A Cumulant Subspace Projection Method for Blind MIMO FIR Identification

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***Abstract*— In this paper, we developed a subspace projection method for blind MIMO FIR identification based on fourth order statistics of the output signals. The proposed method employs a subspace projection technique to identify the channel. The method identifies MIMO channel up to a nonsingular matrix, and it does not require the exact knowledge of each user's channel order. Simulation results are included to verify our theoretical claims.**

I. Introduction

Blind identification is based solely on the channel output signals and some apriori knowledge of the statistics of the channel input signals. Most MIMO identification algorithms are based on SOS (second order statistics) [1]-[4] and HOS (higher order statistics) [5]-[9]. As two different research directions, these two approaches have their own merits and drawbacks. It is shown [5] that, by properly exploiting higher order statistics, it is possible to relax the channel identifiability conditions. Typically, HOS based methods can be classified either linear or nonlinear. Linear approaches [6][8] are simple and non-iterative, and they admit closed-form solutions, while some nonlinear methods [7] can generate more reliable estimates. In this paper, we propose a new linear HOS-based method for blind MIMO channel identification that utilizes two particular matrices constructed by a set of cumulant matrices. The proposed algorithm was developed by exploiting an orthogonal projector formed by the column space of one of two particular matrices. As in [5], the method presented in this paper also relaxed the channel identifiability conditions. Simulation results show the strength of the proposed method against that in [5].

II. Preliminaries

A. Signal Model and Basic Assumptions

We begin by considering an p-input q-output linear time-invariant FIR MIMO system with a series of $q \times p$ impulse response matrix, $\mathbf{H}(l) \triangleq \{h_{ij}(l)\}$, where $h_{ij}(l)$ denotes the impulse response between the j^{th} input and the i^{th} output. Let $\mathbf{s}(n) \triangleq [s_1(n) \ \ s_2(n) \ \ ... \ \ s_p(n)]^T$ denotes p source signals, $\mathbf{x}(n) \triangleq [x_1(n) \ \ x_2(n) \ \ ... \ \ x_q(n)]^T$ is the q output signals and $\mathbf{w}(n) \triangleq [w_1(n) \ \ w_2(n) \ \ ... \ \ w_q(n)]^T$ is a vector of additive q dimensional white Gaussian noise. The MIMO FIR model is then given as follows:

$$\mathbf{x}(n) \triangleq \mathbf{H}(n) * \mathbf{s}(n) + \mathbf{w}(n) \triangleq \sum_{l=0}^{L} \mathbf{H}(l)\mathbf{s}(n-l) + \mathbf{w}(n) \quad (1)$$

where L denotes the length of the longest element of $\mathbf{H}(l)$. If we define vectors $\vec{\mathbf{s}} \triangleq [\mathbf{s}^T(n) \ \ \mathbf{s}^T(n-1) \ \ \dots \ \ \mathbf{s}^T(n-N-L)]^T$, $\vec{\mathbf{x}} \triangleq [\mathbf{x}^T(n) \ \ \mathbf{x}^T(n-1) \ \ \dots \ \ \mathbf{x}^T(n-N)]^T$ and $\vec{\mathbf{w}} \triangleq [\mathbf{w}^T(n) \ \ \mathbf{w}^T(n-1) \ \ \dots \ \ \mathbf{w}^T(n-N)]^T$, we can express Eqn.(1) as:

$$\vec{\mathbf{x}} = \mathcal{H}\vec{\mathbf{s}} + \vec{\mathbf{w}} \quad (2)$$

where the channel convolution matrix $\mathcal{H}$ is an $(N+1)q \times (N+L+1)p$ block Toeplitz matrix written as follows:

$$\mathcal{H} \triangleq \begin{bmatrix} \mathbf{H}(0) & \dots & \mathbf{H}(L) & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}(0) & \dots & \mathbf{H}(L) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{H}(0) & \dots & \mathbf{H}(L) \end{bmatrix} \quad (3)$$

In this paper, we define the following notations. If $\mathbf{A}$ is an $m \times n$ matrix, then $\mathbf{A}^T$ represents matrix transpose, $\mathbf{A}^H$– matrix conjugate transpose, $\mathbf{A}^*$– matrix conjugate. The system model assumptions are: $\mathbf{A1}$) All sources are spatially independent, temporally white and are non-Gaussian source signals with zero means. $\mathbf{A2}$) Additive noises are spatially and temporally white Gaussian noises, and they are statistically independent of the sources. $\mathbf{A3}$) The number of channel outputs is no less than the number of inputs, i.e., $q \geq p$. $\mathbf{A4}$) Channel $\mathbf{H}(z)$ is irreducible, i.e., rank($\mathbf{H}(z)$) = $p \ \forall z \neq 0$. Under $\mathbf{A4}$), we know that rank($\mathbf{H}(0)$) = p (consider $z = \infty$) and $\mathbf{H}(0)$ does not contain all-zero columns. Our objective is to estimate the channel impulse response $\mathbf{H}(z)$ by utilizing fourth order statistics of the observed data $\mathbf{x}(n)$.

B. Cumulant Matrices

We define a series of fourth order cumulant matrices of the channel output signals. Let $x_l(n)$ denote the l^{th} channel output signal, we define $\mathbf{C}_l[k]$ as:

$$\mathbf{C}_l[k] \triangleq \text{cum}(\vec{\mathbf{x}}, \vec{\mathbf{x}}^H, x_l(n-k), x_l^*(n-k)) \quad (4)$$

where $\mathbf{C}_l[k]$ is an $q(N+1)\times q(N+1)$ matrix with its ij^{th} element written as: $\mathbf{C}_l^{ij}[k] \triangleq \text{cum}(\vec{\mathbf{x}}(i), \vec{\mathbf{x}}^H(j), x_l(n-k), x_l^*(n-k))$, in which $\vec{\mathbf{x}}(i)$ represents the i^{th} element of vector $\vec{\mathbf{x}}$. Invoking the cumulant properties and assumptions **A1**)-**A2**), we have

$$\mathbf{C}_l[k] = \mathcal{H}\Lambda_l[k]\mathcal{H}^H \tag{5}$$

where $\Lambda_l[k] \triangleq \text{diag}(\underbrace{\mathbf{0},\ldots,\mathbf{0}}_{k \text{ blocks}}, D_l[0],\ldots,D_l[L], \underbrace{\mathbf{0},\ldots,\mathbf{0}}_{(N-k) \text{ blocks}})$ and $D_l[i] \triangleq \text{diag}(\gamma_1|h_{l1}(i)|^2,\ldots,\gamma_p|h_{lp}(i)|^2)$, in which γ_i denotes the fourth-order kurtosis of source signal $s_i(n)$. By exploiting the spatial diversity, we define $\mathbf{C}[k]$ as:

$$\mathbf{C}[k] \triangleq \sum_{l=1}^{q}\mathbf{C}_l[k] = \mathcal{H}\Lambda[k]\mathcal{H}^H \tag{6}$$

where $\Lambda[k] \triangleq \text{diag}(\underbrace{\mathbf{0},\ldots,\mathbf{0}}_{k \text{ blocks}}, D[0],\ldots,D[L], \underbrace{\mathbf{0},\ldots,\mathbf{0}}_{(N-k) \text{ blocks}})$ and $D[i] \triangleq \text{diag}(\gamma_1\sum_{l=1}^{q}|h_{l1}(i)|^2,\ldots,\gamma_p\sum_{l=1}^{q}|h_{lp}(i)|^2)$. It is clear that $D[0]$ is a full rank diagonal matrix since any diagonal element $\gamma_i\sum_{l=1}^{q}|h_{li}(0)|^2 \ (1\le i\le p)$ in this matrix could not be zero under **A4**).

III. Proposed Identification Algorithm

A. Problem Formulation

We consider the following choice of k and N:

$$\boxed{L\le k\le 2L \qquad N\ge 3L} \tag{7}$$

Under this condition, we can rewrite Eqn.(8) as:

$$\mathbf{C}[k] \triangleq \mathbf{H}_s\Sigma[k]\mathbf{H}_s^H \tag{8}$$

where

$$\mathbf{H}_s \triangleq \begin{bmatrix} \mathbf{H}(L) & \mathbf{0} & \ldots & \mathbf{0} \\ \vdots & \mathbf{H}(L) & \ddots & \vdots \\ \mathbf{H}(0) & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}(0) & \ddots & \mathbf{H}(L) \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \ldots & \mathbf{0} & \mathbf{H}(0) \end{bmatrix} \tag{9}$$

$$\Sigma[k] \triangleq \text{diag}(\underbrace{\mathbf{0},\ldots,\mathbf{0}}_{(k-L) \text{ blocks}}, D[0],\ldots,D[L], \underbrace{\mathbf{0},\ldots,\mathbf{0}}_{(N-L-k) \text{ blocks}}) \tag{10}$$

Notice that $\mathbf{H}_s$ is a convolution matrix with block Toeplitz structure and is a tall matrix with dimension $(N+1)q\times(N-L+1)p$. From [5] we know that $\mathbf{H}_s$ has full column rank under **A3**)-**A4**). We construct two particular matrices $\mathbf{S}_1$ and $\mathbf{S}_2$, which are defined as:

$$\mathbf{S}_1 \triangleq \mathbf{C}[k_1] = \mathbf{H}_s\Gamma_1\mathbf{H}_s^H \tag{11}$$

$$\begin{aligned} \mathbf{S}_2 &\triangleq \begin{bmatrix} \mathbf{C}[k_2] & \mathbf{C}[k_2+1] & \ldots & \mathbf{C}[k_3] \end{bmatrix} \\ &= \mathbf{H}_s\Gamma_2 \begin{bmatrix} \mathbf{H}_s^H & \ldots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \ldots & \mathbf{H}_s^H \end{bmatrix} \\ &= \mathbf{H}_s\Gamma_2\mathbf{M}^H \end{aligned} \tag{12}$$

By choosing $\boxed{k_1 = L,\ \ k_2 = L+1,\ \ k_3 = 2L}$, we have

$$\Gamma_1 \triangleq \text{diag}(D[0],\ldots,D[L], \underbrace{\mathbf{0},\ldots,\mathbf{0}}_{(N-2L) \text{ blocks}}) \tag{13}$$

$$\Gamma_2 \triangleq \begin{bmatrix} \Sigma[k_2] & \Sigma[k_2+1] & \ldots & \Sigma[k_3] \end{bmatrix} \tag{14}$$

and further written as:

$$\Gamma_2 = \begin{bmatrix} \text{diag}(\mathbf{0}, D[0],\ldots,\ldots, D[L], \underbrace{\mathbf{0},\ldots,\mathbf{0}}_{N-2L-1}) \\ \vdots \\ \text{diag}(\underbrace{\mathbf{0},\ldots,\mathbf{0}}_{L}, D[0],\ldots,D[L], \underbrace{\mathbf{0},\ldots,\mathbf{0}}_{N-3L}) \end{bmatrix}^T \tag{15}$$

The structure of Γ_2 guarantees that all nonzero rows are linearly independent.

B. Channel Identification

We denote $R(\star)$ the subspace spanned by the columns of matrix $\star$. For notational convenience, let R_1 and R_2 respectively denote the range of $\mathbf{S}_1$ and $\mathbf{S}_2$. We note that $R_1 \subset R(\mathbf{H}_s)$ and $R_2 \subset R(\mathbf{H}_s)$ since both Γ_1 and Γ_2 include all-zero rows. In order to study R_1 and R_2, we partition $\mathbf{H}_s$ into three parts: $\mathbf{H}_s \triangleq [\mathbf{H}_r\ \ \mathbf{H}_t\ \ \mathbf{H}_v]$, where $\mathbf{H}_r$ denotes the first block column, i.e., first p columns, of $\mathbf{H}_s$, $\mathbf{H}_t$ is the middle part from second block column to $(L+1)^{th}$ block column of $\mathbf{H}_s$ and $\mathbf{H}_v$ is the last $(N-2L)$ block columns of $\mathbf{H}_s$. Hence we have

$$\mathbf{S}_1 = \begin{bmatrix} \mathbf{H}_r & \mathbf{H}_t \end{bmatrix} \begin{bmatrix} D[0] & \mathbf{0} \\ \mathbf{0} & \Psi_1 \end{bmatrix} \begin{bmatrix} \mathbf{H}_r^H \\ \mathbf{H}_t^H \end{bmatrix} \tag{16}$$

where

$$\Psi_1 \triangleq \text{diag}(D[1], D[2],\ldots,D[L]) \tag{17}$$

where $D[0]$ is full rank and Ψ_1 may include zero diagonal elements, therefore subspace R_1 is spanned by the following two parts: $R(\mathbf{H}_r)$ and $R(\tilde{\mathbf{H}}_t)$, where $R(\tilde{\mathbf{H}}_t)$ is the partial space of $R(\mathbf{H}_t)$ and we have $R(\tilde{\mathbf{H}}_t) \subseteq R(\mathbf{H}_t)$. As for subspace R_2, we have

$$R_2 = R(\mathbf{C}[k_2]) \cup R(\mathbf{C}[k_2+1]) \cup \ldots \cup R(\mathbf{C}[k_3]) \tag{18}$$

The subspace $R(\mathbf{C}[i])$ is formed by columns of $\mathbf{H}_s$ whose corresponding diagonal elements in $\Sigma[i]$ are non-zero. Therefore subspace R_2 is spanned by columns of $\mathbf{H}_s$ whose corresponding rows in Γ_2 are not all-zero rows. We rewrite $\mathbf{S}_2$ as:

$$\mathbf{S}_2 = \begin{bmatrix} \mathbf{H}_r & \mathbf{H}_t & \mathbf{H}_v \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \Psi_2 \\ \Psi_3 \end{bmatrix} \mathbf{M}^H \tag{19}$$

where Ψ_2 is the middle part from second block row to the $(L+1)^{th}$ block row of Γ_2, Ψ_3 is the last $(N-2L)$ block rows of Γ_2. Since $D[0]$ in Γ_2 shifts from second block row to $(L+1)^{th}$ block row, Ψ_2 is a full row rank matrix without any all-zero rows. Therefore subspace R_2 is spanned by the following two parts: $R(\mathbf{H}_t)$ and $R(\tilde{\mathbf{H}}_v)$, where $R(\tilde{\mathbf{H}}_v)$ is the partial space of $R(\mathbf{H}_v)$. Since $\mathbf{H}_s$ is full column rank, we can conclude that

$$R_1 \cap R_2 = R(\tilde{\mathbf{H}}_t) \qquad R(\mathbf{H}_r) \cap R_2 = \mathbf{0} \tag{20}$$

Let us consider the orthogonal projection from subspace R_1 onto subspace R_2. In essence, this is equivalent to the orthogonal projection from subspace $R(\mathbf{H}_r)$ onto subspace R_2 because $R(\tilde{\mathbf{H}}_t) \subset R_2$. Suppose $\mathbf{T} \triangleq [\mathbf{t}_1 \;\; \mathbf{t}_2 \;\; \ldots \;\; \mathbf{t}_p]$ is a basis for subspace $R(\mathbf{H}_r)$, $\mathbf{A}$ is defined as a basis for subspace R_2. Therefore the orthogonal projector is $\mathbf{P} \triangleq \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$ and $\mathbf{T}$ can be written as: $\mathbf{T} = \mathbf{T}_s + \mathbf{T}_n$, where $\mathbf{T}_s \triangleq \mathbf{PT}$ and $\mathbf{T}_n \triangleq \mathbf{T} - \mathbf{T}_s$. It is known that $R(\mathbf{T}_s) \subset R_2$ and $R(\mathbf{T}_n) \perp R_2$. Hence we have

$$\begin{aligned} \mathbf{T}_n^T\mathbf{S}_1 &= \mathbf{T}_n^T \left[\begin{array}{cc} \mathbf{H}_r & \mathbf{H}_t \end{array}\right] \left[\begin{array}{cc} D(0) & \mathbf{0} \\ \mathbf{0} & \Psi_1 \end{array}\right] \left[\begin{array}{c} \mathbf{H}_r^H \\ \mathbf{H}_t^H \end{array}\right] \\ &= \mathbf{Q}\mathbf{H}_r^H \end{aligned} \tag{21}$$

where $\mathbf{Q} \triangleq \mathbf{T}_n^T\mathbf{H}_r D[0]$ is an $p \times p$ matrix. Notice that $\mathbf{H}_r = [\mathbf{H}^T(L) \;\; \ldots \;\; \mathbf{H}^T(0) \;\; \mathbf{0} \;\; \ldots \;\; \mathbf{0}]^T$, by deleting the all-zero blocks which include no channel information in $\mathbf{H}_r$, we can identify channel up to an instantaneous matrix $\mathbf{Q}$.

C. Discussions

We discuss the singularity of $\mathbf{Q}$. Since $D[0]$ has full rank, the rank of $\mathbf{Q}$ is determined by $\tilde{\mathbf{Q}} \triangleq \mathbf{T}_n^T\mathbf{H}_r$. Let $\mathbf{T}_n \triangleq [\mathbf{t}_{n1} \;\; \mathbf{t}_{n2} \;\; \ldots \;\; \mathbf{t}_{np}]$, and it is easy to show that $\mathbf{t}_{ni}^T\mathbf{H}_r \neq \mathbf{0}$. Let $\tilde{\mathbf{Q}} \triangleq [\tilde{\mathbf{q}}_1^T \;\; \ldots \;\; \tilde{\mathbf{q}}_p^T]^T$, λ_j be p complex numbers. We define ξ and ν as:

$$\xi \triangleq \lambda_1\tilde{\mathbf{q}}_1 + \lambda_2\tilde{\mathbf{q}}_2 + \ldots + \lambda_p\tilde{\mathbf{q}}_p \tag{22}$$

$$\nu \triangleq \lambda_1\mathbf{t}_{n1} + \lambda_2\mathbf{t}_{n2} + \ldots + \lambda_p\mathbf{t}_{np} \tag{23}$$

we have $\nu^T\mathbf{H}_r = \xi$. It is clear that $\xi = 0$ if and only if ν belongs to the left null space of $\mathbf{H}_r$. Therefore we know that the probability of $\xi = 0$ is equal to zero. We can almost assure that matrix $\mathbf{Q}$ is invertible. Another important issue is that the proposed method does not require to know the exact knowledge of channel order. Only a upper bound is needed. Theoretically, channel order overestimation makes no difference to our method.

D. Multiple Estimations Combination

We can generalize our method by choosing:

$$\boxed{k_1 \geq L, \;\; k_2 = k_1 + 1, \;\; k_3 = k_1 + L, \;\; N \geq k_3 + L} \tag{24}$$

Thus the $(k_2 - L)^{th}$ block column in $\mathbf{H}_s$ can be extracted and identified up to a nonsingular matrix. By deleting the all-zero blocks which include no channel information in the $(k_2 - L)^{th}$ block column of $\mathbf{H}_s$, we can get multiple estimations of the same channel matrix, $\mathbf{H} \triangleq [\mathbf{H}^T(0) \;\; \mathbf{H}^T(1) \;\; \ldots \;\; \mathbf{H}^T(L)]^T$. Therefore, we need to find an approach to combine these estimations, hence obtain a better estimate of the channel matrix. For simplicity, assume we have two estimates $\hat{\mathbf{H}}_1$ and $\hat{\mathbf{H}}_2$ of $\mathbf{H}$, where, in theory, $\hat{\mathbf{H}}_1$ and $\hat{\mathbf{H}}_2$ respectively identify $\mathbf{H}$ up to unknown nonsingular matrices $\hat{\mathbf{Q}}_1$ and $\hat{\mathbf{Q}}_2$. We concatenate $\hat{\mathbf{H}}_1$ and $\hat{\mathbf{H}}_2$ to obtain $\mathbf{B} \triangleq [\hat{\mathbf{H}}_1 \;\; \hat{\mathbf{H}}_2]$. Our task is to find a matrix $\mathbf{C}$ to satisfy the following equation:

$$\mathbf{C} = \arg \min_{\mathrm{rank}(\mathbf{C})=p} \|\mathbf{B} - \mathbf{C}\|_F \tag{25}$$

It turns into a *multidimensional total least squares* problem. Compute the SVD of $\mathbf{B}$, we have $\mathbf{B} = \mathbf{U}\Sigma\mathbf{V}^H$. Then the optimum solution is

$$\mathbf{C} = \sum_{i=1}^{p} \sigma_i u_i v_i^H \tag{26}$$

where σ_i, u_i, v_i are respectively the i^{th} diagonal element or column vector of Σ, $\mathbf{U}$, $\mathbf{V}$. We can take a basis for the column space of $\mathbf{C}$ as the combined estimate of the channel matrix $\mathbf{H}$.

IV. Simulation Results

Now we present simulation results to illustrate the performance of our algorithm. We compare our method, namely *MIMO Cumulant Subspace Projection* (MCSP) algorithm, to the *MIMO Cumulant Subspace* (MCS) algorithm presented in [5]. In our simulations, source signals are mutually independent i.i.d QPSK signals. Channel outputs are added with complex white Gaussian noise with zero-mean. We consider 2-input/2-output FIR-MIMO channels with channel order $L = 2$. The performance is measured by the Overall Normalized Mean Square Error (ONMSE), which is defined in [5]. Results are averaged over 50 Monte Carlo runs.

Example 1. We consider a 2-input/2-output system with transfer function $H(z)$ as

$$\left[\begin{array}{cc} 0.7 + z^{-1} + 0.7z^{-2} & 1.4 + 1.82z^{-1} + 0.6593z^{-2} \\ 2.7 + 0.8z^{-2} & 0.5 + 1.2z^{-1} + 0.7426z^{-2} \end{array}\right] \tag{27}$$

In Figure 1, we show the performance of MCSP and MCS at different SNR with true channel order case and overestimated channel order case. It can be seen that both MCSP and MCS have a mild performance degradation when channel order is overestimated. Moreover, in both cases MCS surpasses MCSP a bit in performance. It shows that MCS algorithm performs very well when channel order of each user are equal.

Example 2. Let us consider a example in which the channel order of each user are not equal. The channel transfer function $H(z)$ is given as

$$\left[\begin{array}{cc} 1 + 0.5z^{-1} + 0.5z^{-2} & 1.6 + 0.64z^{-1} \\ 0.4 + 0.6z^{-1} + z^{-2} & 0.7263z^{-1} \end{array}\right] \tag{28}$$

Figure 2 shows the performance of MCSP and MCS when channel order are exactly known. Compared with example 1, the performance of MCS has some degree of degradation. The reason is that only partial nullspace can be estimated in this example. In such case, MCSP outperforms MCS. However, we also can see that MCSP is more sensitive to noise and estimation errors. The performance of MCSP degrades more rapidly than MCS when data samples is less or SNR becomes low. In Figure 3, we show the performance of MCSP and MCS when channel order is overestimated. Both algorithms show mild performance degradation in this case and MCSP also outperforms MCS.

V. Conclusion

In this paper, we present a new linear HOS method for blind identification of FIR MIMO channel. The proposed method inherits the advantage of other existing HOS algorithms in relaxing the identifiability conditions. Also, to some extent, the proposed method is robust to channel order overestimation. Simulation results show that our algorithm is potential to achieve better performance than MCS under a more general channel condition. Our future work tries to extend our method to a more relaxing identifiability conditions where $p > q$, that is, number of sources are greater than number of sensors.

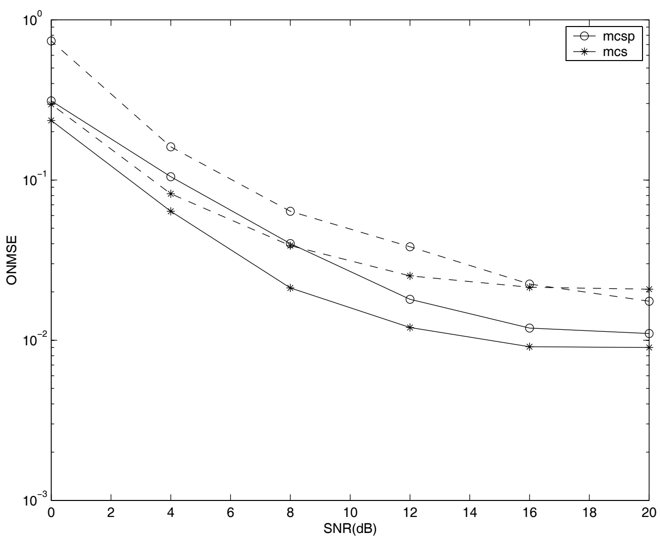

Fig. 1. Performance comparison at different SNR. Solid lines are for true order; dashed lines are for overestimated order by 1. Data samples are 8000.

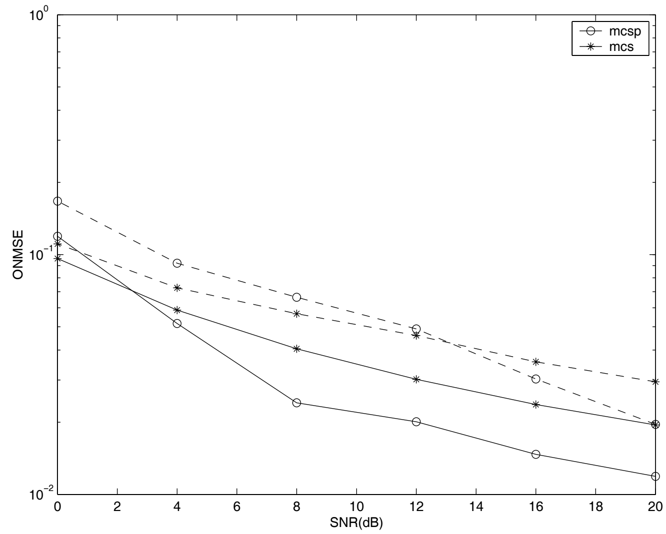

Fig. 2. Performance comparison at different data samples. Solid lines are for 8000 samples; dashed lines are for 6000 samples.

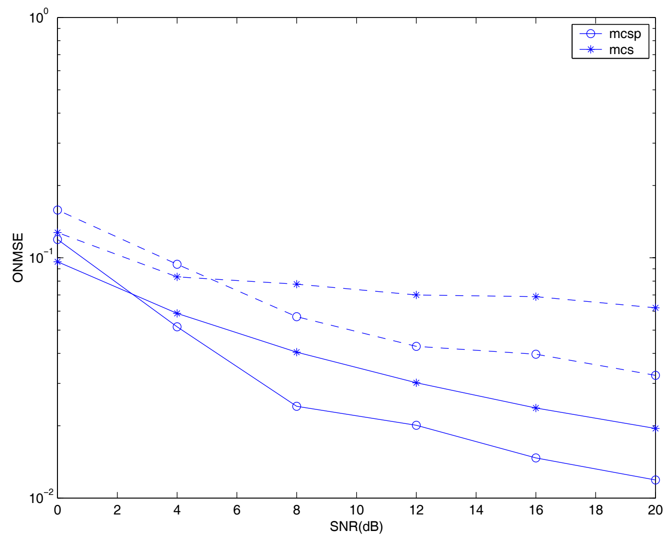

Fig. 3. Performance comparison at different SNR. Solid lines are for true order; dashed lines are for overestimated order by 1. Data samples are 8000.

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