A ROBUST ALGORITHM FOR BLIND SPACE-TIME EQUALIZATION

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ABSTRACT

The efficient separation of signals is a frequent problem in multiuser communication systems. Among many algorithms to blind deconvolution of a multiple-input multiple-output (MIMO) systems, the one that utilizes higher-order cumulants has advantages in regards of convergence rate. Inspired on this algorithm and on a stochastic gradient approach, we propose an algorithm with capacity of recovering simultaneously all sources, denoted as MU-SWA (Multiuser Shalvi-Weinstein Algorithm). Based on the steady-state analysis, recently presented by Luo and Chambers for the Multiuser Constant Modulus Algorithm, we derive the expression for the mean-square error of MU-SWA. Simulation results show that MU-SWA presents a more robust behavior with respect to convergence rate and tracking capability when compared to others known algorithms for blind multiuser equalization.

1. INTRODUCTION

Nowadays, the ever growing demand for mobile communications is constantly increasing the need for better convergence and improved capacity. A typical problem that frequently arises in multiuser communication systems is the separation of linear mixtures of signals. In this context, several different approaches have been considered. Recently blind adaptation algorithms for channel equalization using time-space diversity with multiuser signal separation capacity have been proposed [1, 2]. The Multiuser Constant Modulus Algorithm (MU-CMA) is based on a stochastic gradient approach [1]. The Quasi-Newton Cross-Correlation Constant Modulus Algorithm (QN-CCCMA) [2] is based on Quasi-Newton methods. Compared to MU-CMA, it has faster convergence rate, but higher complexity. Moreover, it suffers from problems of numerical instability.

A higher-order cumulant-based algorithm for blind deconvolution of a MIMO system, capable of extracting the input signals, was proposed in [3]. In this paper we present an extension of this algorithm considering the fourth order cumulant and a stochastic gradient approach. The proposed algorithm, called Multiuser Shalvi-Weinstein Algorithm (MU-SWA), is able to simultaneously recover the input signals. It is compared to MU-CMA and QN-CCCMA. Based on the MU-CMA steady-state analysis [4], on the energy conservation relation [5], and on the equivalence between Godard and Shalvi-Weinstein schemes [6], we present a mean-square error (MSE) steady-state analysis for MU-SWA. Simulation results show good tracking capability of the algorithm when compared to others.

In the sequel data model is presented and the fourth order cumulant-based method [7] is revisited. Then we introduce MU-SWA and the MSE steady-state analysis. Next we present simulation results. We close the paper making some concluding remarks.

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2. ISSUES ON BLIND EQUALIZATION

A MIMO system with N sources and with an antenna array which has $L > N$ sensors has been considered. The source sequences $a_i(n)$, $i = 1, \ldots, N$ are assumed zero-mean, non gaussian, i.i.d., and independent on each other. The transmitted signals suffer inter-symbol and co-channel interferences. The channel from the i^{th} source to the j^{th} sensor is modelled by an FIR filter with K_c coefficients. The output of the L sensors are processed with N parallel space-time FIR equalizers, each one with K_t time diversity and $M = L\,K_t$ taps. The blind equalizer must mitigate the channel effects without accessing the data training. In a noise free environment, i^{th} equalizer's output can be written as $y_i(n) = \mathbf{w}_i^T(n-1)\mathbf{u}(n)$, where $\mathbf{u}(n)$ and $\mathbf{w}_i(n-1)$ are the input and the weight equalizer vectors, respectively.

Now suppose that we wish to extract only the i^{th} source. The update equation of the equalizer coefficients can be done with the Constant Modulus Algorithm (CMA) [8]

$$\mathbf{w}_i(n) = \mathbf{w}_i(n-1) - \mu e_i(n)\mathbf{u}(n) \tag{1}$$

in which $e_i(n) = (|y_i(n)|^2 - R_2^a)y_i(n)$ and μ is the step size. Assuming the sources with the same statistics, the expected modulus is equal for all of them and is defined as $R_2^a = \mathrm{E}\{|a(n)|^4\}/\mathrm{E}\{|a(n)|^2\}$. This algorithm is derived as a stochastic gradient method for minimizing the Godard cost function

$$J_{G_i} = \mathrm{E}\left\{(|y_i(n)|^2 - R_2^a)^2\right\}. \qquad (2)$$

Another blind equalization criterion is the Super-Exponential $J_{SE4i} = cum_4[y_i(n)]/cum_2[y_i(n)]^2$ where $cum_j[\cdot]$ denotes the cumulant of order j of the argument [7, 6]. The equalizer taps are adjusted to find the most negative value of J_{SE4i}, which is equivalent to minimize the cost function (2). An extremum of J_{SE4i} can be approached by using a gradient search procedure [6]. In the particular step size choice, the gradient algorithm in the equalizer coefficient domain can be written as [7, 3, 6]

$$\mathbf{w}_i(n) = \gamma^{-1}\mathbf{R}^{-1}\mathbf{d}_i(n) \qquad (3)$$

being $\mathbf{R} = \mathrm{E}\{\mathbf{u}^*(n)\mathbf{u}^T(n)\}$ the autocorrelation matrix, $\mathbf{d}_i = \mathrm{E}\{|y_i|^2 y_i \mathbf{u}^*\} - \beta\mathrm{E}\{|y_i|^2\}\mathrm{E}\{y_i\mathbf{u}^*\}$ the higher-order cumulants, $\beta = 2$ $(= 3)$ in the complex (real) case, and γ a constant that controls the radial factor of the equalizer [6]. Let $\gamma = cum_4[a(n)]/cum_2[a(n)] = R_2^a - \beta cum_2[a(n)]$, a realizable form of (3) can be done with the estimates

$$\mathbf{d}_i(n) = \lambda\mathbf{d}_i(n-1) + \Delta_i(n), \quad \text{and}$$

$$\mathbf{R}^{-1}(n) = \lambda^{-1}\mathbf{R}^{-1}(n-1) + \lambda^{-1}\mathbf{v}(n)\mathbf{v}^H(n), \qquad (4)$$

where $\Delta_i(n) = \left[|y_i(n)|^2 - \beta\mathrm{E}\{|y_i(n)|^2\}\right] y_i(n)\mathbf{u}^*(n)$, $\mathbf{v}(n) = \mathbf{R}^{-1}(n-1)\mathbf{u}(n)\left(\lambda + \|\mathbf{u}\|^2_{\mathbf{R}^{-1}(n-1)}\right)^{-1/2}$, $\|\mathbf{x}\|^2_{\mathbf{A}} = \mathbf{x}^H\mathbf{A}\mathbf{x}$, and $0 \ll \lambda < 1$ is the forgetting factor. Replacing these estimates in (3), assuming $\mathrm{E}\{y_i^2\} = \mathrm{E}\{a_i^2\}$, after some algebraic manipulations, we obtain the Shalvi-Weinstein Algorithm (SWA) [7]

$$\mathbf{w}_i(n) = \mathbf{w}_i(n-1) + \gamma^{-1}e_i(n)\mathbf{R}^{-1}(n)\mathbf{u}^*(n). \qquad (5)$$

Considering $-\mathbf{R}^{-1}(n)\gamma^{-1} = \mu$ as an adaptation step, it can be interpreted as a stochastic gradient algorithm like (1). Considering $\gamma\mathbf{R}(n)$ as a hessian matrix approximation of (2), it can be interpreted as a Quasi-Newton algorithm.

3. SPACE-TIME EQUALIZATION ALGORITHMS

In the case of joint blind simultaneous recovery of all input signals, the Godard cost function is given by [1]

$$J_G = \sum_{i=1}^{N}\left[J_{G\,i} + \frac{\xi}{2}\sum_{j=1,j\neq i}^{N}\sum_{\delta=-\delta_1}^{\delta_1}|r_{ij}(\delta)|^2\right] \qquad (6)$$

in which $J_{G\,i}$ is the cost function (2) for the i^{th} user, $r_{ij}(\delta) = \mathrm{E}\left\{y_i(n)y_j^*(n-\delta)\right\}$ and $\delta_1 = K_t + K_c - 1$. The second term of the right side of (6) is introduced to penalize, with weight ξ, the cross-correlations between different users [1]. The gradient vector of this cost function related to the i^{th} user is given by

$$\nabla_{\mathbf{w}_i}J_G = \mathrm{E}\{e_i(n)\mathbf{u}^*(n)\} + $$
$$+\frac{\xi}{2}\sum_{j=1,i\neq j}^{N}\sum_{\delta=-\delta_1}^{\delta_1}\mathrm{E}\{y_j(n)\mathbf{u}^*(n)\}\,r_{ij}(\delta). \qquad (7)$$

A stochastic gradient algorithm can be obtained by using convenient estimates of the gradient vector. It is usual to estimate the cross-correlation $r_{ij}(\delta)$ with an exponential window, considering a forgetting factor λ, and the other expectations with instantaneous estimates [1]. With these estimates, the stochastic gradient algorithms can be characterized by the following equations

$$\mathbf{w}_i(n) = \mathbf{w}_i(n-1) - \mu(n)\breve{e}_i(n)\mathbf{u}^*(n), \qquad (8)$$

in which $\breve{e}_i(n) = e_i(n) + \rho_i(n)$ and

$$\rho_i(n) = \frac{\xi}{2}\sum_{j=1,i\neq j}^{N}\sum_{\delta=-\delta_1}^{\delta_1}y_j(n)r_{ij}(\delta). \qquad (9)$$

In MU-CMA [1], the adaptation step size is a constant scalar $\mu(n) = \mu$. In QN-CCCMA [2], the approximation of hessian matrix of the cost function (6) can be interpreted as the step size. Inspired on (5), another possible candidate for the step size is $\mu(n) = -\gamma^{-1}\mathbf{R}^{-1}(n)$ with the inverse autocorrelation matrix $\mathbf{R}^{-1}(n)$ updated as (4). In this case the algorithm denoted MU-SWA is written as

$$\mathbf{w}_i(n) = \mathbf{w}_i(n-1) + \bar{e}_i(n)\mathbf{R}^{-1}(n)\mathbf{u}^*(n) \qquad (10)$$

with $\bar{e}_i(n) = \breve{e}_i(n)/\gamma$. It can be interpreted as an extension of SWA (5) for the multiuser environment with capacity to simultaneously recover all source sequences.

4. THE MU-SWA STEADY-STATE ANALYSIS

In this section we extend the steady-state analysis of MU-CMA [4] to MU-SWA. Proofs and justifications of the approximations used here have been omitted and can be found in the references [4, 5]. In this analysis, we assume a noise free environment and that the i^{th} equalizer converges asymptotically to the i^{th} source with delay τ_d^i. These assumptions are usual in steady-state analysis [4, 5].

Let $\mathbf{w}_i^{\mathrm{o}}$ be the zero-forcing solution for the i^{th} equalizer and $\widetilde{\mathbf{w}}_i(n-1) = \mathbf{w}_i^{\mathrm{o}} - \mathbf{w}_i(n-1)$. The *a priori* error for the i^{th} equalizer is given by $e_i^a(n) = \mathbf{u}^T(n)\widetilde{\mathbf{w}}_i(n-1)$. One measure of filter performance is the steady-state mean-square error $\mathrm{MSE}_i = \lim_{n\to\infty}\mathrm{E}\left\{|e_i^a(n)|^2\right\}$ which is the quantity we wish to determine for MU-SWA.

Considering the adaptation equation of MU-SWA for the i^{th} equalizer (10) and subtracting both sides of this equation from $\mathbf{w}_i^{\mathrm{o}}$, we obtain the error equation

$$\widetilde{\mathbf{w}}_i(n) = \widetilde{\mathbf{w}}_i(n-1) - \bar{e}_i(n)\mathbf{R}^{-1}(n)\mathbf{u}^*(n). \qquad (11)$$

All adaptive schemes of the form (11) obey the energy conservation relation and can be described by a lossless mapping and a feedback loop [5]. By equating the squared weighted norms on both sides of (11), using $\mathbf{R}(n)$ as a weighting matrix, we obtain

$$\|\widetilde{\mathbf{w}}(n)\|^2_{\mathbf{R}(n)} + (\bar{e}_i(n)e_i^a(n)^* + \bar{e}_i(n)^*e_i^a(n)) =$$
$$= \|\widetilde{\mathbf{w}}(n-1)\|^2_{\mathbf{R}(n)} + \kappa(n)|\bar{e}_i(n)|^2 \qquad (12)$$

where $\kappa(n) = \|\mathbf{u}(n)\|^2_{\mathbf{R}^{-1}(n)}$. By taking expectations of both sides of (12) and using the same assumptions of [5], i.e., $\mathrm{E}\left\{\|\widetilde{\mathbf{w}}(n)\|^2_{\mathbf{R}(n)}\right\} \approx \mathrm{E}\left\{\|\widetilde{\mathbf{w}}(n-1)\|^2_{\mathbf{R}(n)}\right\}$ and the independence between $\kappa(n)$ and $|\bar{e}_i(n)|^2$, we get

$$\mathrm{E}\left\{\kappa(n)\right\} \mathrm{E}\left\{|\bar{e}_i(n)|^2\right\} = \mathrm{E}\left\{\bar{e}_i(n) e_i^a(n)^* + \bar{e}_i(n)^* e_i^a(n)\right\}. \quad (13)$$

Considering the approximations (eq. (47) and (50) of [4]):

$$\gamma^2 \mathrm{E}\left\{|\bar{e}_i(n)|^2\right\} \approx aux(n) + K\ \mathrm{E}\{|a(n)|^2\}^3 \quad \text{and} \quad (14)$$

$$E\left\{\bar{e}_i(n) e_i^a(n)^* + \bar{e}_i(n)^* e_i^a(n)\right\} \approx 2\mathrm{MSE}_i, \quad (15)$$

with $aux(n) = \mathrm{E}\{|a(n)|^6 - (R_2^a)^2 |a(n)|^2\}$ and

$$K = \frac{\xi^2}{4}\left[2\left(K_t + K_c\right) - 1\right]\frac{1-\lambda}{1+\lambda}(N-1),$$

Eq. (13) can be rewritten as

$$\mathrm{MSE}_i \approx \frac{\mathrm{E}\left\{\kappa(n)\right\}}{2\gamma^2}\left\{aux(n) + KE\{|a(n)|^2\}^3\right\}. \quad (16)$$

This result is related to MSE_i obtained for MU-CMA in [4] by the following relation

$$\frac{\mathrm{MSE}_i^{\mathrm{MU-SWA}}}{\mathrm{MSE}_i^{\mathrm{MU-CMA}}} \approx -\frac{\mathrm{E}\left\{\|\mathbf{u}(n)\|^2_{\mathbf{R}^{-1}(n)}\right\}}{\gamma\mu \mathrm{E}\left\{\|\mathbf{u}(n)\|^2\right\}}. \quad (17)$$

Notice that the differences between these MSE_i's are in the adaptation step, which is a matrix for MU-SWA, and in the factor γ. It is possible to adjust the parameters of these algorithms to reach the same MSE in steady-state

$$\mu \approx -\mathrm{E}\left\{\|\mathbf{u}(n)\|^2_{\mathbf{R}^{-1}(n)}\right\}\left(\gamma \mathrm{E}\{\|\mathbf{u}(n)\|^2\}\right)^{-1}. \quad (18)$$

Now assuming $\mathrm{E}\left\{\|\mathbf{u}(n)\|^2_{\mathbf{R}^{-1}(n)}\right\} \approx M(1-\lambda)$ and replacing $\mathrm{E}\{\|\mathbf{u}(n)\|^2\} = M\sigma_u^2$, Eq. (18) simplifies to

$$\mu \approx -(1-\lambda)/(\gamma\sigma_u^2), \quad (19)$$

where σ_u^2 is the variance of the input signal [5].

In the single-input single-output (SISO) case, the steady-state MSE of CMA and SWA can be obtained by making $\xi = 0$ in the expressions of the MIMO case.

5. SIMULATION RESULTS

In this section we verify the validity of the MSE steady-state analysis for MU-SWA and also compare the performance of this algorithm to MU-CMA [1] and QN-CCCMA [2]. We consider a MIMO system with $N = 2$ users, $L = 3$ sensors, and channel models $H_{ij}(z) = h_0 + h_1 z^{-1} + h_2 z^{-2}$, $i = 1, \ldots, N$, $j = 1, \ldots, L$, shown in Table 1. For the time-varying sub-channel, H_{22} of Channel 1, coefficients $h_0(n)$, $h_1(n)$, and $h_2(n)$ are generated by passing a Gaussian white noise through a second order Butterworth filter designed to simulate a fade rate of 0.1 Hz [9].

In order to examine the steady-state MSE of MU-SWA, we consider 2 sub-equalizers with $M = 6$ taps and the Channel 2. We assume a zero mean circularly V29 non-constant modulus source [7] with statistics $\mathrm{E}\{|a|^2\} = 1.150$, $\mathrm{E}\{|a|^4\} = 1.650$, $\mathrm{E}\{|a|^6\} = 2.626$. By fixing $\xi = 4$ and initializing the sub-equalizers taps with only two non-null elements in the second and fourth positions respectively, the steady-state MSE is measured with varying forgetting factor λ. Table 2 shows that simulation results are in excellent agreement with the analysis. For modulus constant sources, most of steady-state MSE is due to cross-correlation term. In this case, there is a relatively large gap between analysis and simulation, as observed in [4].

To compare the tracking capability of MU-SWA to MU-CMA [1] and QN-CCCMA [2], we consider 2 sub-equalizers with $M = 15$ taps and the Channel 1. The sub-equalizers taps are initialized with only two non-null elements at fifth and seventh positions, respectively. Fig. 1 shows the equalizer-2's output error for MU-CMA, QN-CCCMA and MU-SWA. The absolute values of the roots of $h_0(n)x^2 + h_1(n)x + h_2(n)$ are shown in Fig.1-d so that bursts of errors can be associated with rapid changes of these roots. Particularly, the bursts near iterations 5000, 27000 and 37000 are due to strong spectral nulls (absolute value equal one is indicated by a straight line). MU-SWA shows the faster convergence and is not affected by all rapid changes of the channel compared to MU-CMA and QN-CCCMA. The last one presents the worst tracking capability and suffers with instability. The corresponding residual interference (RI) curves are presented in Fig. 2. In this simulation, the adaptation step size of MU-CMA was adjusted to reach the same steady-state MSE of MU-SWA using (19), and the parameter α of QN-CCCMA was chosen to ensure faster convergence of this algorithm without introducing instability. Although this channel was not very realistic, it is interesting to compare the behavior of the algorithms in critical situations: spectral nulls and rapid changes. About the stability, the hessian matrix of QN-CCCMA can lose its positive definite nature which causes divergence [2]. On the other hand, MU-SWA is more robust although in order to ensure its stability and good tracking capability, the forgetting factor must be properly chosen.

Table 3 shows the computational complexity of the algorithms. MU-SWA requires less operations at each iteration than QN-CCCMA. It is relevant to notice that the first does not require square root operations and has less divisions than the other one. Comparing it to MU-CMA, both algorithms have a higher computational complexity, which is the price for faster convergence.

6. CONCLUSIONS

We have proposed an extension of the Shalvi-Weinstein algorithm for multiuser environments with capacity of simultaneous recovery of all sources. Based on the MU-CMA steady-state analysis we derived the expression for MSE in steady-state which has good agreement with experimental results for non constant modulus sources. By means of simulations we showed that MU-SWA has a better tracking capability than MU-CMA and QN-CCCMA. Moreover, it presents less operations and is more stable than the latter.

7. REFERENCES

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	H_{ij} of Channel 1			H_{ij} of Channel 2	
ij	h_0	h_1	h_2	h_0	h_1
11	-0.50	$+0.48$	-0.03	-0.6	$+1.2$
21	$+0.15$	-0.03	$+0.10$	$+0.1$	-0.2
12	-0.26	-0.44	$+0.19$	$+0.5$	-1.0
22	$h_0(n)$	$h_1(n)$	$h_2(n)$	-0.6	$+0.9$
13	$+1.00$	-1.00	$+0.41$	$+0.4$	-0.2
23	$+1.00$	$+1.60$	$+0.68$	-0.1	$+0.4$

Table 1. Communication channel models.

λ	MSE analysis	MSE simulation
0.99000	1.16×10^{-2}	1.09×10^{-2}
0.99500	5.50×10^{-3}	6.10×10^{-3}
0.99900	1.00×10^{-3}	1.10×10^{-3}
0.99950	5.21×10^{-4}	6.03×10^{-4}
0.99975	2.60×10^{-4}	2.55×10^{-4}

Table 2. MSE of V29 at equalizer-2 output.

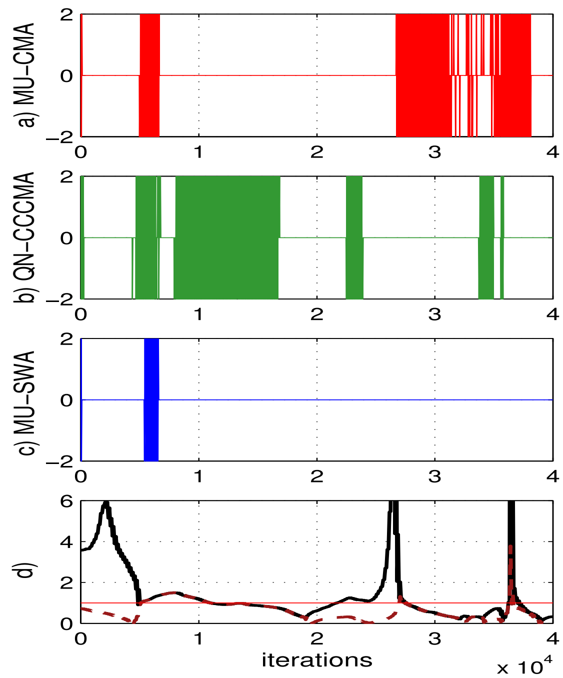

Fig. 1. Equalizer-2 output erros of a) MU-CMA ($\mu = 0.001$, $\xi = 3$), b) QN-CCCMA ($\alpha = 0.01$, $\epsilon = 0.5$, $\xi = 3$), c) MU-SWA ($\lambda = 0.995$, $\xi = 3$) d) Absolute roots value of the polynomial $h_0(n)x^2 + h_1(n)x + h_2(n)$.

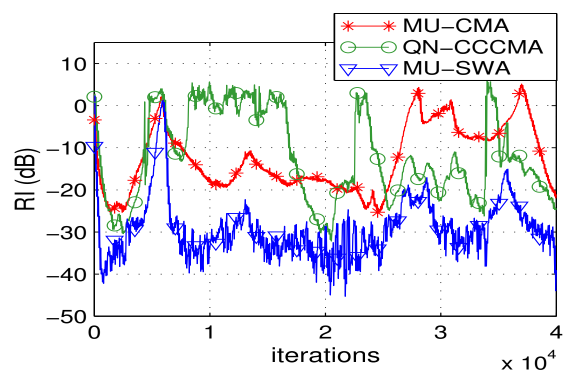

Fig. 2. RI curves for equalizer-2 considering MU-CMA ($\mu = 0.001$, $\xi = 3$), QN-CCCMA ($\alpha = 0.01$, $\epsilon = 0.5$, $\xi = 3$), and MU-SWA ($\lambda = 0.995$, $\xi = 3$). For 2PAM, N=2, L=3, M=15, SNR=30 dB, and Channel 1.

Op.	QN-CCCMA	MU-SWA	MU-CMA
$\times$	$6M^2+M(N+D+1)+2D+3N+2$	$4M^2+M(2N+1)+N(3D+6)$	$M(N+1)+N(3D+6)$
$\div$	4	1	–
SQRT	M	–	–

Table 3. Computational complexity of the algorithms for real signals, $D=(N-1)(2\delta_1+1)$.