EM ALGORITHM-BASED MULTIUSER SYNCHRONIZATION IN TURBO RECEIVERS

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ABSTRACT

The current paper addresses the issue of estimating the users propagation delays, received carrier phase offsets and received amplitudes in an asynchronous DS-CDMA environment with frequency non-selective propagation channels. The proposed synchronizer is based on the expectation- maximization (EM) algorithm and takes benefit from the soft information delivered by the receiver which is of turbo type. Performance of the proposed synchronizer is illustrated by simulation results. In particular, the mean and the mean squared error of the estimator as well as the bit error rate reached by the synchronized system are reported.

1. INTRODUCTION

Direct-sequence code division multiple access (DS-CDMA) is a major air interface for third-generation mobile communication systems. The optimal detector for multiuser systems was proposed by Verdu [1]. Since then, researchers have proposed a variety of much less complex receivers that provide near-optimal performance (e.g. references in [2]). Their performance relies on the availability of accurate estimates of parameters like the users channels cofficients, propagation delays and carrier phase offsets. Consequently many estimator schemes (references in [2]) resistant to multiple access interference (MAI) have been developed.

The goal of this paper is to propose an EM algorithm-based[3] estimator of the pre-cited parameters in the case of users frequency non-selective propagation channels. In this estimator the soft information provided by a multiuser turbo receiver will help the synchronizer to provide good estimation parameters. Actually, [4] proposed such an estimator for BPSK users data modulation and synchronous DS-CDMA. The interest of the present paper is to extend those developments to multilevel data modulation and asynchronous DS-CDMA. Moreover our paper generalizes to the multiuser case the synchronizer proposed in [5] where timing of one single user had to be estimated. We also point out that unlike many existing schemes our estimator assumes bandlimited chip waveforms using square root raised cosine pulse instead of rectangular waveforms.

The sequel of this paper will be organized as follows. In section 2, the system model will be presented. The proposed EM-based synchronizer will be developed in section 3. Finally, in section 4, the performance of the synchronizer will be illustrated by some simulation results.

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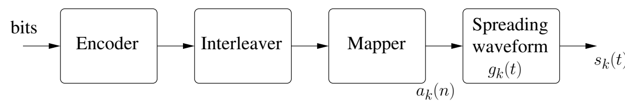

Fig. 1. Transmitter of user k

2. SYSTEM MODEL

Let us consider a K-user asynchronous DS-CDMA system. At the transmitter, the branch of each user (Fig. 1) consists of a bit-interleaved coded modulation (BICM) scheme, which is made up of a binary convolutional encoder and a constellation mapper separated by a bit interleaver. In the baseband formalism, the signal transmitted by user k may then be written as

$$s_k(t) = \sum_{n=1}^{N} a_k(n)\, g_k(t - nT), \tag{1}$$

where $a_k(n)$ is the nth ($n = 1, \ldots, N$) symbol of user k ($k = 1, \ldots, K$) belonging to constellation alphabet $\mathcal{A}$, T is the symbol period and $g_k(t)$ is the spreading waveform of user k. We consider here short spreading sequences, e.g. Gold sequences. We have then

$$g_k(t) = \frac{1}{M} \sum_{m=0}^{M-1} c_k(m)\, u(t - mT_c), \tag{2}$$

where T_c is the chip period, $M = T/T_c$ is the spreading gain, $c_k(m) \in \{+1, -1\}$ denotes the mth chip of the spreading sequence for user k and $u(t)$ is a unit energy square-root raised-cosine pulse with roll-off α.

Assuming that each user transmits through a frequency non-selective channel, the total baseband received signal is given by

$$r(t) = \sum_{k=1}^{K} \sum_{n=1}^{N} A_k\, a_k(n)\, g_k(t - nT - \tau_k)\, e^{j\theta_k} + z(t), \quad (3)$$

where A_k is the received signal amplitude of user k, τ_k is the kth user propagation delay, θ_k is the kth user carrier phase offset and $z(t)$ is the complex envelope of an additive white gaussian noise with passband power spectral density $N_0/2$. At the receiver, after anti-aliasing filtering, $r(t)$ is sampled with period $T_s = T_c/2$ leading to samples $r_s \triangleq r(sT_s)$ where $z_s \triangleq z(sT_s)$ is a white gaussian noise with variance $2N_0/T_s$. Those samples are then passed through a bank of K discrete-time filters, each of them matched to the spreading waveform of one particular user. The output at time v of the kth filter is given by

$$y_k(v) = \sum_s r_s\, g_k(sT_s - v). \quad (4)$$

Finally, we assume that statistics $\hat{A}_k^{-1}\, y_k(nT + \hat{\tau}_k)\, e^{-j\hat{\theta}_k}$ ($1 \le n \le N$), $\hat{A}_k$, $\hat{\tau}_k$ and $\hat{\theta}_k$ being respectively the estimated amplitude, propagation delay and phase offset of user k, are processed in a multiuser turbo demodulator. Such a device is described in [6] for the multiple antennas case but may be easily generalized to the multiuser case if replacing antennas by users. It will perform iterative joint demodulation and decoding through the exchange of extrinsic information between a soft-input soft-ouput (SISO) multiuser demodulator and K SISO decoders.

As we mentioned earlier, inaccurate estimates of A_k, θ_k and τ_k ($k = 1, \ldots, K$) will degrade the performance of the multiuser receiver. For this reason, the problem addressed in the sequel will be the estimation of those parameters. Our approach will be based on the EM algorithm.

3. EM ALGORITHM-BASED SYNCHRONIZER

3.1. Generalities

Let $\mathbf{r}$ denote a random vector and let $\mathbf{b}$ indicate a deterministic vector of parameters to be estimated from the observation of the received vector $\mathbf{r}$. Assume that $\mathbf{r}$ also depends on a random nuisance vector $\mathbf{a}$ independent of $\mathbf{b}$. In this case, it is shown in [7] that the well-known $\mathcal{Q}$-function of the EM algorithm reduces at iteration i to

$$\mathcal{Q}(\tilde{\mathbf{b}}, \hat{\mathbf{b}}^{(i-1)}) = \int_{\mathbf{a}} p(\mathbf{a}|\mathbf{r}, \hat{\mathbf{b}}^{(i-1)})\ \ln p(\mathbf{r}|\mathbf{a}, \tilde{\mathbf{b}})\, d\mathbf{a}. \quad (5)$$

The EM algorithm states that the sequence $\hat{\mathbf{b}}^{(i)}$ defined by

$$\hat{\mathbf{b}}^{(i)} = \arg\max_{\tilde{\mathbf{b}}} \{\mathcal{Q}(\tilde{\mathbf{b}}, \hat{\mathbf{b}}^{(i-1)})\}, \quad (6)$$

converges under fairly general conditions towards the maximum likelihood (ML) estimate of vector $\mathbf{b}$.

3.2. EM algorithm applied to multiuser synchronization

In this subsection we will apply the EM algorithm to estimate the users propagation delays, received amplitudes and received phase offsets in the case of a multiuser BICM tranmission. In this context, vector $\mathbf{r}$ contains all the samples r_s, vector $\mathbf{a}$ contains the KN users symbols and vector $\mathbf{b}$ consists of the $3K$ parameters to be estimated, i.e. $\mathbf{b} = (A_1, \ldots, A_K, \theta_1, \ldots, \theta_K, \tau_1, \ldots, \tau_K)^T$.

Let now $w_k \triangleq A_k\, e^{j\theta_k}$ denote the complex amplitude of user k. Using this definition, the expression of the received samples r_s, and neglecting the terms independent of $\tilde{\mathbf{b}}$, the log-likelihood function $\ln p(\mathbf{r}|\mathbf{a}, \tilde{\mathbf{b}})$ present in (5) can then be written as

$$\ln p(\mathbf{r}|\mathbf{a}, \tilde{\mathbf{b}}) = 2\,\mathrm{Re}\Big\{\sum_{k=1}^{K}\sum_{n=1}^{N} \tilde{w}_k^*\, a_k^*(n)\, y_k(nT + \tilde{\tau}_k)\Big\}$$
$$- \sum_{k=1}^{K}\sum_{k'=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} \tilde{w}_k^*\, \tilde{w}_{k'}\, a_k^*(n)\, a_{k'}(n')\, R_{k,k'}(n'-n), \quad (7)$$

where * denotes the complex conjugate and $R_{k,k'}(n)$ is the cross-correlation between users k and k' evaluated at time $nT + \tilde{\tau_{k'}} - \tilde{\tau_k}$ i.e.

$$R_{k,k'}(n) = \sum_{s=-\infty}^{+\infty} g_k(sT_s) g_{k'}(sT_s - [nT + \tilde{\tau_{k'}} - \tilde{\tau_k}]). \quad (8)$$

Similarly as in [5], let us define for each transmitted symbol $a_k(n)$

$$\eta_k^n(\mathbf{r}, \hat{\mathbf{b}}^{(i-1)}) \triangleq \sum_{a\,\in\mathcal{A}} a\, p(a_k(n) = a|\mathbf{r}, \hat{\mathbf{b}}^{(i-1)}). \quad (9)$$

$$\rho_{k,k'}^{n,n'}(\mathbf{r}, \hat{\mathbf{b}}^{(i-1)}) \triangleq \sum_{a_1\,\in\mathcal{A}}\sum_{a_2\,\in\mathcal{A}} a_1^*\, a_2$$
$$p(a_k(n) = a_1, a_{k'}(n') = a_2|\mathbf{r}, \hat{\mathbf{b}}^{(i-1)}). \quad (10)$$

Using (9)-(10) and replacing $\ln p(\mathbf{r}|\mathbf{a}, \tilde{\mathbf{b}})$ by (7) in (5),

$$\mathcal{Q}(\tilde{\mathbf{b}}, \hat{\mathbf{b}}^{(i-1)})$$
$$= 2\,\mathrm{Re}\Big\{\sum_{k=1}^{K}\sum_{n=1}^{N} \tilde{w}_k^*\, \eta_k^{n*}(\mathbf{r}, \hat{\mathbf{b}}^{(i-1)})\, y_k(nT + \tilde{\tau}_k)\Big\}$$
$$- \sum_{k=1}^{K}\sum_{k'=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} \tilde{w}_k^*\, \tilde{w}_{k'}\, \rho_{k,k'}^{n,n'}(\mathbf{r}, \hat{\mathbf{b}}^{(i-1)})\, R_{k,k'}(n'-n). \quad (11)$$

3.3. EM algorithm implementation in a turbo multiuser receiver

We see from (9) and (10) that the calculation of $\eta_k^{n*}(\mathbf{r}, \hat{\mathbf{b}}^{(i-1)})$ and $\rho_{k,k'}^{n,n'}(\mathbf{r}, \hat{\mathbf{b}}^{(i-1)})$ respectively requires the knowledge of the marginal posterior probabilities $p(a_k(n)|\mathbf{r}, \hat{\mathbf{b}}^{(n-1)})$ ($\forall k = 1, \ldots, K$ and $\forall n = 1, \ldots, N$) and the joint posterior probabilities $p(a_k(n), a_{k'}(n')|\mathbf{r}, \hat{\mathbf{b}}^{(n-1)})$. However, these probabilities are not directly available since our multiuser turbo demodulator only computes posterior probabilities on bits rather than on symbols. Hopefully the presence of the interleaver enables to consider that the bits transmitted in one symbol are independent. Therefore, we have

$$p(a_k(n)|\mathbf{r}, \hat{\mathbf{b}}^{(i-1)}) \simeq \prod_{q=1}^{Q} p(u_k^q|\mathbf{r}, \hat{\mathbf{b}}^{(i-1)}), \qquad (12)$$

where Q is the number of bits contained in symbol $a_k(n)$, u_k^q is the q^{th} bit of $a_k(n)$ and $p(u_k^q|\mathbf{r}, \hat{\mathbf{b}}^{(i-1)})$ is the bit a posteriori probabilities delivered by the SISO decoder at the latest turbo iteration. Moreover we will assume

$$\rho_{k,k'}^{n,n'}(\mathbf{r}, \hat{\mathbf{b}}^{(i-1)}) \simeq \eta_k^{n*}(\mathbf{r}, \hat{\mathbf{b}}^{(i-1)})\, \eta_{k'}^{n'}(\mathbf{r}, \hat{\mathbf{b}}^{(i-1)}) \quad \text{if } k \neq k' \text{ or } n \neq n' \qquad (13)$$

whereas

$$\rho_{k,k}^{n,n}(\mathbf{r}, \hat{\mathbf{b}}^{(i-1)}) = \sum_{a \in \mathcal{A}} |a|^2\, p(a_k(n) = a|\mathbf{r}, \hat{\mathbf{b}}^{(i-1)}) \qquad (14)$$

Like in [5], we perform a new EM-step at each turbo iteration and therefore merge the synchronization iterations (EM) into those of the turbo process.

Unlike the one-user case [7], the parameter joint maximization problem of (6) in the asynchronous multiuser case leads to a system of coupled equations.

If we have to estimate both the users complex amplitudes and the propagation delays, in order to overcome the coupling of the equations, we may maximize (11) for $\tilde{\tau} = (\tilde{\tau}_1, \ldots, \tilde{\tau}_k)^T$ assuming that $\tilde{\mathbf{w}} = (\tilde{w}_1, \ldots, \tilde{w}_k)^T$ is equal to $\hat{\mathbf{w}}^{(i-1)}$. Then we re-estimate the complex amplitudes with the new estimations of the propagation delays and so on. Actually it amounts to applying the ECM algorithm [3]. If we have only to estimate the users propagation delays, the known complex amplitudes must be introduced in (11) which has then to be maximized for vector $\tilde{\tau}$.

For the simulations in the next section, we will focus on the case where the users propagation delays are assumed to be known (i.e. $\forall k\ \hat{\tau}_k = \tau_k$) and only the users complex amplitudes have to be estimated[1]. Maximizing (11) amounts then to solving a linear system of K complex equations with

[1]Due to lack of space we will explain in a next paper how to practically maximize (11) when the propagation delays have also to be estimated.

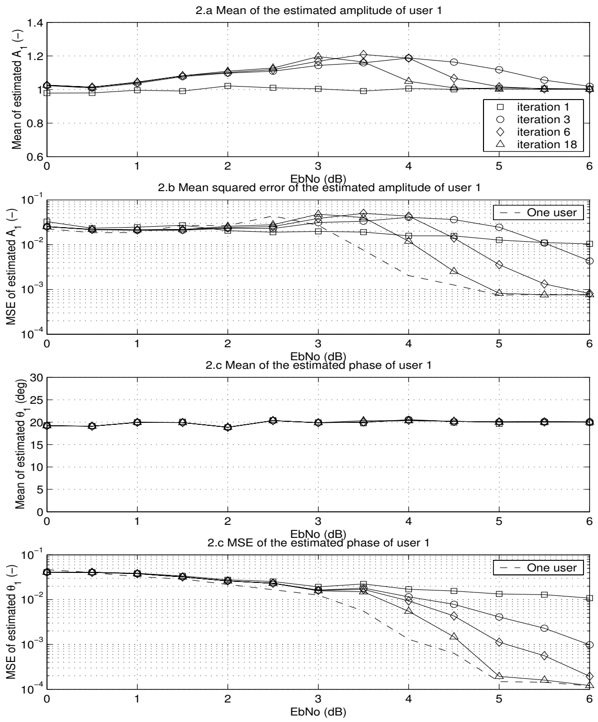

Fig. 2. Mean and mean-squared error of the amplitude and phase estimator versus E_b/N_0. User 1.

K complex unknowns, the lth ($1 \leq l \leq K$) equation being

$$\sum_{k=1, k\neq l}^{K} \Big\{ \sum_{n=1}^{N} \sum_{n'=1}^{N} \rho_{l,k}^{n,n'}(\mathbf{r}, \hat{\mathbf{b}}^{(i-1)})\, R_{l,k}(n'-n) \Big\}\, w_k + \sum_{n=1}^{N} \rho_{l,l}^{n,n}(\mathbf{r}, \hat{\mathbf{b}}^{(i-1)})\, w_l = \sum_{n=1}^{N} \eta_l^{n*}(\mathbf{r}, \hat{\mathbf{b}}^{(i-1)})\, y_l(nT + \tau_l). \qquad (15)$$

4. SIMULATION RESULTS

In this section the performance of the proposed synchronization method will be studied through simulation results. A three-user system with respective propagation delays equal to $0\,T_c$, $1.75\,T_c$ and $5.5\,T_c$ is considered. At the transmitter, we consider for each user a rate-$\frac{1}{2}$ non-systematic convolutional encoder with polynomial generators $(g_1, g_2) = (5, 7)_8$ and use 16-QAM modulation. A mapping proposed by ten Brink in [8] and referred to as medium unconditioned bit-wise mutual information mapping is used. The interleaver is totally random and a different permutation is used at each frame. The users spreading waveforms are built with three 31-chip Gold sequences and a square-root raised cosine with roll-off 0.2.

The simulations have been run for frames of 513 16-QAM symbols and 18 turbo iterations have been performed. For each new frame, at iteration 1, a joint ML estimate [4]

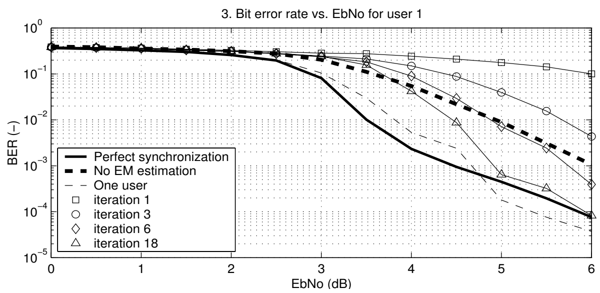

Fig. 3. BER versus E_b/N_0. User 1.

of the complex amplitudes is obtained using 6 training symbols. Then, at the next iterations, the synchronizer provides an estimate which combines (refer to the "combining method" of [4]) the updated estimate resulting from the maximization of the $\mathcal{Q}$-function with the training-based estimate.

The users amplitudes will be chosen all equal to 1.0 (same E_b/N_0) and their phase offsets all equal to 20 degrees. Fig. 2 represents, for iterations 1, 3, 6 and 18 and for E_b/N_0 ranging from 0 to 6dB, the synchronizer performance for user 1, which is with the chosen propagation delays the user with worst performance. More precisely, it illustrates the mean and the mean squared error (MSE) of the amplitude and phase estimates. The dashed curves in Fig. 2.b and 2.d represent the results at iteration 18 of one single user with unit amplitude and a 20-degree phase offset.

At iteration 18, we notice that for values of $E_b/N_0 \geq$ 5dB the amplitude and phase estimates are both unbiased and their MSEs reach their values in the single-user case. For $E_b/N_0 \leq$ 5dB, the gap between the 18th-iteration curve and the single-user curve represents the degradation of the estimator due to MAI. In order to prevent senseless values for the amplitude estimate at low E_b/N_0's, whenever the estimate was greater than 1.3, we did not take it into account and kept the value obtained at the previous iteration. It explains why in Fig. 2.a and 2.b quite flat curves are obtained for $E_b/N_0 \leq$ 2dB : there is no convergence of the estimate throughout the iterations for any $E_b/N_0 \leq$ 3.5dB and for $E_b/N_0 \leq$ 2dB the amplitude estimate is so bad that we most of the time retain the training-based estimate obtained at the first iteration.

Fig. 3 shows for iterations 1, 3, 6 and 18 the BER of user 1 for E_b/N_0 values ranging from 0 to 6dB. The thin dashed curve is again for the single-user case. The thick dashed curve shows the BER if we performed 18 turbo iterations while always keeping the initial ML estimate of complex amplitudes (EM estimation is not used). Consequently the gap between this curve and the 18th-iteration curve shows the interest of EM estimation of the complex amplitudes. We also notice that, for $E_b/N_0 \geq$ 5dB, the BER of our system almost reaches that of the three-user system with perfect knowledge of the complex amplitudes, represented by the continuous bold curve.

Analogous conclusions, not reported here, are obtained for the two other users. Promising results were also obtained when the users amplitudes or carrier phase offsets are not equal but they were not reported here for lack of space.

5. CONCLUSION

In this paper we derived from the EM algorithm an estimator of the users propagation delays, received carrier phase offsets and received amplitudes in an asynchronous DS-CDMA environment with users frequency non-selective propagation channels. This synchronizer utilizes the soft information provided by a multiuser turbo receiver.

Simulation results show that, when the propagation delays are perfectly known, the proposed estimator is unbiased and its performance converges to that of the single-user case for values of E_b/N_0 greater than a very reasonable threshold value. Over this range, the bit error rate reached by the synchronized system does not suffer from any degradation with respect to the system with perfect knowledge of the complex amplitudes.

6. REFERENCES

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