF} = \mathbf{y} = \mathbf{S}^H \mathbf{r} \tag{2}$$

where $[\bullet]^H$ is the conjugate transpose, or hermitian, operation. The matched filter performs poorly when the cross-correlations between user signature codes are high, when there are several simultaneous users, or when there is a large difference in the received powers among users, known as the "near-far effect".

The seminal work of Verdu [2] demonstrated that, in terms of bit error rate (BER), an optimum multiuser detector exists but requires a complexity that is exponential in the number of users, *K*. This has led to the development of sub-optimum detectors that attempt to approach the performance of the optimum detector while maintaining a practical computational cost.

Lupas and Verdu [3] proposed a family of sub-optimum detectors called decorrelators and demonstrated that a linear mapping of the matched filter output can result in a substantial improvement in estimation accuracy. This mapping is based on maximum likelihood estimation and is found by maximizing the likelihood of $\mathbf{d}$ for *K* superimposed CDMA signals in AWGN

$$p(\mathbf{r}\,|\,\mathbf{d}) = C_z^N \exp\left(\frac{-1}{\sigma_z^2}(\mathbf{r} - \mathbf{S\Lambda d})^H(\mathbf{r} - \mathbf{S\Lambda d})\right) \tag{3}$$

where $C_z = 1/(\pi\sigma_z^2)$. The decorrelating detector is quite robust in that it requires no knowledge of the received amplitudes $\mathbf{\Lambda}$. However, it may suffer from noise enhancement when the correlation matrix is poorly conditioned.

To reduce the effects of noise enhancement in the decorrelating detectors, minimum mean square-error (MMSE) detectors were proposed in [4] that, because they take background noise into account, balance the trade-off between reducing MAI and minimizing noise in the detector output.

Another class of detectors is known as subtractive interference cancellers whose principal objective is to estimate the MAI generated by each user so that it can be subtracted from other users. Interference subtraction can be performed either serially, as a successive interference canceller (SIC) [5]-[6], or in parallel, as a parallel interference canceller (PIC) [7]-[8].

The approach we propose differs significantly from those previously taken. Past approaches have either treated the received symbols as deterministic parameters and used the *a priori* knowledge about the symbol constellation only for projection after estimation or have treated them as stochastic parameters but limited them to belong to a finite set of discrete values. We relax the latter constraint by formulating the received symbols as stochastic *a priori* information in the form of a continuously differentiable distribution that can be viewed as an approximation to the discrete distribution, and use this *a priori* knowledge during the estimation process to improve performance. As depicted in Fig. 1 for QPSK, the discrete distribution for a constellation of M symbols is essentially comprised of impulses located at the true symbol values each with probability $1/M$. A straightforward approximation would then be to allow each impulse to become a complex Gaussian distribution which is also illustrated in Fig. 1.

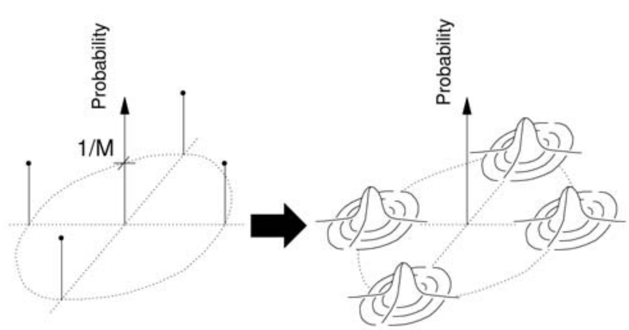

Fig. 1. Approximating the discrete symbol constellation with symbol-centered Gaussians

The motivation for this approximation is that the symbol constellation PDF is now differentiable so it can be combined with the likelihood function in (3) to yield a maximum *a posteriori* formulation which is known to provide better results than maximum likelihood estimation. From this formulation we derive an iterative technique that is capable of improving the BER significantly over other multiuser detection techniques.

This paper is organized as follows. Section II introduces the model used for the continuous approximation of the discrete symbol constellation. Section III develops the proposed iterative MAP multiuser detection algorithm. Section IV provides simulation results for binary, quadrature, and 8-ary PSK and compares the MAP performance against other multiuser detection techniques. Finally, Section V is the conclusion.

2. MODELING THE DISCRETE SYMBOL CONSTELLATION

The probability distribution of a particular discrete symbol constellation can be viewed as impulses located at the M complex symbol values, each with probability $1/M$. When the signal passes through a stochastic channel, each symbol value tends to become a continuous random variable due to many unaccountable effects. Therefore, instead of an impulse at each symbol value, we model the received symbol constellation as having a complex Gaussian distribution centered at each discrete symbol value. Note that a Rayleigh distribution is not appropriate here because we are not modeling the received amplitude. Instead, this constellation model is analogous to the effects generated by errors in estimating the received signals.

The complex Gaussian approximation of the discrete symbol g_i for user k is

$$p(d_k)_i = \frac{1}{\pi\sigma_k^2}\exp\left\{\frac{-1}{\sigma_k^2}(\lambda_k d_k - \lambda_k g_i)^*(\lambda_k d_k - \lambda_k g_i)\right\} \quad (4)$$

where $(\bullet)^*$ is the complex conjugate operator, σ_k^2 is the variance of the k^{th} user's symbol estimate, and the subscript k in $g_{i,k}$ is suppressed for simplicity. The user variance σ_k^2 is dependent on the proximity of the symbol estimate $\hat{d}_k$ to the discrete symbol values g_i and is discussed in detail in [9].

To determine the complete *a priori* PDF for the k^{th} user, for M equally likely symbols, we have

$$p(d_k) = \frac{1}{M}\sum_{i=1}^{M} C_k \exp\left\{\frac{-\lambda_k^2}{\sigma_k^2}(d_k - g_i)^*(d_k - g_i)\right\} \quad (5)$$

where $C_k = 1/(\pi\sigma_k^2)$.

Finally, because the symbols from different users are independent, the *a priori* PDF for the collection of all K users is found by multiplying the PDFs for all respective users as

$$p(\mathbf{d}) = \prod_{k=1}^{K}\frac{C_k}{M}\left(\sum_{i=1}^{M}\exp\left\{\frac{-\lambda_k^2}{\sigma_k^2}(d_k - g_i)^*(d_k - g_i)\right\}\right). \quad (6)$$

This PDF is combined with (3) to generate a maximum *a posteriori* (MAP) multiuser detection estimator.

3. MAP MULTIUSER DETECTION

The maximum *a posteriori* estimate for multiuser detection is found by maximizing the posterior PDF, $p(\mathbf{d}|\mathbf{r})$, or equivalently $p(\mathbf{r}|\mathbf{d})p(\mathbf{d})$. Ignoring the constant terms, the cost function becomes

$$J = \exp\left(\frac{-1}{\sigma_z^2}\left|\mathbf{r}-\tilde{\mathbf{S}}\mathbf{d}\right|^2\right)\prod_{k=1}^{K}\left(\sum_{i=1}^{M} h_k(g_i)\right) \tag{7}$$

where $h_k(g_i) = \exp\left\{\frac{-\lambda_k^2}{\sigma_k^2}\left|d_k - g_i\right|^2\right\}$ and $\tilde{\mathbf{S}} = \mathbf{S\Lambda}$.

Differentiating J with respect to $\mathbf{d}^*$ yields

$$\nabla J = \frac{J}{\sigma_z^2}\tilde{\mathbf{S}}^H\left(\mathbf{r}-\tilde{\mathbf{S}}\mathbf{d}\right) + \exp\left(\frac{-1}{\sigma_k^2}\left|\mathbf{r}-\tilde{\mathbf{S}}\mathbf{d}\right|^2\right)\hat{\beta} \tag{8}$$

where the j^{th} element of the $K \times 1$ vector $\hat{\beta}$ is

$$\hat{\beta}_j = \frac{-\lambda_j^2}{\sigma_j^2}\left(\sum_{i=1}^{M}(d_j - g_i)h_j(g_i)\right)\prod_{\substack{k=1\\k\neq j}}^{K}\left(\sum_{i=1}^{M}h_k(g_i)\right). \tag{9}$$

If we then multiply and divide $\hat{\beta}_j$ by $\sum_{i=1}^{M} h_j(g_i)$, for $j = 1,2,\ldots,K$, it can be rewritten as

$$\hat{\beta}_j = \frac{-\lambda_j^2}{\sigma_j^2}\left(\frac{\sum_{i=1}^{M}(d_j - g_i)h_j(g_i)}{\sum_{i=1}^{M}h_j(g_i)}\right)\prod_{k=1}^{K}\left(\sum_{i=1}^{M}h_k(g_i)\right). \tag{10}$$

Employing this result, we can therefore express the gradient as

$$\nabla J = \frac{J}{\sigma_z^2}\left((\tilde{\mathbf{y}} - \mathbf{Qd}) + \begin{Bmatrix}\lambda_1^2\left(\frac{\sigma_z^2}{\sigma_1^2}\right)\tilde{\beta}_1\\ \vdots \\ \lambda_K^2\left(\frac{\sigma_z^2}{\sigma_K^2}\right)\tilde{\beta}_K\end{Bmatrix}\right) \tag{11}$$

where $\tilde{\mathbf{y}} = \tilde{\mathbf{S}}^H\mathbf{r}$, $\mathbf{Q} = \left(\tilde{\mathbf{H}} + \sigma_z^2\mathbf{\Sigma}^{-1}\mathbf{\Lambda}^2\right)$ in which $\mathbf{\Sigma} = diag\left(\sigma_1^2 \ldots \sigma_K^2\right)$, $\tilde{\mathbf{H}} = \tilde{\mathbf{S}}^H\tilde{\mathbf{S}}$, and

$$\tilde{\beta}_k = \frac{\sum_{i=1}^{M} g_i h_k(g_i)}{\sum_{i=1}^{M} h_k(g_i)}. \tag{12}$$

The first term in (11), $(\tilde{\mathbf{y}} - \mathbf{Qd})$, is the maximum likelihood (ML) gradient and the second term represents the stochastic *a priori* information regarding the values of the discrete symbols.

This can be further simplified for a constant modulus symbol constellation such that each $h_k(g_i)$ becomes

$$f_k(g_i) = \exp\left\{\frac{-\lambda_k^2}{\sigma_k^2}\left(d_k^* g_i + d_k g_i^*\right)\right\} \tag{13}$$

and (11) becomes

$$\nabla J = \frac{J}{\sigma_z^2}\left((\tilde{\mathbf{y}} - \mathbf{Qd}) + \sigma_z^2\mathbf{\Sigma}^{-1}\mathbf{\Lambda}^2\boldsymbol{\beta}\right) \tag{14}$$

where the k^{th} element of the new vector $\boldsymbol{\beta}$ is

$$\beta_k = \frac{\sum_{i=1}^{M} g_i f_k(g_i)}{\sum_{i=1}^{M} f_k(g_i)}. \tag{15}$$

This is the general MAP framework requiring only the specific values of g_i for a given symbol constellation in order to compute β_k and obtain the constellation-specific MAP detector.

Note that a direct solution to (14) cannot be found explicitly. We therefore find the solution to maximize J through steepest-ascent iteration as

$$\hat{\mathbf{d}}^{(m+1)} = \hat{\mathbf{d}}^{(m)} + \mu\,\nabla J^{(m)} \tag{16}$$

where μ is the adaptation step-size. The optimal step-size can be shown [9] to be $\mu_{MAX} = \left[(\nabla J)^H(\nabla J)\right]/\left[(\nabla J)^H\mathbf{Q}(\nabla J)\right]$. Also note that the term J/σ_z^2 in (14) is simply a scaling factor and can be absorbed into μ. Also, the complexity is $O\left(K^2\right)$ [9].

For BPSK modulation, it is straightforward to show that

$$\beta_k = \tanh\left(\frac{\lambda_k^2}{\sigma_k^2}\hat{d}_k^{(m)}\right). \tag{17}$$

For QPSK modulation, with discrete symbol values $g_1 = 1$, $g_2 = j$, $g_3 = -1$, and $g_4 = -j$, (15) can be simplified as

$$\beta_k = \frac{(1+j)}{2}\tanh\left(\frac{\lambda_k^2}{\sigma_k^2}\left(\mathrm{Re}\left\{\hat{d}_k^{(m)}\right\} + \mathrm{Im}\left\{\hat{d}_k^{(m)}\right\}\right)\right) - \frac{(1-j)}{2}\tanh\left(\frac{\lambda_k^2}{\sigma_k^2}\left(-\mathrm{Re}\left\{\hat{d}_k^{(m)}\right\} + \mathrm{Im}\left\{\hat{d}_k^{(m)}\right\}\right)\right). \tag{18}$$

Finally, for 8PSK modulation the *a priori* term is

$$\beta_k = \frac{\sinh(x_1) + \frac{1+j}{\sqrt{2}}\sinh(x_2) + j\sinh(x_3) + \frac{1-j}{\sqrt{2}}\sinh(x_4)}{\cosh(x_1) + \cosh(x_2) + \cosh(x_3) + \cosh(x_4)} \tag{19}$$

where $x_1 = \frac{2\lambda_k^2}{\sigma_k^2}\mathrm{Re}\left\{\hat{d}_k^{(m)}\right\}$, $x_2 = \frac{\sqrt{2}\lambda_k^2}{\sigma_k^2}\left(\mathrm{Re}\left\{\hat{d}_k^{(m)}\right\} + \mathrm{Im}\left\{\hat{d}_k^{(m)}\right\}\right)$, $x_3 = \frac{2\lambda_k^2}{\sigma_k^2}\mathrm{Im}\left\{\hat{d}_k^{(m)}\right\}$ and $x_4 = \frac{\sqrt{2}\lambda_k^2}{\sigma_k^2}\left(\mathrm{Re}\left\{\hat{d}_k^{(m)}\right\} - \mathrm{Im}\left\{\hat{d}_k^{(m)}\right\}\right)$.

4. SIMULATION RESULTS

For each of the different symbol constellations discussed, we compare the performance of the MAP detector with the MMSE, decorrelating, and matched filter detectors. Due to space restriction we limit our comparison to the case of equal power for all users. In each case we consider 5 users with signature codes derived from Gold sequences of length $N = 7$ and we iterate the update equation (16) of the MAP detector 20 times with the step-size $0.8\,\mu_{MAX}$.

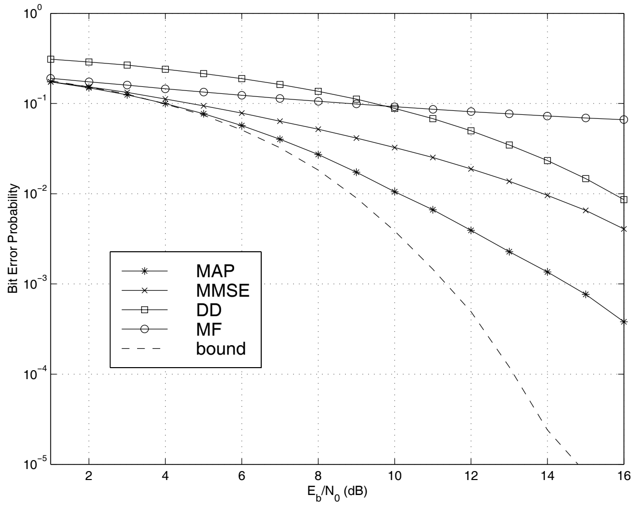

Fig. 2. System BER for BPSK with equal-power users

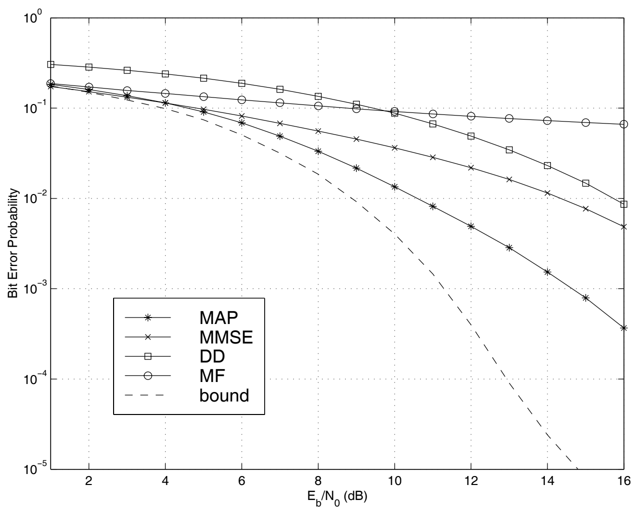

Fig. 3. System BER for QPSK with equal-power users

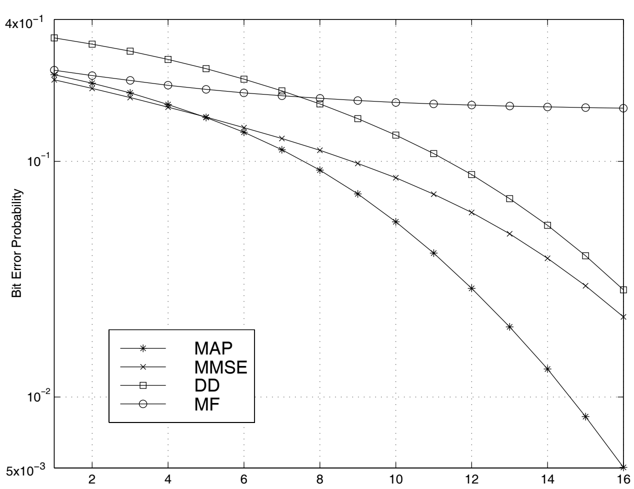

Fig. 4. System BER for 8PSK with equal-power users

For BPSK modulation, Fig. 2 illustrates the average BER over all 5 users. Obviously, the MAP detector performs substantially better than the other three detectors with an order of magnitude improvement in BER over MMSE at 16 dB E_b/N_0. Figure 3 depicts the average BER for QPSK modulation. Again the MAP detector exhibits significant BER performance improvement over the other detectors with an order of magnitude improvement in BER over MMSE at 16 dB E_b/N_0. Finally, Fig. 4 presents the average BER results for 8PSK modulation. While the improvement is not quite as dramatic, the MAP detector is still noticeably better than the other detectors with less than ½ the BER of MMSE at 12 dB E_b/N_0.

5. CONCLUSIONS

We have proposed an iterative algorithm for multiuser detection that incorporates the *a priori* knowledge of a constant modulus symbol constellation such as *M*-PSK into the optimization process to improve the estimation accuracy of the transmitted user symbols. This is accomplished by approximating the discrete PDF of the constellation in the transmitter by a continuous PDF at the receiver that is comprised of complex Gaussians centered at the true symbol locations. This continuous PDF is combined with the likelihood function of the received data to generate a differentiable maximum *a posteriori* objective function that can be maximized iteratively to yield a dramatically lower BER than the MMSE algorithm.

REFERENCES

[1] S.D. Blunt and K.C. Ho, "An iterative maximum a posteriori (MAP) estimator for multiuser detection in synchronous CDMA systems," in Proc. *IEEE ICASSP*, pp. 2313-2316, Orlando, FL, May, 2002.

[2] S. Verdu, "Minimum probability of error for asynchronous Gaussian multiple access channels," *IEEE Trans. Inform. Theory*, Vol. IT-32, pp. 85-96, Jan. 1986.

[3] R. Lupas and S. Verdu, "Linear multiuser detectors for synchronous code-division multiple-access channels," *IEEE Trans. Inform. Theory*, Vol. 35, pp. 123-136, Jan. 1989.

[4] Z. Xie, *et al.*, "A family of suboptimum detectors for coherent multi-user communication," *IEEE J. Select. Areas Commun.,* Vol. 8, pp. 683-690, May, 1990.

[5] P. Patel and J. Holtzman, "Analysis of a simple successive interference cancellation scheme in a DS/CDMA system," *IEEE J. Select. Areas Commun.,* Vol. 12, pp. 796-807, June 1994.

[6] K.-C. Lai and J.J. Shynk, "Steady-state analysis of the adaptive successive interference canceller for DS/CDMA signals," *IEEE Trans. Signal Processing*, Vol. 49, pp. 2345-2362, Oct. 2001.

[7] D. Divsalar, M.K. Simon, and D. Raphaeli, "Improved parallel interference cancellation for CDMA," *IEEE Trans. Commun.,* Vol. 46, pp. 258-268, Feb. 1998.

[8] M.K. Varanasi and B. Aazhang, "Multistage detection asynchronous code-division multiple access communications," *IEEE Trans. Commun.,* Vol. 38, pp. 509-519, Apr. 1990.

[9] S.D. Blunt and K.C. Ho, "Iterative MAP symbol estimation for synchronous CDMA," under review *IEEE Trans. Wireless Commun.*