# SEMI-BLIND EQUALIZATION FOR GMSK-BASED MOBILE COMMUNICATIONS

*Raphaël Gallego, Alexandre Rouxel*

{Raphael.Gallego, Alexandre.Rouxel}@wavecom.com

Wavecom S.A.
Advanced Baseband Department
3, Esplanade du Foncet
92442 Issy-les-Moulineaux Cedex, France

*Florence Alberge, Pierre Duhamel*

{Florence.Alberge, Pierre.Duhamel}@lss.supelec.fr

LSS - CNRS
Signal Processing Department
Plateau de Moulon - 3, rue Joliot-Curie
91192 Gif-sur-Yvette Cedex, France

**ABSTRACT**

Our work focuses on the semi-blind equalization techniques when applied to GMSK-based mobile communication systems. Starting from the works developed in [1], a new semi-blind block algorithm is proposed to provide reliable communication through a typical mobile multi-path channel. The algorithm is shown to have performances similar to those of the Viterbi algorithm in a fixed setting, while being more efficient on time-varying situations.

## 1. INTRODUCTION

In mobile communications systems (MCS), signals are transmitted through multi-path mixed-phase time-variant channels. Because of the introduction of distortion in the received signal, state-of-the-art utilizes a training sequence to estimate the channel parameters. Since then, blind equalization methods have been applied in MCS in order to obtain satisfactory performances without using any training symbols.

This paper is concerned with semi-blind equalization applied to GMSK-based mobile communications without using any extra antenna neither oversampling technique at the receiver.

The GSM norm specifies a maximum speed $v$ (depending on the frequency of carrier $f_c$) between the transmitter and the receiver under which the system must guarantee a certain level of performance. Viterbi-based receivers provide a comfortable performance at this (low) speed but, nowadays, more and more situations with higher speeds applications are involved. Thus, our task is as follows: to develop a semi-blind block algorithm that i) attains equivalent performance w.r.t. the Viterbi algorithm when time-invariant channels ($v = 0\ km/h$) are used w; and ii) is well suited for its adaptation in variant channel context, specially in high speed environments ($v \gg 0\ km/h$), where the Viterbi-based receiver can't afford reliable performance.

The paper is organized as follows. The background and problem setup are given in Section 2, where the basis of our algorithms are presented. Our contribution is detailed in Section 3. Section 4 show the behavior of our algorithms when simulating in a simplified GSM environment. Finally, conclusion and further research lines are detailed in Section 5.

## 2. BACKGROUND AND PROBLEM SETUP

### 2.1. Diversity from derotation

In order to apply blind equalization algorithms that are usually utilized for linear modulations, a linear approximation of GMSK modulation is derived. This linear approximation is based on the Laurent's decomposition [2]. The general outlines of the method are presented below.

Let denote $s_n = \pm 1$ the transmitted binary data. Considering the GMSK modulation like defined in the GSM norm ([3]), the received signal can be written as:

$$x_k = \sum_n h_n \cdot s_{k-n} \cdot j^{k-n} \tag{1}$$
$$h_n = h_0 * h_c * g \tag{2}$$

where $h_0$ denotes the first order filter from the GMSK linear decomposition, $h_c$ stands for the physical propagation channel and $g$ is used for the reception filter. Once a derotation operation is performed, the received signal results on:

$$\widehat{x}_k = j^{-k} \cdot x_k = \sum_n h_n \cdot s_{k-n} \cdot j^{-n} = \sum_n \widehat{h}_n \cdot s_{k-n} \tag{3}$$

where $\widehat{h}_n = h_n \cdot j^{-n}$ is the global channel impulse response after derotation. Taking into account that $\widehat{h}_n$ is complex and $s_n$ a real BPSK, the derotated signal $\widehat{x}_k$ sampled at the baud rate can be seen as the output of a two-subchannel system. Derotation allows the use of algorithms that are usually applied in a SIMO (Single-Input-Multiple-Output) model, as in [4] for blind channel identification or in [5] for GSM semi-blind equalization.

### 2.2. SIMO Model. Definitions and notations

Thanks to the approximations above, diversity has been extracted from the rotation technique. Now, we can deal with a SIMO model with two outputs. Let $N$ denote the number of received symbols, $M$ the channel order (so $M+1$ is the length of the channel impulse response) and $L$ the diversity order ($L = 2$). Thus, we define:

The transmitted symbols sequence of size $(M+N) \times 1$,

$\mathbf{s}_{M+N} \triangleq \left[ s(n),\ n = -M \ldots 0 \ldots N-1 \right]^T$

The channel impulse response of size $(M+1) \times 1$,

$\mathbf{h}_n^{\mathbf{c}} \triangleq \left[ h_n^{\mathbf{c}}(k),\ k = M \ldots 0,\ c = \{1, \ldots, L\} \right]^T$

and its SIMO equivalent, of size $L \cdot (M+1) \times 1$,

$\mathbf{h}_n \triangleq \left[ \left[\mathbf{h}_n^{\mathbf{1}}\right]^T \ \ldots\ \left[\mathbf{h}_n^{\mathbf{L}}\right]^T \right]^T$

Finally, the received symbols sequence of size $(N) \times 1$,

$\mathbf{X}_N^{\mathbf{c}} \triangleq \left[ x^{\mathbf{c}}(n),\ n = 0 \ldots N-1,\ c = \{1, \ldots, L\} \right]^T$

and its SIMO equivalent, of size $L \cdot (N) \times 1$,

$\mathbf{X}_N \triangleq \left[ x^{\mathbf{1}}(0), .., x^{\mathbf{L}}(0), \ldots, x^{\mathbf{1}}(N-1), .., x^{\mathbf{L}}(N-1) \right]^T$

(where $n$ is the time index)

Now assume that we have a unique channel impulse response "viewed" by the transmitted data $\mathbf{s}_{M+N}$, that is, $\mathbf{h}_n = \mathbf{h}\ \forall n$. In this block approach, the basic filtering equation of a SIMO system can be written as follows.

$$\mathbf{X}_N = \mathcal{T}_{N,\ M+N}(\mathbf{h}) \cdot \mathbf{s}_{M+N} \tag{4}$$

where $\mathcal{T}_{N,\ M+N}(\mathbf{h})$ is a Sylvester matrix of size $(L \cdot N) \times (M + N)$ defined as:

$$\mathcal{T}_{N,\ M+N}(\mathbf{h}) = \begin{pmatrix} h^{\mathbf{1}}(M) & \dots & h^{\mathbf{1}}(0) & 0 & \dots & 0 \\ h^{\mathbf{2}}(M) & \dots & h^{\mathbf{2}}(0) & 0 & \dots & 0 \\ & \ddots & & \ddots & \ddots & \vdots \\ 0 & & & & & \\ \vdots & \ddots & \ddots & & \ddots & 0 \\ 0 & \dots & 0 & h^{\mathbf{1}}(M) & \dots & h^{\mathbf{1}}(0) \\ 0 & \dots & 0 & h^{\mathbf{2}}(M) & \dots & h^{\mathbf{2}}(0) \end{pmatrix} \tag{5}$$

### 2.3. Conditional Maximum Likelihood (CML) approach

#### *2.3.1. Maximum Likelihood approach*

Our criterion belongs to the family of Maximum Likelihood (ML) criteria. If no statistics are assumed for the source, then both symbols and channel need to be estimated by the receiver, leading to a Deterministic ML (DML) approach. The DML criterion reads:

$$\mathcal{J}_{\mathrm{B}}^{\mathrm{DML}}(\mathbf{h}, \mathbf{s}_{M+N}) = \left\|\mathcal{T}_{N,\ M+N}(\mathbf{h}) \cdot \mathbf{s}_{M+N} - \mathbf{X}_N\right\|^2 \tag{6}$$

The SML approach can be understood as "plugging" the finite alphabet constraint on $\mathbf{s}$ to the DML criterion. Adding new hypothesis improves the performances at the cost of increasing the number of local minima. The SML criterion is expressed as:

$$\mathcal{J}_{\mathrm{B}}^{\mathrm{SML}}(\mathbf{h}, \mathbf{s}_{M+N}) = \left\|\mathcal{T}_{N,\ M+N}(\mathbf{h}) \cdot \mathbf{s}_{M+N} - \mathbf{X}_N\right\|^2, \quad \text{with} \quad \mathbf{s}_{M+N} \in \{-1, +1\} \tag{7}$$

#### *2.3.2. The CML criterion: from DML to SML*

The criterion used in our work increases the possible tradeoff between local minima and performances. Starting from DML approach, the idea consists of introducing new hypothesis that will allow to improve the performances while controlling the local minima problem.

In DML approach, both symbols and channel are unknown variables and the criterion derived from the joint minimization is non-convex, leading to the apparition of local minima. However, the criterion is convex w.r.t. each variable (s or h) separately. Furthermore, it has been shown in [1] that, provided the convexity is property is kept, even if new local minima appear, these local minima can be controlled.

In our CML approach symbols are considered as random variables with a given probability density function (pdf), different from the real one, but reflecting our a priori knowledge of the symbols. This method has been first introduced by De Carvalho and Slock in [6] resulting on the GML (Gaussian ML) approach. In our case, a exponential truncated function is used that keep the above mentioned convexity as shown below:

$$p(s(n)) = 0 \quad \text{if} \quad |s(n)| > 1$$
$$p(s(n)) = \tfrac{1}{Z} e^{ks^2(n)} \quad \text{if} |s(n)| \leq 1, \quad p(\mathbf{s}_{M+N}) = \Pi_n p(s(n))$$

where $Z$ is a normalization factor.

When $k \to 0$, the pdf tends to a constant function on the interval $[-1, +1]$. When $k \to \infty$, the criterion corresponds to an SML approach, since symbols are constrained to be $(\{-1, +1\})$. The choice of $k$ follows from a compromise between the quantity of a priori information used and the respect of the convexity property. Our preferred value for $\gamma = 2k\sigma^2$, with $\sigma^2$ the noise variance, is the upper bound for the criterion to remain convex w.r.t. each variable separately. Thereby, the CML criterion can be written as:

$$\mathcal{J}_{\mathrm{B}}^{\mathrm{CML}}(\mathbf{h}, \mathbf{s}_{M+N}) = \left\|\mathcal{T}_{N,\ M+N}(\mathbf{h}) \cdot \mathbf{s}_{M+N} - \mathbf{X}_N\right\|^2 - \gamma \cdot \left\|\mathbf{s}_{M+N}\right\|^2, \quad \mathbf{s}_{M+N} \in C^{M+N} \tag{8}$$

$$\text{with} \quad \gamma = \lambda_{min}\{\mathcal{T}_{N,\ M+N}(\mathbf{h})^H \mathcal{T}_{N,\ M+N}(\mathbf{h})\} \tag{9}$$

where $\lambda_{min}(\mathbf{A})$ denotes the minimum eigenvalue of $\mathbf{A}$ and where $C^K = \{\mathbf{s} \in \mathbb{R}^K; -1 \leq s_i \leq +1, 1 \leq i \leq K\}$.

## 3. APPLICATION OF CML ALGORITHMS TO GMSK-BASED MOBILE COMMUNICATIONS

In practical situations, a burst contains $M + N$ symbols among which $N_u$ are unknown and $N_k$ are known from the receiver. This allows the full use of the known symbols in our semi-blind approach, as presented below.

### 3.1. CML Block Algorithm (CMLBA)

The block algorithm can be derived from the CML criterion above and adapted for the semi-blind context. Thus, the semi-blind criterion can be expressed as:

$$\begin{aligned} &\mathcal{J}_{\mathrm{B}}^{\mathrm{CML}}(\mathbf{h}, \mathbf{s}_{N_u}) = \\ &= \left\|\mathcal{T}_{N,\ M+N}(\mathbf{h})\mathbf{s}_{M+N} - \mathbf{X}_N\right\|^2 - \gamma\left\|\mathbf{s}_{N_u}\right\|^2 \\ &= \left\|\left[\mathcal{T}_{N,\ N_k}(\mathbf{h}) \mid \mathcal{T}_{N,\ N_u}(\mathbf{h})\right] \begin{bmatrix} \mathbf{s}_{N_k} \\ \mathbf{s}_{N_u} \end{bmatrix} - \mathbf{X}_N\right\|^2 - \gamma\left\|\mathbf{s}_{N_u}\right\|^2 \\ &= \left\|\mathcal{T}_{N,\ N_u}(\mathbf{h})\mathbf{s}_{N_u} - \left(\mathbf{X}_N - \mathbf{I}_N(\mathbf{s}_{N_k})\right)\right\|^2 - \gamma\left\|\mathbf{s}_{N_u}\right\|^2 \end{aligned} \tag{10}$$

$$\text{with} \quad \gamma = \lambda_{min}\{\mathcal{T}_{N,\ N_u}(\mathbf{h})^H \mathcal{T}_{N,\ N_u}(\mathbf{h})\} \tag{11}$$

$$\text{and} \quad \mathbf{I}_N(\mathbf{s}_{N_k}) = \mathcal{T}_{N,\ N_k}(\mathbf{h})\mathbf{s}_{N_k} \tag{12}$$

In [7], a new form for the filtering equation (4) is proposed, that is equivalent to classical one.

$$\mathbf{X}_N = \mathcal{U}_{N,\ M+1}(\mathbf{s}_{M+N})\mathbf{h} \tag{13}$$

where $\mathcal{U}_{N,\ M+1}(\mathbf{s}_{M+N})$ is a *Symbols* matrix of size $(L \cdot N) \times (L \cdot (M + 1))$ defined as:

$$\mathcal{U}_{N,\ M+1}(\mathbf{s}_{M+N}(n)) = \begin{pmatrix} \mathbf{Id}_L \bigotimes \mathbf{s}_M(n)^T \\ \mathbf{Id}_L \bigotimes \mathbf{s}_M(n-1)^T \\ \vdots \\ \mathbf{Id}_L \bigotimes \mathbf{s}_M(n-N+1)^T \end{pmatrix}$$

where $\mathbf{Id}_L$ denotes the identity matrix of size $L \times L$ and where $\bigotimes$ stands for the Kronecker product; this results in:

$$\begin{aligned} &\mathcal{J}_{\mathrm{B}}^{\mathrm{CML}}(\mathbf{h}, \mathbf{s}_{M+N}) = \\ &= \left\|\mathcal{U}_{N,\ M+1}(\mathbf{s}_{M+N})\mathbf{h} - \mathbf{X}_N\right\|^2 - \gamma\left\|\mathbf{s}_{M+N}\right\|^2 \end{aligned} \tag{14}$$

Thus, the block algorithm consists on the separate minimization of s and h variables, using an iterative procedure given on the algorithm that follows:

**for k = 1:ite$_\mathrm{B}$**

$$\widehat{\mathbf{s}}_{N_u}^{(k)} = \arg\min_{\mathbf{s}_{N_u} \in C^{N_u}} \mathcal{J}_{\mathrm{B}}^{\mathrm{CML}}(\widehat{\mathbf{h}}^{(k-1)}, \mathbf{s}_{N_u}) \tag{15}$$

$$\widehat{\mathbf{s}}_{M+N}^{(k)} = \left[\left[\mathbf{s}_{N_k}\right]^T \left[\widehat{\mathbf{s}}_{N_u}^{(k)}\right]^T\right]^T \tag{16}$$

$$\widehat{\mathbf{h}}^{(k)} = \arg\min_{\mathbf{h}} \mathcal{J}_{\mathrm{B}}^{\mathrm{CML}}(\mathbf{h}, \widehat{\mathbf{s}}_{M+N}^{(k)}) \quad \mathbf{end} \tag{17}$$

The inequality constraint (for symbols minimization) is resolved by a relaxation method (see [8] for details). The most computationally intensive part of the algorithm resides on the optimization method, so effective cost and sub-optimal methods should be used in a real-time application.

### 3.2. The partitioning procedure

As already explained on the CML criterion, the $\gamma$ parameter plays a very important role, because it controls the amount of prior information taken into account. Thereby, our interest is to achieve the largest $\gamma$ parameter as possible. Theorem 1 below show that a segmentation of (15) into a set of equations increase $\lambda_{min}$.

**Theorem 1** *Let $\mathbf{T_1}$ and $\mathbf{T_2}$ be two sub-matrices of a matrix $\mathbf{T}$ such that $\mathbf{T} = [\mathbf{T_1}|\mathbf{T_2}]$ then*

$$\lambda_{min}^{\mathbf{T}} \leq \lambda_{min}^{\mathbf{T_1}} \quad \text{and} \quad \lambda_{min}^{\mathbf{T}} \leq \lambda_{min}^{\mathbf{T_2}}$$

*where $\lambda_{min}^{\mathbf{T}}$ resp. $\lambda_{min}^{\mathbf{T_p}}$ stands for the smallest eigenvalue of $\mathbf{T^H T}$ resp. $\mathbf{T_p^H T_p}$, p=1,2.*

***Proof*** *Let $N$ resp. $N_p$ be the number of columns of $\mathbf{T}$ resp $\mathbf{T_p}$.*

$$\lambda_{min}^{\mathbf{T}} = \arg\min_{\mathbf{v}\in\mathbb{C}^N} \frac{\mathbf{v^H T^H T v}}{\mathbf{v^H v}} \leq \frac{\mathbf{v^H T^H T v}}{\mathbf{v^H v}} \quad \forall\, \mathbf{v} \in \mathbb{C}^N$$

*Let $\mathbf{v} = \left[\mathbf{v_1^H}\, \mathbf{0}_{N_2}^H\right]^H$ where $\mathbf{v_1} \in \mathbb{C}^{N_1}$ and $\mathbf{0}_{N_2} \in \mathbb{C}^{N_2}$, then*

$$\lambda_{min}^{\mathbf{T}} \leq \frac{\mathbf{v_1^H T_1^H T_1 v_1}}{\mathbf{v_1^H v_1}} \quad \forall\, \mathbf{v_1} \in \mathbb{C}^{N_1}$$

*In particular:* $\quad \lambda_{min}^{\mathbf{T}} \leq \arg\min_{\mathbf{v_1}\in\mathbb{C}^{N_1}} \frac{\mathbf{v_1^H T_1^H T_1 v_1}}{\mathbf{v_1^H v_1}} = \lambda_{min}^{\mathbf{T_1}}$

*The same line of arguments leads to:* $\quad \lambda_{min}^{\mathbf{T}} \leq \lambda_{min}^{\mathbf{T_2}}$ ∎

This method is easily extended to more than two partitions. In this case, the minimum eigenvalues corresponding to the associated sub-matrices increase, as well as the amount of a priori information. Of course, the number of columns must remain larger than the channel order $M$ for the sub-systems to be solvable. So, the optimal number of columns is $M+1$.

Thus, the partitioning procedure consists on the (vertical) division of the global filtering matrix $\mathcal{T}_{N,\,M+N}(\mathbf{h})$. Apart from the known symbols, the rest of the matrix who is related to the unknown symbols is partitioned in $P$ small sub-matrices of $M+1$ columns. This procedure not only allows to take profit from the $\lambda_{min}$ condition, but also permits to realize $P$ small optimizations instead of a large one, leading to computational savings.

### 3.3. CML Block Algorithm w/Partitions (CMLBAP)

The resulting algorithm of the above mentioned procedure is now based on the following criterion:

$$\begin{aligned}
&\mathcal{J}_{\text{BP}}^{\text{CML}}(\mathbf{h}, \mathbf{s}_{N_p}) = \\
&= \left\|\mathcal{T}_{N,\,M+N}(\mathbf{h})\mathbf{s}_{M+N} - \mathbf{X}_N\right\|^2 - \gamma\left\|\mathbf{s}_{N_p}\right\|^2 \\
&= \left\|\mathcal{T}_{N,\,N_u}(\mathbf{h})\mathbf{s}_{N_u} - \left(\mathbf{X}_N - \mathbf{I}_N(\mathbf{s}_{Nk})\right)\right\|^2 - \gamma\left\|\mathbf{s}_{N_p}\right\|^2 \\
&= \left\|\left[\mathcal{T}_{N,\,N_1}(\mathbf{h}) \mid \ldots \mid \mathcal{T}_{N,\,N_P}(\mathbf{h})\right]\begin{bmatrix}\mathbf{s}_{N_1}\\ \vdots \\ \mathbf{s}_{N_P}\end{bmatrix}\right. \\
&\qquad \left. -\left(\mathbf{X}_N - \mathbf{I}_N(\mathbf{s}_{Nk})\right)\right\|^2 - \gamma\left\|\mathbf{s}_{N_p}\right\|^2 \\
&= \left\|\mathcal{T}_{N,\,N_p}(\mathbf{h})\mathbf{s}_{N_p} - \left(\mathbf{X}_N - \mathbf{I}_N(\mathbf{s}_{Nk}) - \mathbf{I^c}_N(\mathbf{s}_{N_p})\right)\right\|^2 \\
&\qquad -\gamma\left\|\mathbf{s}_{N_p}\right\|^2,
\end{aligned} \tag{18}$$

$$\text{with:} \quad \gamma = \lambda_{min}\{\mathcal{T}_{N,\,N_p}(\mathbf{h})^H \mathcal{T}_{N,\,N_p}(\mathbf{h})\} \tag{19}$$

$$\begin{aligned}
\mathbf{I^c}_N(\mathbf{s}_{N_p}) &= \mathcal{T}_{N,\,N_1}(\mathbf{h})\mathbf{s}_{N_1} + \ldots + \mathcal{T}_{N,\,N_{p-1}}(\mathbf{h})\mathbf{s}_{N_{p-1}} \\
&\quad + \mathcal{T}_{N,\,N_{p+1}}(\mathbf{h})\mathbf{s}_{N_{p+1}} + \ldots + \mathcal{T}_{N,\,N_P}(\mathbf{h})\mathbf{s}_{N_P}
\end{aligned} \tag{20}$$

Index $p$ denotes the partition we deal with. The $\mathcal{T}_{N_u,\,N_p}(\mathbf{h})$ matrix corresponds to the associated channel matrix of the $p^{th}$ partition. $P$ is calculated from the division of the total number of symbols to be estimated ($N_u$) by the size of these small matrices ($N_p = M+1$). If the calculated $P$ is not a rational number, the size of the last partition ($N_P$) is modified for convenience to fit an integer number of partitions into the $Nu$ symbols. Finally, $\mathbf{I^c}_N(\mathbf{s}_{N_p}^{(p)})$ is introduced to simplify notation, and is defined as the *complementary* interference for the partition $p$, that is, the subtracted interference to the partition we deal with, considering the symbols of the rest of the partitions as known.

The algorithm is similar to the CMLBA one, but the minimization of s variable is now realized by $P$ minimizations corresponding to the $P$ small sub-matrices $\mathcal{T}_{N_u,\,N_p}(\mathbf{h})$. Iteration $k$ of the CMLBAP is then:

```
for p = 1:P
```

$$\begin{aligned}
\widehat{\mathbf{s}}_{N_p}^{(k)} &= \arg\min_{\mathbf{s}_{N_p}\in C^{N_p}} \mathcal{J}_{\text{BP}}^{\text{CML}}\left(\widehat{\mathbf{h}}^{(k-1)}, \mathbf{s}_{N_p}\right) \\
&= \arg\min_{\mathbf{s}_{N_p}\in C^{N_p}} \left\|\mathcal{T}_{N,\,N_p}(\widehat{\mathbf{h}}^{(k-1)})\mathbf{s}_{N_p}\right. \\
&\quad \left. -\left(\mathbf{X}_N - \widehat{\mathbf{I}}_N(\mathbf{s}_{Nk}) - \widehat{\mathbf{I}^c}_N(\mathbf{s}_{N_p})\right)\right\|^2 - \widehat{\gamma}\left\|\mathbf{s}_{N_p}\right\|^2
\end{aligned} \tag{21}$$

$$\text{with:} \quad \widehat{\gamma} = \lambda_{min}\{\mathcal{T}_{N,\,N_p}(\widehat{\mathbf{h}})^H \mathcal{T}_{N,\,N_p}(\widehat{\mathbf{h}})\} \tag{22}$$

$$\begin{aligned}
\widehat{\mathbf{I}^c}_N(\mathbf{s}_{N_p}) &= \mathcal{T}_{N,\,N_1}(\widehat{\mathbf{h}}^{(k-1)})\widehat{\mathbf{s}}_{N_1}^{(k)} + \ldots \\
&\quad + \mathcal{T}_{N,\,N_{p-1}}(\widehat{\mathbf{h}}^{(k-1)})\widehat{\mathbf{s}}_{N_{p-1}}^{(k)} + \mathcal{T}_{N,\,N_{p+1}}(\widehat{\mathbf{h}}^{(k-1)})\widehat{\mathbf{s}}_{N_{p+1}}^{(k-1)} \\
&\quad + \ldots + \mathcal{T}_{N,\,N_P}(\widehat{\mathbf{h}}^{(k-1)})\widehat{\mathbf{s}}_{N_P}^{(k-1)}
\end{aligned} \tag{23}$$

$$\widehat{\mathbf{s}}_{N_u}^{(k)} = \left[\left[\widehat{\mathbf{s}}_{N_1}^{(k)}\right]^T \ \ldots \ \left[\widehat{\mathbf{s}}_{N_P}^{(k)}\right]^T\right]^T \tag{24}$$

```
end
```

The minimization w.r.t. the channel remains unchanged (17).

## 4. SIMULATION RESULTS

### 4.1. Simulation conditions

A simplified version of the GSM system is here presented. The whole GSM burst is divided in two, each one composed of $N_k + N_u$ symbols. The known symbols of the Training Sequence (TS) located at the beginning of the semi-burst are used for initializing all the algorithms. Indeed, a channel estimate ($\mathbf{h}_{ini}$) is derived from a straightforward Least Square Estimation (LSE) of this known sequence. Refer to [9] for details.

Simulation conditions are based in model presented in (1). Details are related at following. Two random binary data (BPSK) semi-bursts are generated, each one with $N_u = 61$ symbols. A pseudo-random sequence of $N_k = 26$ known symbols corresponds to the TS. After a rotation operation, the whole burst is then transmitted over a random generated channel (with $L = 2$) of length $M+1 = 5$. Once a derotation operation is performed by the receiver, equalization algorithms are applied. Soft decision symbols are then sliced and compared with the transmitted ones. No coding techniques are applied. The raw bit (symbol) error rate (Raw BER) is shown in order to evaluate the performance (see Fig.1).

The simulated channel impulse responses $\{h_n, \forall n\}$, as defined in (2), are considered as normalized gaussian random variables.

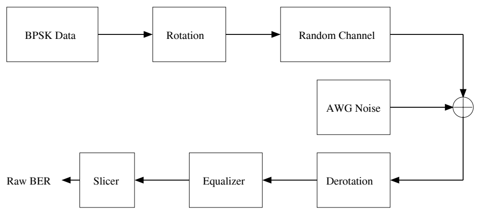


**Fig. 1**. The simplified GSM model

### 4.2. Performance comparison

First, a performance comparison is given when time-invariant channels ($v = 0\ km/h$) are used (Fig.2). In this situation, the Viterbi (block) algorithm, as is used in most GSM receivers, excels and is considered as a reference.

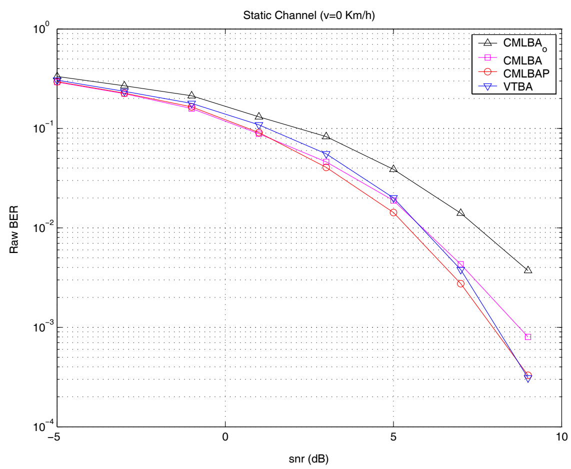


**Fig. 2**. Performance comparison (time-invariant channel)

As we can observe, the partitioning procedure applied in CMLBAP provides a substantial gain of performance w.r.t. the CMLBA, and despite of its reduced complexity. Performances of CMLBA when any known symbols are used (named CMLBA$_o$ in the figure) is also given to compare the gain obtained by the partitioning procedure from the gain from the known symbols usage; indeed, just the $\widehat{\mathbf{h}}_{ini}$ from the TS is used to initialize the algorithm. Furthermore, no degradation is observed when comparing the CMLBAP to the VTBA (Viterbi Block Algorithm) in the targeted zone (SNR -Signal to Noise Ratio- under 10 dB). Even more, at low SNR, our algorithms take some advantage w.r.t. the Viterbi because of the iterated processing that improves channel estimates, while VTBA only uses the estimate based on the TS. This behavior is encouraging taking into account that GSM norm is designed for this latest.

Next, a simulation with time-varying channel is shown (Fig.3). This multi-path fast fading channel follows a sinus profile and can be considered as a very high speed case, since the simulated conditions ($v = 300\ km/h$ and $f_c = 1800\ MHz$) are more than two times faster than the worst case considered by the GSM norm.

As we can see, both algorithms are limited by very fast fading of the channel, and over a signal to noise ratio larger than 25 dB no improvement is reached. However, for a raw BER of $8{\cdot}10^{-2}$, that can be considered as a typical target performance in GSM before coding, CMLBAP outperforms VTBA with a gain equal to 5 dB.

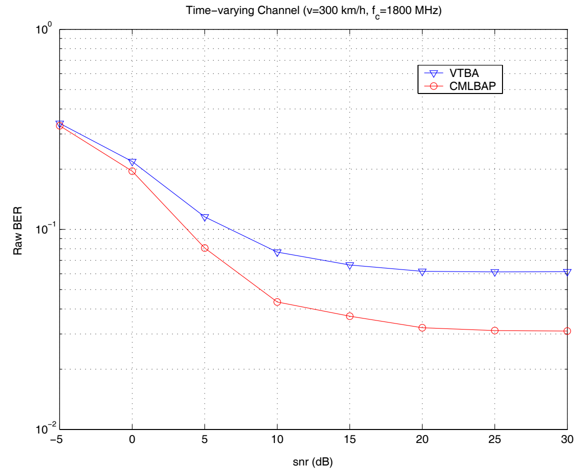


**Fig. 3**. Performance comparison (time-varying channel)

## 5. CONCLUSION

A new algorithm for semi-blind estimation have been presented in this paper. From the works on the CML approach proposed in [1], a new criterion have been developed in order to increase performances. Thanks to a partitioning procedure and a full usage of the all the available symbols, a new block algorithm have been derived. This new algorithm provides encouraging performance when comparing to VTBA used in GSM. Moreover, the computational cost of the algorithm has been drastically reduced and it's well suited for deriving adaptive versions. Simulations on real GSM environment will be considered in further work. Finally, these algorithms can be also applied to (E)GPRS systems using the GMSK modulation.

## 6. REFERENCES

[1] Florece Alberge, Pierre Duhamel, and Mila Nikolova, "Blind identification/equalization using deterministic maximum likelihood and a partial information on the input," *SPAWC'99*.

[2] Pierre A. Laurent, "Exact and Approximative Construction of Digital Phase Modulations by Superposition of Amplitude Modulated Pulses (AMP)," *IEEE Trans. on Comm.*, vol. 34, no. 2, pp. 150–162, February 1986.

[3] 3GPP Technical Specification 05.04 R'99, "Modulation," .

[4] M. Kristensson, B. Ottersten, and D. Slock, "Blind subspace identification of a BPSK communication channel," *Asilomar'96*, vol. 2, pp. 828–832, November 1996.

[5] Zhi Ding and Ge Li, "Single-channel blind equalization for GSM cellular systems," *Selec. Areas in Comm., IEEE Journal on*, vol. 16, no. 8, pp. 1493–1505, October 1998.

[6] E. de Carvalho and D.T.M. Slock, "Maximum-likelihood blind FIR multi-channel estimation with Gaussian prior for the symbols," *ICASSP'97*, vol. 5, pp. 3593–3596, April 1997.

[7] D. Gesbert, Pierre Duhamel, and S. Mayrargue, "Blind least-squares approaches for joint data/channel estimation," *DSPW'96*, pp. 450–453, September 1996.

[8] Philippe G. Ciarlet, *Introduction to Numerical Linear Algebra and Optimisation*, Cambridge University Press, Nov. 1989.

[9] Michel Mouly and Marie-Bernadette Pautet, *The GSM system for mobile communications*, Europe Media Duplication, 1992.