REVISITING THE LINEAR PREDICTION ALGORITHM FOR BLIND CHANNEL EQUALIZATION AND IDENTIFICATION

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ABSTRACT

The Linear Prediction (LP) algorithm can be regarded as one that leads to the estimation of a set of zero-forcing equalizers. A final step in the LP algorithm consists in (linearly) combining these equalizers to form one that restores the transmitted symbols ISI-free, unattenuated and not delayed. We show that the way they were originally combined in not justified. We propose and justify a different combination of the original LP equalizers. The modified LP algorithm shows enhanced performances for both equalization and identification tests. In this work, we also comment on the often questioned robustness of the LP algorithm to channel order over estimation. We argue here that the LP algorithm is truly robust to order over estimation and we give a formal proof to this.

1. INTRODUCTION

Channel identification and equalization are classic topics in signal processing. The objective is to identify and/or remedy to the inter-symbol interference effect of a (linear) channel. They are referred to as blind if achieved without knowing about the transmitted symbols. They are then based on the statistical properties of these symbols. Those based on Second Order Statistics (SOS) are preferred because the associated moments are much easier to estimate. SOS-based blind algorithms are made possible if the channel output is over sampled and/or received using multiple sensors.

The Linear Prediction (LP) algorithm [10, 2] was among the first SOS algorithms. It is, in deed, among the most popular, due to its claimed (but sometimes questioned) robustness to channel order over estimation. We will be addressing this aspect in the second part of the paper. In the first part, we partially rewrite the LP algorithm. The original LP algorithm is based on the fact that the channel output is an auto-regressive process and that the associated innovation is equal (up to a multiplicative vector) to the transmitted symbol. The latter can, hence, be restored using the linear predictor, hence (almost) achieving a Zero-Forcing (ZF) equalization. The so-computed LP ZF equalizer is obtained by linearly combining the columns of the predictor. We show that, in fact, each of these columns can be regarded as a separate ZF equalizer and each can lead to a different channel estimate. The way they were combined in the original LP algorithm is not justified. This is particularly true since we can prove that these ZF equalizers have different behaviors in the presence of observation noise.

We rewrite the LP algorithm by proposing an alternative linear combination of the individual ZF equalizers. This modification is inspired by [1, 5] where the problem of optimally combining ZF equalizers has been posed and has lead to the introduction of the Equalization Peak Criterion (EPC). Simulation results confirm that the modified LP algorithm outperforms the original one for equalization and also for identification tests.

2. THE CHANNEL MODEL

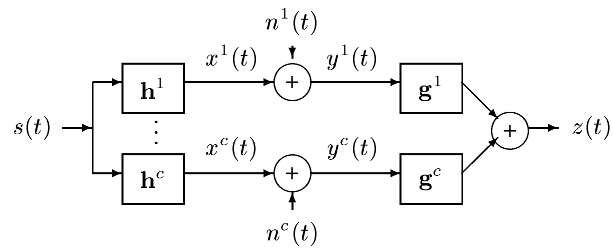

Fig. 1. Single input multiple output channel

A fractionally spaced and/or multi-sensor receiver is often modeled by a Single Input Multiple Output (SIMO) scheme as depicted in Fig. 1. A set of c filters are driven by a common scalar input $s(t)$. The impulse response of the c'-th filter is given by $\mathbf{h}^{c'} \stackrel{\text{def}}{=} \left[h^{c'}(0), h^{c'}(1), \cdots\right]^T$. The order m is defined as the maximum among those of $\mathbf{h}^1 \cdots \mathbf{h}^c$. We define the c-dim taps $\mathbf{h}(k) \stackrel{\text{def}}{=} \left[h^1(k) \cdots h^c(k)\right]^T, k = 0, \cdots, m$. where $\mathbf{h}(k) = \mathbf{0}$ for $k < 0$ or $k > m$. The SIMO impulse response is defined as $\mathbf{h} \stackrel{\text{def}}{=} \left[\mathbf{h}^T(0) \cdots \mathbf{h}^T(m)\right]^T$. The noise corrupted output $\mathbf{y}(t) \stackrel{\text{def}}{=} \left[y^1(t) \cdots y^c(t)\right]^T$ equals $\mathbf{x}(t) + \mathbf{n}(t) = \mathbf{H}\mathbf{s}_{m+1}(t) + \mathbf{n}(t)$ where $\mathbf{H} \stackrel{\text{def}}{=} [\mathbf{h}(0) \cdots \mathbf{h}(m)]$ and $\mathbf{s}_k(t) \stackrel{\text{def}}{=} [s(t) \cdots s(t-(k-1))]^T$ for any k. The output is observed over many (say l) symbol periods to form $\mathbf{y}_l^T(t) \stackrel{\text{def}}{=} \left[\mathbf{y}^T(t) \cdots \mathbf{y}^T(t-(l-1))\right]^T$. We have $\mathbf{y}_l(t) = \mathbf{H}_l \mathbf{s}_{l+m}(t) + \mathbf{n}_l(t)$ where $\mathbf{H}_l \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{H}\,\mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0}\,\mathbf{H} & \cdots & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & \cdots & \mathbf{0}\,\mathbf{H} \end{bmatrix}$ is the $cl \times (l+m)$ filtering matrix, $\mathbf{n}_l(t)$ is defined similarly as $\mathbf{y}_l(t)$ and $\mathbf{0}$ is the c-dim null vector. The channel SOS are completely described by the correlation terms $\gamma(k) \stackrel{\text{def}}{=} \mathbf{E}\left[\mathbf{x}(t+k)\mathbf{x}^H(t)\right]$. $\gamma(k) = \mathbf{0}$ for $|k| > m$. We define the correlation matrix

$$\begin{aligned} \mathbf{R}_l &\stackrel{\text{def}}{=} \mathbf{E}\left[\mathbf{y}_l(t)\mathbf{y}_l^H(t)\right] \\ &= \begin{bmatrix} \gamma(0) & \gamma(1) & \cdots & \gamma(l-1) \\ \gamma^H(1) & \gamma(0) & \ddots & \gamma(l-2) \\ \vdots & \ddots & \ddots & \vdots \\ \gamma^H(l-1) & \gamma^H(l-2) & \cdots & \gamma(0) \end{bmatrix} \end{aligned}$$

$$= \quad \sigma_s^2 \mathbf{H}_l \mathbf{H}_l^H + \sigma_n^2 \mathbf{I}$$

The last equation (where $\mathbf{I}$ stands for the identity matrix of appropriate dimensions) is valid assuming the symbols to be i.i.d. and uncorrelated from the white noise components. σ_s^2 and σ_n^2 refer to symbol and noise powers, respectively.

3. ZERO-FORCING EQUALIZATION

An $(l-1)$-order d-delay ZF equalizer $\mathbf{g}$ verifies $\mathbf{g}^T \mathbf{H}_l = [0 \cdots 0 u 0 \cdots 0]$ for some scalar u. His output

$$z(t) = us(t-d) + \mathbf{n}_l^T(t)\mathbf{g}$$

has signal and noise parts with respective powers $\sigma_s^2 |u|^2$ and $\sigma_n^2 \|\mathbf{g}\|^2$. The associated MSE on the equalized (N-length) symbol sequence is defined [4] as

$$\frac{1}{\sigma_s^2 N} \mathbf{E} \left[\min_\beta \|\mathbf{s}_N(N-1-d) - \beta \mathbf{z}_N(N-1)\|^2 \right] \qquad (1)$$

where β is complex-valued. Following this definition, two equalizers are equivalent if they are equal up to some multiplicative factor. A ZF equalizer can also be used to estimate the channel response [4, (4)]. When this is a no-delay equalizer, we have

$$\mathbf{h} = \begin{bmatrix} \gamma(0) - \sigma_n^2 \mathbf{I} & \gamma(1) & \cdots & \gamma(l) \\ \gamma(1) & \gamma(2) & \cdots & \gamma(l+1) \\ \vdots & \vdots & & \vdots \\ \gamma(m) & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} \mathbf{g}^*$$

Obviously, any linear combination of ZF equalizers of the same delay is equally a ZF equalizer of that same delay. Hence, in algorithms where many ZF equalizers are estimated, one is faced with the problem of choosing the *best* equalizer within the so-defined linear subspace. Fortunately, for algorithms such as the MRE [6] and the GRDA [5], these ZF equalizers are estimated as eigen vectors associated to some SOS-expressed matrices, they (as well as any unit-norm linear combination of them) are unit-norm and so are equivalent from the noise enhancement point of view. Consequently, the equalizer that maximizes the signal part (and hence the SNR) at the equalizer output was considered to be the *best* one, hence defining the EPC criterion [1, 5]. Formally, if $\mathbf{g}_1, \cdots, \mathbf{g}_w$ are orthonormal ZF equalizers that restore the transmitted symbol with the same delay and with the attenuation $u_1, \cdots, u_w$, then the best equalizer built up from $\mathbf{g}_1, \cdots, \mathbf{g}_w$ is given by $[\mathbf{g}_1 \cdots \mathbf{g}_w] [u_1 \cdots u_w]^H$. $[u_1 \cdots u_w]$, unknown, is estimated in [1] as the eigen vector associated to the unique non-zero eigenvalue of $[\mathbf{g}_1 \cdots \mathbf{g}_w]^H \left(\mathbf{R}_l^T - \sigma_n^2 \mathbf{I} \right) [\mathbf{g}_1 \cdots \mathbf{g}_w]$.

4. THE LINEAR PREDICTION ALGORITHM

In [10, 2] , the (noise-free) channel output, an m-order moving average process, is proved to be an m-order auto-regressive process. Furthermore, the associated innovation equals $s(t)\mathbf{h}(0)$. Consequently, the linear predictor, obtained by solving the Yule-Walker (YW) equation, restores the transmitted symbols (up to a multiplicative factor), hence (almost) achieving a ZF equalization.

Formally, the linear prediction $c \times c$ coefficients $\mathbf{A}_1, \cdots, \mathbf{A}_m$, given by $\left[\mathbf{A}_1^T \cdots \mathbf{A}_m^T \right]^T \stackrel{\text{def}}{=} - \left(\left(\mathbf{R}_m - \sigma_n^2 \mathbf{I} \right)^\dagger \right)^T [\gamma(1) \cdots \gamma(m)]^T$, are such that $\mathbf{x}(t) - \sum_{k=1}^m \mathbf{A}_k \mathbf{x}(t-k) = s(t)\mathbf{h}(0)$ or equivalently $\begin{bmatrix} \mathbf{I} \\ - \left(\left(\mathbf{R}_m - \sigma_n^2 \mathbf{I} \right)^\dagger \right)^T [\gamma(1) \cdots \gamma(m)]^T \end{bmatrix}^T \mathbf{x}_{m+1}(t) = s(t)\mathbf{h}(0)$. Hence, each of the columns of

$$\mathbf{G} \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{I} \\ - \left(\left(\mathbf{R}_m - \sigma_n^2 \mathbf{I} \right)^\dagger \right)^T [\gamma(1) \cdots \gamma(m)]^T \end{bmatrix} \qquad (2)$$

achieves a no-delay ZF equalization. Following [2], the unknown $\mathbf{h}(0)$ is determined using

$$\begin{aligned} &\sigma_s^2 \mathbf{h}(0)\mathbf{h}^H(0) = \gamma(0) - \sigma_n^2 \mathbf{I} \\ &- [\gamma(1) \cdots \gamma(m)] \left(\mathbf{R}_m - \sigma_n^2 \mathbf{I} \right)^\dagger [\gamma(1) \cdots \gamma(m)]^H \quad (3) \end{aligned}$$

We let $\mathbf{l}$ be the unit-norm eigen vector associated to the unique non-zero eigenvalue l of the matrix in the RHS of (3) or more conveniently the one associated to the largest eigenvalue of $\gamma(0) - [\gamma(1) \cdots \gamma(m)] \left(\mathbf{R}_m - \sigma_n^2 \mathbf{I} \right)^\dagger [\gamma(1) \cdots \gamma(m)]^H$. Then

$$\frac{1}{\sqrt{l}} \mathbf{G}\mathbf{l}^* \qquad (4)$$

is proposed as a no-delay ZF equalizer [2, (14)] that restores the transmitted symbols not attenuated and ISI-free.

5. A MODIFIED LINEAR PREDICTION ALGORITHM

First, notice that the multiplicative factor $\frac{1}{\sqrt{l}}$ in (4) has no effect on the equalization performance. One should simply apply the equalizer $\mathbf{G}\mathbf{l}^*$. Second, the equalizer in (4) is obtained by combining the columns of $\mathbf{G}$ following $\mathbf{l}^*$ i.e. following $\mathbf{h}(0)^*$. This, in fact, maximizes (the power of) the signal part at the equalizer output. By doing so, the SNR is not necessarily maximized because the equalizers which are the columns of $\mathbf{G}$ do not have the same norm and so do not result in the same noise enhancement.

Consequently, we propose to modify the original LP algorithm as follows. Let $\mathbf{G}^\dagger$ be a matrix whose columns form an orthonormal basis of the column span of $\mathbf{G}$. It can be computed using the Gram-Shmidt algorithm [7]. The complexity of the Gram-Shmidt algorithm, $O(c^2 M)$ [3], will not affect significantly the complexity of the modified LP algorithm which is mainly due to the resolution of the YW equation ($O(c^3 M^3)$). In practice, the (sub)channel responses are expected to be long and their number to be limited. For every unit-norm vector $\mathbf{x}$, $\mathbf{G}^\dagger \mathbf{x}$ is a no-delay unit-norm ZF equalizer. The application of the EPC requires the EVD of

$$\left(\mathbf{G}^\dagger \right)^H \left(\mathbf{R}_m^T - \sigma_n^2 \mathbf{I} \right) \mathbf{G}^\dagger \qquad (5)$$

Alternatively, (5) also equals $\sigma_s^2 \left(\mathbf{G}^\dagger \right)^H \mathbf{H}_{c(m+1)}^* \mathbf{H}_{c(m+1)}^T \mathbf{G}^\dagger$. Only the first row of $\mathbf{H}_{c(m+1)}^T \mathbf{G}^\dagger$ is non-zero. It equals $\mathbf{h}^T(0)\mathbf{G}^\dagger(0)$, $\mathbf{G}^\dagger(0)$ is formed by the first c rows of $\mathbf{G}^\dagger$. So, (5) is also equal to

$$\sigma_s^2 \left(\mathbf{G}^\dagger \right)^H \mathbf{h}^*(0)\mathbf{h}^T(0)\mathbf{G}^\dagger = \left(\mathbf{G}^\dagger \right)^H \times \left(\gamma(0) - \sigma_n^2 \mathbf{I} - \begin{bmatrix} \gamma^H(1) \\ \vdots \\ \gamma^H(m) \end{bmatrix}^H \mathbf{R}_m^\dagger \begin{bmatrix} \gamma^H(1) \\ \vdots \\ \gamma^H(m) \end{bmatrix} \right)^* \mathbf{G}^\dagger$$

Hence, $\mathbf{x}$ can be chosen as the eigen vector associated to the largest eigenvalue of

$$\left(\mathbf{G}^{\dagger}\right)^{H}\left(\gamma(0)-\begin{bmatrix}\gamma^{H}(1)\\ \vdots\\ \gamma^{H}(m)\end{bmatrix}^{H}\mathbf{R}_{m}^{\dagger}\begin{bmatrix}\gamma^{H}(1)\\ \vdots\\ \gamma^{H}(m)\end{bmatrix}\right)^{*}\mathbf{G}^{\dagger} \quad (6)$$

(6) involves c-sized matrices while (5) involves much larger $c(m+1)$ ones. Smaller sizes imply not only lower complexity (the matrices have to undertake EVDs), but also less error propagation. (6) is more efficiently estimated when only a finite sample size is available.

6. ROBUSTNESS TO ORDER OVER ESTIMATION

The LP algorithm involves the computation of the pseudo-inverse of the channel correlation matrix. When the channel order is over estimated, the eigenvalues of the (augmented) signal subspace are inverted including some zero eigenvalues. This has mislead to conclude that the LP algorithm is not robust to channel order over estimation [8, 5]. In this section, we prove the contrary.

For a sufficiently large smoothing factor l, $\mathbf{H}_l$ is full column rank [11]. We let λ_k be the k-th largest eigenvalue of the estimate of $\mathbf{R}_l$ and $\mathbf{s}_l^k$ the associated unit-norm eigen vector.

Let $m' > m$ be the assumed (over estimated) channel order. $\left(\mathbf{R}_{m'}-\sigma_n^2\mathbf{I}\right)^{\dagger}$ is estimated by $\sum_{k=1}^{l+m'}\frac{1}{\lambda_k-\sigma_n^2}\mathbf{s}_{m'}^k\left(\mathbf{s}_{m'}^k\right)^H$. Because the $(c-1)l-m$ lowest eigenvalues asymptotically approach σ_n^2, this lead [8, 5] to conclude that the computation of the pseudo-inverse is badly conditioned and that the LP algorithm is not robust to order over estimation. The first conclusion is true, but the second one is not. In fact, by virtue of (2) and (3), what is required by the LP algorithm is the computation of $[\gamma(1)\cdots\gamma(m')]\left(\mathbf{R}_{m'}-\sigma_n^2\mathbf{I}\right)^{\dagger}$ rather than that of the pseudo-inverse. Contrarily to $\left(\mathbf{R}_{m'}-\sigma_n^2\mathbf{I}\right)^{\dagger}$, the computation of $[\gamma(1)\cdots\gamma(m')]\left(\mathbf{R}_{m'}-\sigma_n^2\mathbf{I}\right)^{\dagger}$ turns to be robust to channel order over estimation as proved in the sequel.

For $k > l+m$, we focus on the term

$$\begin{aligned}\left[\gamma(1)\cdots\gamma(m')\right]\mathbf{s}_{m'}^k &= \mathbf{E}\left[\mathbf{x}(n+1)\mathbf{x}_{m'}^H(n)\right]\mathbf{s}_{m'}^k\\ &= \mathbf{E}\left[\left(\mathbf{x}(n+1)\right)\left(\mathbf{x}_{m'}^H(n)\mathbf{s}_{m'}^k\right)\right]\end{aligned}$$

$\mathbf{s}_{m'}^k$ is (almost) orthogonal to $\mathbf{R}_{m'}\overset{\text{def}}{=}\mathbf{E}\left[\mathbf{x}_{m'}(n)\mathbf{x}_{m'}^H(n)\right]$ which means that $\left(\mathbf{x}_{m'}^H(n)\mathbf{s}_{m'}^k\right)$ is (almost) uncorrelated from $\mathbf{x}_{m'}(n)$ and so from $\mathbf{x}_{m'}(n+1)$ too. Hence, $[\gamma(1)\cdots\gamma(m')]\,\mathbf{s}_{m'}^k$ is close to zero. Consequently, the fact that we are inverting (the close to zero) noise subspace eigenvalues is compensated by the fact that the associated eigen vectors are (almost) orthogonal to $[\gamma(1)\cdots\gamma(m')]$. Hence, channel order over estimation degrades the performances of the LP algorithm without resulting in its complete failure. This conclusion holds for the original as well as for the modified LP algorithm.

7. SIMULATIONS

A series of simulations has been conducted to compare the modified LP algorithm to the original one. In the simulations, the channel, taken from [9], is driven by unit-variance i.i.d. 4-QAM symbols and corrupted by AWG noise. The SNR is defined as $\frac{\mathbf{E}\left[\|\mathbf{x}(t)\|^2\right]}{\mathbf{E}\left[\|\mathbf{n}(t)\|^2\right]}=\frac{\sigma_n^2\|\mathbf{h}\|^2}{c\,\sigma_n^2}$. The noise power is estimated as the average of the $(c-1)l-m'$ (m' being the assumed channel order) lowest eigenvalues of the estimated correlation matrix. The simulation results are averaged over 100 Monte Carlo runs.

An estimated ZF equalizer is tested with a sequence $s(0),\cdots,s(N-1)$ of randomly generated source symbols (in the simulations, $N=200$). The equalizer outputs $z(0),\cdots,z(N-1)$ are compared to the transmitted symbols in terms of the MSE defined as in (1) which can be proved to be equal to [4]

$$\frac{1}{\sigma_s^2 N}\left[\|\mathbf{s}_N(N-1)\|^2-\left(\frac{\mathbf{s}_N^H(N-1)\mathbf{z}_N(N-1)}{\|\mathbf{z}_N(N-1)\|}\right)^2\right]$$

Identification tests are also conducted. The MSE on the identified channel response is defined similarly as that on equalized symbols. The MSE associated to the estimate $\hat{\mathbf{h}}$ (whose order m' is possibly over estimated) of $\mathbf{h}$ is defined as [5]

$$1-\left(\frac{\max\left(\left|\left[\mathbf{0}_{1,c(m'-m)}\mathbf{h}^H\right]\hat{\mathbf{h}}\right|,\left|\left[\mathbf{h}^H\mathbf{0}_{1,c(m'-m)}\right]\hat{\mathbf{h}}\right|\right)}{\|\mathbf{h}\|\|\hat{\mathbf{h}}\|}\right)^2$$

Equalization results, presented in Fig. 2, confirm that the modified LP algorithm outperforms the original one. The enhancement is valid for short data as well as asymptotically (cf. Fig. 2.a). As showed in Fig. 2.b, the improvement is more important for low SNR where the noise enhancement effect is more significant. Eventhough there is no evidence of optimality of the EPC criterion w.r.t. to the estimation of the channel response, an improvement of the channel estimate is also observed in Fig. 3.a. Finally, we test the (original and modified) LP algorithm with deliberately over estimated values of the channel order. Simulations results, as shown in Fig. 4, show that such modeling errors result in the deterioration of the equalization performance and not complete failure of it.

8. CONCLUSION

Contrarily to a number of SOS-based blind identification algorithms such as the subspace algorithm [11, 9], the LP algorithm first starts by estimating a (zero-forcing) equalizer. This step does not explicitly require the knowledge of the channel order except for the computation of the correlation matrix pseudo-inverse. We here formally prove that order over estimation has no catastrophic effect on the LP algorithm as was often suspected. Furthermore, the LP algorithm leads to more than one equalizer. These can be combined differently than the in original LP algorithm leading to a new equalizer with better performances in the presence of observation noise.

9. REFERENCES

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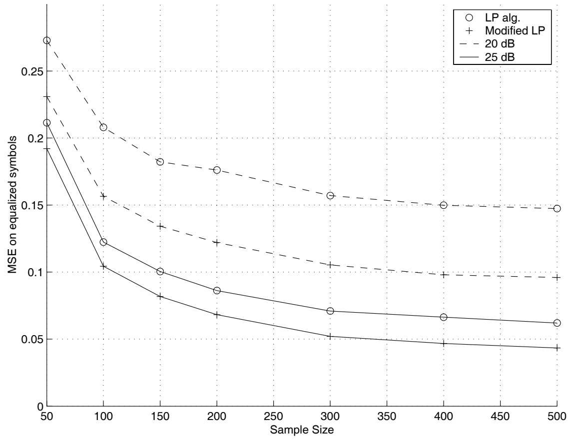
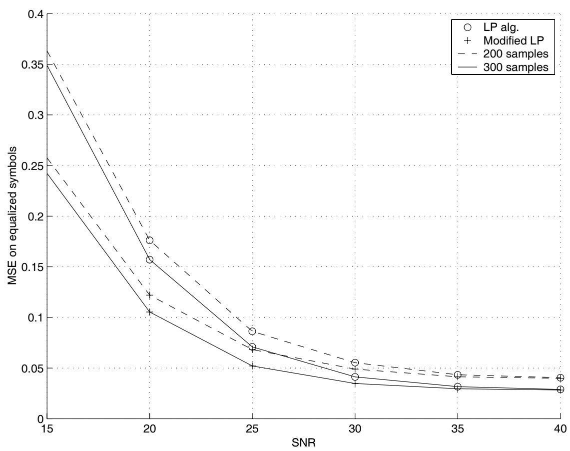


(a)



(b)

Fig. 2. Equalization tests

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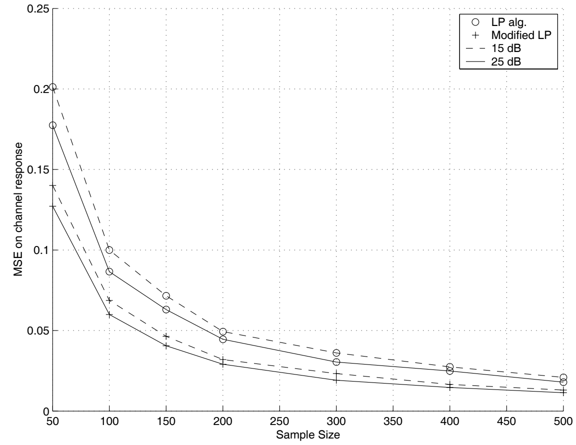

Fig. 3. Identification tests

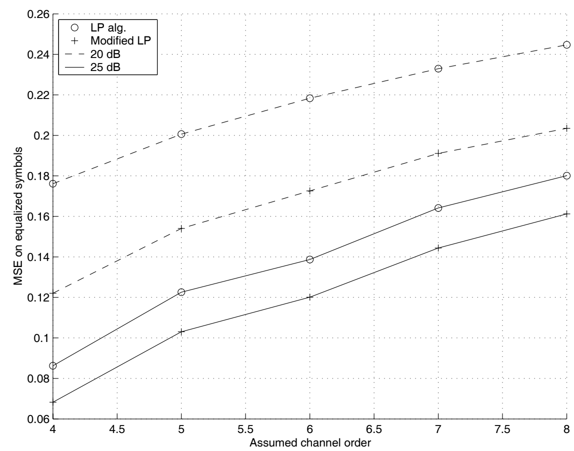

Fig. 4. Channel order over estimation.

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