VARIABLE-STEP-SIZE MULTIMODULUS BLIND DECISION-FEEDBACK EQUALIZATION FOR HIGH-ORDER QAM BASED ON BOUNDARY MSE ESTIMATION

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ABSTRACT

We consider blind decision-feedback equalization (DFE) under high-order modulation, which presents more difficult operating requirements than lower-order modulation. We base our design on the multimodulus algorithm (MMA). To attain fast convergence speed and low steady-state mean-square error (MSE), we consider varying the adaptation step size according to the presently achieved MSE. For this we investigate the properties of the MSE under blind MMA-based DFE and, based on the results, propose a method to estimate its value. The estimate is obtained by analyzing those equalizer filter outputs whose values fall outside the boundary of the modulation's constellation. Simulation results demonstrate the effectiveness of the proposed scheme.

1. INTRODUCTION

Blind equalization is of use in transmission systems where there exist no (or insufficient) known signal patterns that can be used for equalizer training. An example is downlink cable modem transmission where the known signal patterns are quite sparse that, if they are used to adapt equalizers in conventional training-sequence-based ways, the convergence may be very slow. A number of blind equalization algorithms have been proposed in the last few decades, of which many are of the stochastic gradient type. We consider the recently proposed multimodulus algorithm (MMA) [1], [2], which has relatively good performance.

A well-known design issue of stochastic-gradient type of adaptive algorithms is the choice of the adaptation step size, which has to strike a balance between convergence speed and steady-state SNR. In QAM-based transmission, this issue is more acute for higher-order modulations than for lower-order ones, because the former require higher SNR values to attain a given error performance than the latter and thus the convergence speed has to be sacrificed more. A way to alleviate this problem is to employ a variable step size (VSS) [3], [4]. For automatic adjustment of the step size, however, a mechanism to determine the current state of convergence is required, where the state of convergence may be characterized, for example, by the estimated mean-square error (MSE) at equalizer output [4]. Herein lies another problem that is more serious for blind equalization under higher-order modulations than under lower-order ones. That is, the equalizer output errors are relatively large before final convergence. Thus, for higher-order modulations, tentative decisions are liable to greater error probabilities and simplistic MSE estimates may suffer greater inaccuracy. In this work, we propose a method for reliable MSE estimation and an associated VSS multimodulus algorithm for blind decision-feedback equalization (DFE) under high-order QAM-based transmission, such as 1024-QAM over downlink digital cable channels.

The remainder of the paper is organized as follows. Section 2 presents the MMA algorithm for blind DFE and motivates our VSS design based on MSE estimation. Section 3 discusses the proposed MSE estimator. Section 4 further discusses the design of the VSS MMA algorithm and elucidates it with an example that also illustrates the superiority of the VSS MMA over a single-stage MMA. And Section 5 is the conclusion.

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2. BLIND DFE EMPLOYING MMA

2.1. System Structure

The MMA [1], [2] seeks to minimize a cost function given by

$$\Psi(y) = E[(y_r^L - R_m^L)^2 + (y_i^L - R_m^L)^2] \quad (1)$$

with

$$R_m^L = \frac{E\left(a_r^{2L}\right)}{E\left(|a_r|^L\right)} = \frac{E\left(a_i^{2L}\right)}{E\left(|a_i|^L\right)}, \quad (2)$$

where y is the filter output in the equalizer, with y_r being its real part and y_i its imaginary part, a_r and a_i are, respectively, the real part and the imaginary part of the QAM symbol a, L is a postive integer, and R_m is called the constraint value of the algorithm. In practice, $L = 2$ is a good choice to compromise between implementation complexity and performance [2]. Letting $L = 2$ and taking the gradient of $\Psi(y)$ with respect to y yield

$$\psi(y) = y_r\left(y_r^2 - R_m^2\right) + j\, y_i\left(y_i^2 - R_m^2\right). \quad (3)$$

With fractionally-spaced DFE, we have

$$y(n) = \sum_{i=0}^{L-1}\sum_{k=0}^{N_f-1} f_{k,i}\, x_i(n-k) - \sum_{k=1}^{N_b} b_k\, \widehat{a}(n-k) \quad (4)$$

where we now associate a time index n (in number of QAM symbols) with the equalizer filter output y, $f_{k,i}$ is the kth coefficient of the ith phase of the feedforward filter (FFF), b_k is the kth coefficient of the feedback filter (FBF), $x_i(n)$ is the equalizer input in phase i of symbol n, L is the oversamping factor, N_f is the length of the FFF in number of symbols, N_b is the number of taps in the FBF, and $\widehat{a}(n)$ is the decision output of the equalizer. From (3), we obtain the MMA blind DFE adaptation equations as

$$\begin{aligned} f_{k,i}(n) &= f_{k,i}(n-1) - \mu\,\psi(y)\,x_i^*(n-k), &(5)\\ b_k(n) &= b_k(n-1) + \mu\,\psi(y)\,\widehat{a}^*(n-k), &(6)\end{aligned}$$

where μ is the step size.

2.2. Steady-State MSE of the MMA

We start by considering the simpler case of linear equalization. For this, let $\underline{w}$ be the vector of equalizer coefficients and $\underline{x}$ be the vector of input signal samples stored in the equalizer tapped delay line. The adaptation equation is given by

$$\underline{w}(n) = \underline{w}(n-1) - \mu\,\psi(y(n))\,\underline{x}^*(n). \tag{7}$$

Let $\underline{w}_{opt}$ be the optimal filter coefficient vector and define $\underline{\tilde{w}}(n) = \underline{w}_{opt} - \underline{w}(n)$. Then the a priori and the a posteriori estimation errors are given by, respectively,

$$e_a(n) = \underline{x}(n)\,\underline{\tilde{w}}(n-1), \tag{8}$$

$$e_p(n) = \underline{x}(n)\,\underline{\tilde{w}}(n) = e_a(n) - \mu\,\|\underline{x}(n)\|^2\,\psi(y(n)). \tag{9}$$

In the steady state where $E\{\underline{\tilde{w}}(n)\} = E\{\underline{\tilde{w}}(n-1)\}$, the mean-squares of the estimation errors are related by [5]

$$E\left\{\frac{|e_a(n)|^2}{\|\underline{x}(n)\|^2}\right\} = E\left\{\frac{\left|e_a(n) - \mu\|\underline{x}(n)\|^2\psi(y(n))\right|^2}{\|\underline{x}(n)\|^2}\right\}, \tag{10}$$

which can be simplified to

$$E\{\Re[e_a(n)\psi(y(n))]\} = \frac{\mu}{2}E\{\|\underline{x}(n)\|^2\;|\psi(y(n))|^2\}. \tag{11}$$

Now, assume that the residual error is small when the equalizer is converged. Then first-order approximation as in [6] gives

$$\psi(y(n)) \approx \psi(a(n)) + \psi'(a(n))\,e_a(n), \tag{12}$$

where $a(n)$ is the transmitted symbol at time n. (Without loss of generality, the transmission and filtering delays are disregarded.) Substituting (12) into (11) and assuming independence among $\psi(a(n))$, $e_a(n)$, and $\|\underline{x}(n)\|^2$ as in [5], we obtain the MSE as

$$E\{|e_a|^2\} = \frac{\mu}{2}\frac{E(\|\underline{x}\|^2)\cdot E(|\psi(a)|^2)}{E\{\Re[\psi'(a)]\}}. \tag{13}$$

Since $E(\|\underline{x}\|^2) = P_x\,N$ where P_x is the mean-square value of the equalizer input and N is the length of the equalizer, we have

$$E\{|e_a|^2\} = \frac{\mu}{2}\frac{P_x\cdot N\cdot E(|\psi(a)|^2)}{E\{\Re[\psi'(a)]\}}. \tag{14}$$

Now we turn to the case of DFE. The update equations (5) and (6) can be combined into

$$\begin{bmatrix}\underline{f}(n)\\ \underline{b}(n)\end{bmatrix} = \begin{bmatrix}\underline{f}(n-1)\\ \underline{b}(n-1)\end{bmatrix} - \mu\,\psi(y(n))\begin{bmatrix}\underline{x}^*(n)\\ \underline{\widehat{a}}^*(n-1)\end{bmatrix}, \tag{15}$$

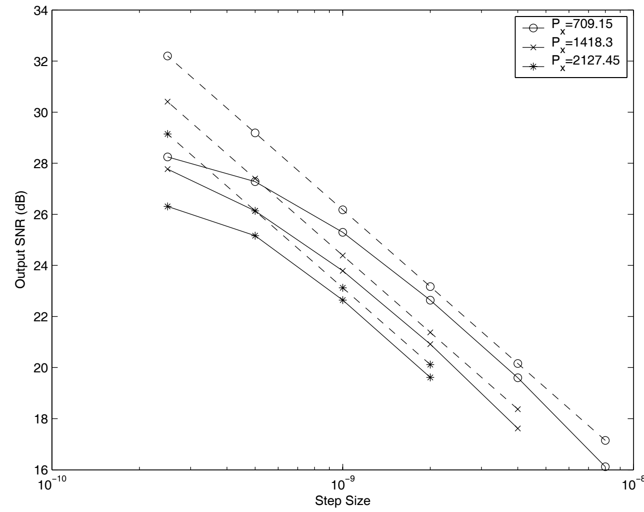

Fig. 1. Steady-state SNR of MMA blind DFE for 1024-QAM transmission over the channel in [7]. Solid lines: simulation; dashed lines: theory.

with obvious definitions for $\underline{f}(n)$, $\underline{b}(n)$, $\underline{x}(n)$, and $\underline{\widehat{a}}$. Extending the result (14) for linear equalization to the case of DFE yields

$$E\{|e_a|^2\} = \frac{\mu}{2}\frac{E\{|\psi(a)|^2\}}{E\{\Re[\psi'(a)]\}}\cdot(N_f L P_x + N_b E_s) \tag{16}$$

where $E_s = E\{|a|^2\}$ is the QAM symbol energy. Now since

$$\psi'(a) = \frac{3}{2}|a|^2 - R_m^2, \tag{17}$$

we get

$$E\{|e_a|^2\} = \frac{\mu\cdot E(|\psi(a)|^2)}{3E_s - 2R_m^2}\cdot(N_f L P_x + N_b E_s). \tag{18}$$

To verify the above results, we simulate 1024-QAM transmission over the (rather bad) cable channel in [7]. Theoretical analysis of the achievable SNR under MMSE (minimum MSE) DFE shows that $N_f = 10$ and $N_b = 20$ should be suitable choices for $L = 2$ (i.e., $T/2$-spaced FFF). The equalizer input SNR is 36 dB. The results for several adaptation step sizes and several equalizer input signal power levels are depicted in Fig. 1. The figure shows that the theory agrees reasonably well with the simluation results at larger step sizes that yield smaller steady-state SNR. The discrepancy at smaller step sizes (larger steady-state SNR) should be due to that the assumptions made earlier do not fully capture the dynamic behavior of the algorithm.

2.3. Approach to Variable-Step-Size MMA

The above results may be used in the following way: At any time, we calculate the actual SNR and compare it with the steady-state SNR for the presently used adaptation step size. If the latter has been reached, then we may switch to a smaller step size. Finally, when the SNR is high enough, we may switch out of blind mode of operation and enter decision-directed (DD) mode of equalizer operation.

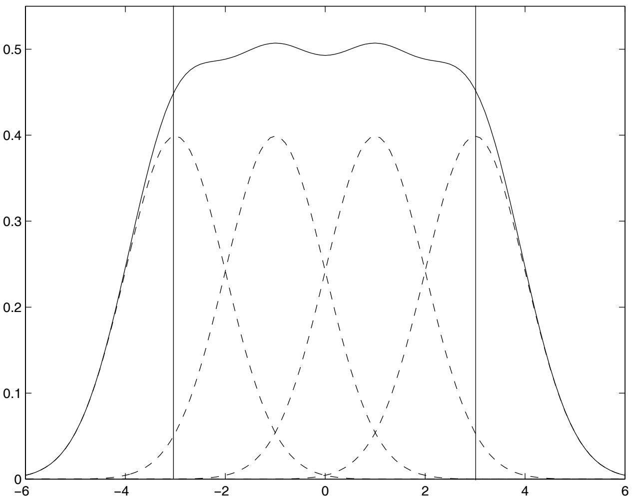

Fig. 2. Motivation and principle of boundary MSE estimation, illustrated for the case of 4-PAM (applicable also to 16-QAM). The constellation points are at ±1 and ±3. Dashed lines illustrate PDFs of equalizer filter output $y(n)$ corresponding to different values of $a(n)$; solid line their sum.

Key in this procedure is the estimation of actual SNR, or equivalently, the estimation of the actual MSE given by $E\{|y(n) - a(n)|^2\}$, the mean-square difference between equalizer filter output and the transmitted QAM symbol. To see how this can be accomplished, consider Figure 2 which illustrates the situation of 4-PAM (applicable also to 16-QAM), where the signal points are at ±1 and ±3. In the figure, the dashed lines illustrate the PDFs of the equalizer filter output $y(n)$ corresponding to different values of $a(n)$ and the solid line is their sum. An estimator of the MSE can be obtained from analyzing the PDF of $y(n)$. However, when the SNR is not high, the center part of the PDF is relatively flat. The variation in this part with changes in SNR (when the SNR is not high) is relatively small. Thus this part does not contribute significantly to the ability of MSE estimation. Numerical results also verify this observation. PDF variation outside the boundary symbol values is greater. Thus we base our MSE estimation on analysis of values of $y(n)$ that fall outside the boundary symbol values. Accordingly, we call this approach boundary MSE estimation. It is further described and analyzed in the next section.

From Fig. 2, we also see that only about $1/M$ of the equalizer input samples will be used in performing the estimate, but not all samples. This is a price we pay to have good sensitivity in MSE estimation.

3. BOUNDARY MSE ESTIMATION

Treat a QAM symbol as the direct sum of two PAM symbols. Let $\bar{y}$ denote the value of either dimension of the equalizer filter output y. The boundary MSE, for PAM, is defined as

$$\text{BMSE} = E\left\{ (\bar{y} - \bar{a}_{\max})^2 \,\middle|\, |\bar{y}| > \bar{a}_{\max} \right\}, \tag{19}$$

where $\bar{a}_{\max}$ is the largest symbol value in the PAM constellation.

For convenience, let the constellation points of M-PAM have values $\pm1, \pm3, \cdots, \pm(M-1)$. Due to symmetry, in theoretical analysis we only need to consider the positive side. Let P_k be the probability that the transmitted symbol value is $(M-1-2k)$ but $\bar{y} > \bar{a}_{\max}$. Then

$$P_k = \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x+2k)^2}{2\sigma^2}\right) dx = \mathrm{Q}\left(\frac{2k}{\sigma}\right), \tag{20}$$

where we have assumed that the sum of the residual intersymbol interference (ISI) and additive noise is Gaussian and let σ^2 denote its variance. Note that σ^2 is the target of estimation. The corresponding mean-square boundary error is given by

$$\begin{aligned} V_k &= \int_0^\infty \frac{x^2}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x+2k)^2}{2\sigma^2}\right) dx \\ &= (4k^2+\sigma^2)P_k - \sqrt{\frac{2}{\pi}}\, k\sigma \exp\left(-\frac{2k^2}{\sigma^2}\right). \end{aligned} \tag{21}$$

Assume all constellation points are transmitted with equal probability. Then the total boundary error probability and total mean-square boundary error, on the positive side, are given by

$$P = \frac{1}{M}\sum_{k=0}^{M-1} P_k \quad \text{and} \quad V = \frac{1}{M}\sum_{k=0}^{M-1} V_k, \tag{22}$$

respectively. The BMSE is thus given by

$$\begin{aligned} \frac{V}{P} &= \sigma^2 + \frac{4\sum_k k^2 Q(2k/\sigma)}{\sum_k Q(2k/\sigma)} \\ &\quad - \frac{\sqrt{2/\pi}\,\sigma \sum_k k \exp(-2k^2/\sigma^2)}{\sum_k Q(2k/\sigma)}. \end{aligned} \tag{23}$$

Figure 3 plots the ratio of σ^2 to BMSE in log scale for 32-PAM (applicable to 1024-QAM) over a range of SNR values (where SNR $= E\{\bar{a}^2\}/\sigma^2$ with $\bar{a}$ being PAM symbol value). Note that the ratio is nearly unity in large SNR. This is because when the true MSE σ^2 is small, the last two terms in the RHS of (23) are close to zero. Even for an SNR as low as 0 dB, the difference is only about 1.65 dB. In any case, this difference is compensated for in our variable-step-size MMA.

Practical estimation of the BMSE, for QAM, may be effected by time averaging, such as

$$\text{BMSE}(n) = \beta\cdot\text{BMSE}(n-1) + (1-\beta)\cdot 2(|\bar{y}(n)| - \bar{a}_{\max})^2, \tag{24}$$

where β is the forgetting factor, the factor 2 is to account for the difference between QAM and PAM, and the recursion is executed only when $|\bar{y}(n)| > \bar{a}_{\max}$.

4. VARIABLE-STEP-SIZE MMA BLIND DFE AND SIMULATION RESULTS

Our proposed variable-step-size method works in a multistage, gear-shifting fashion rather than employing a continuously varying step size as some other researchers have considered. First, we decide an SNR that is safe to switch to DD mode with little concern of divergence afterwards. In the case of 1024-QAM, for example, simulation results indicate that 27 dB appears to be proper. Hence, the objective of blind equalization is set to be 27 dB. Figure 1 can then be used to find a suitable step size. For example, when $P_x = 709$, a suitable step size is around 5×10^{-10}. This constitutes the last stage of blind equalization.

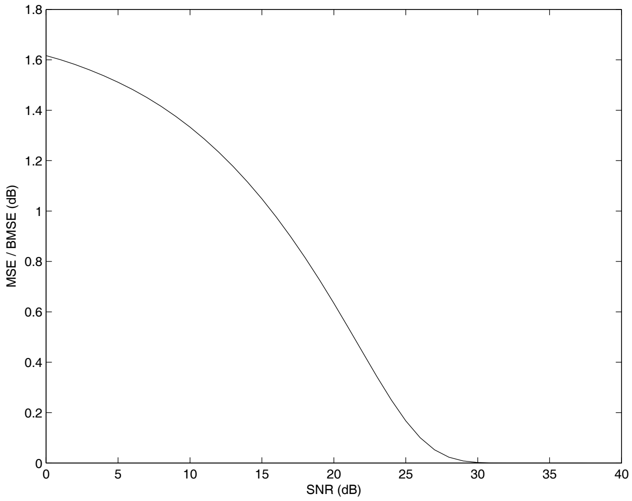

Fig. 3. Ratio of MSE to BMSE for 32-PAM (and 1024-QAM).

Table 1. Algorithm Parameters for the Example Design

Stage Index	1	2	3
Objective SNR (dB)	16	23	27
Step Size	8e–9	2e–9	5e–10
Stage Transition Threshold	13.6	2.9	1.3

Prior to the last blind stage, we can have one or more stages with larger (but diminishing with stage index) adaptation step sizes to effect fast initial convergence. We here present an example using two additional stages. For this, note again from Fig. 1 that a step size of 8×10^{-9} can yield an SNR of 16 dB after convergence and a step size of 2×10^{-9}, 23 dB. The BMSE value corresponding to these SNR values can be obtained as

$$TH = \frac{E_s}{SNR \cdot R(SNR)}, \tag{25}$$

where $R(SNR)$ is the ratio of MSE to BMSE at the given SNR value as can be obtained from (23) and shown in Fig. 3. For example, at SNR = 16 dB the ratio is about 1.1 dB. In the case of 1024-QAM for which $E_s = 682$, we get $TH = 13.6$. Once the estimated BMSE of the first stage reaches this level (call it threshold), we can switch to the second stage. And this continues to the last stage of blind equalization. The resulting algorithm parameters for this example are summarized in Table 1.

Figure 4 shows some simulation results for blind DFE under 1024-QAM transmission over the cable channel in [7] where the equalizer input SNR is 36 dB. The parameters of the multistage, VSS MMA blind DFE are as given above. The forgetting factor used in BMSE estimation is 0.99. The single-stage MMA uses a step size of 5×10^{-10}. The multistage algorithm clearly outperforms the single-stage algorithm. It only requires about 50,000 samples to converge and switch to DD mode whereas the single-stage MMA requires about 125,000 samples. Hence the former can provide a much faster startup speed than the latter.

An alternative algorithm design that sidesteps Fig. 1 is under investigation. Nevertheless, we note that the parameters in Table 1 are found applicable to all the cable channels that we simulated.

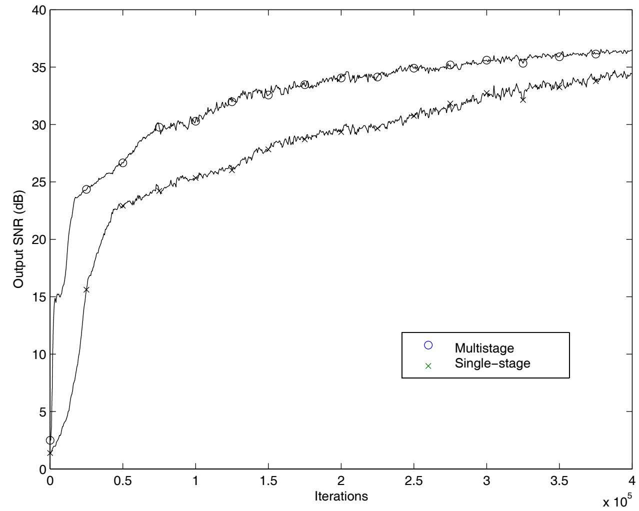

Fig. 4. Convergence of multistage VSS MMA and single-stage MMA.

5. CONCLUSION

Higher-order modulations lead to more stringent design requirements for blind equalization than lower-order ones. We considered the problem of blind DFE employing the multimodulus algorithm under high-order QAM, and developed a novel variable-step-size adaptation scheme based on a new way of MSE estimation. Simulation results show that the proposed method is effective in achieving fast startup.

6. REFERENCES

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