# A PARTICLE FILTER FOR BLIND TIMING RECOVERY AND DATA DETECTION IN FAST FADING WIRELESS CHANNELS

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## ABSTRACT

Accurate estimation of synchronization parameters is a fundamental issue in digital transmission. In this paper, we investigate a novel approach to joint synchronization and blind data detection in frequency non-selective fast fading channels based on the application of sequential Monte Carlo (SMC) techniques. The algorithm is derived by modeling the transmission process as a dynamic system where the channel parameters and the transmitted symbols are unobserved state variables. The performance of the proposed technique is studied through computer simulations that illustrate the accuracy of timing recovery and the overall performance of the resulting receiver in terms of its Symbol Error Rate (SER).

#### 1. INTRODUCTION

There are many practical scenarios where the wireless link can be accurately represented by a frequency non-selective, fast fading channel model which is tied to a set of well-defined physical parameters, namely the received signal attenuation, its relative delay and the carrier frequency and phase offsets. The generalized synchronization problem [1] is a fundamental issue in digital receiver design that consists of the recovery of the physical parameters using the observed signal, and can be interpreted as a particular case of equalization in which the physical channel structure is fully exploited.

Most existing synchronization techniques are based on approximate maximum likelihood arguments [1, 2], because optimal estimators for the parameters of interest are analytically intractable. The sequential Monte Carlo (SMC) methodology (commonly referred to as *particle filtering*) [3] is a powerful tool that can be applied in this context as it provides a means for numerically computing (optimal) Bayesian estimators when closed-form solutions are intractable.

In this paper, we propose the application of particle filtering for blind timing recovery and data detection in a fast fading, frequency-flat wireless channel. The channel time-selectivity is accounted for by modeling the relative delay of the received signal as a first order autoregressive (AR) process [4], while the fading process of the channel is modeled as second order AR, driven by complex white Gaussian noise [5]. These statistical assumptions allow the representation of the communication process as a dynamic system in state-space form that lends itself to the application of an SMC algorithm for joint timing recovery and data detection which operates in a *blind* manner, i.e., without requiring the transmission of pilot data.

The remaining of the paper is organized as follows. Section 2 introduces the signal model and the state-space representation of the communication system. The proposed algorithm is introduced in Section 3, and subsequently applied to design of a closed-loop blind receiver in Section 4. Section 5 presents illustrative computer simulation results and, finally, Section 6 is devoted to conclusions.

#### 2. SIGNAL MODEL

Let us consider a digital communication system that transmits symbols,  $\{s_k\}$ , from a finite alphabet in frames of length K over a non-dispersive time-selective channel. The discrete-time received signal obtained after matched filtering and symbol rate sampling can be compactly written as

$$y_k = h_k \mathbf{s}_k^{\top} \mathbf{g}(\tau_k, \omega_k) + v_k, \tag{1}$$

where  $h_k$  is the complex channel fading process,  $\mathbf{s}_k = [s_{k-L}, ..., s_k]^\top$  is an  $(L+1) \times 1$  symbol vector, and  $\mathbf{g}_k$  is an  $(L+1) \times 1$  vector defined by

$$\mathbf{g}(\tau_k,\omega_k) = e^{j\omega_k k} \left[ g(LT+\tau_k), g((L-1)T+\tau_k), \dots, g(\tau_k) \right]^\top$$

and it represents the discrete-time joint response of the transmit and receive filters (assumed causal), which depends on the relative symbol delay,  $0 \le \tau_k < T$ , and the carrier frequency offset,  $\omega_k$ , and  $v_k$  is an additive white Gaussian noise (AWGN) process. Note that due to imperfect timing, the received signal suffers from Inter-Symbol Interference (ISI), with a spread of L + 1 symbols  $(s_{k-L}, \ldots, s_k)$ .

In order to apply particle filtering, we model the communication process as a dynamic system. This is easily done by using (1) as an observation equation and defining the dynamic system state in order to include the transmitted data, the relative delay and the channel fading process. In the sequel, we assume for simplicity that there is no carrier frequency offset,  $\omega_k = 0$ , although it is straightforward (but notationally cumbersome) to include it in the proposed algorithm.

We need to specify the dynamics of the state variables. Following [4], we model the symbol-timing by a slowly varying first order AR process driven by white Gaussian noise,

$$\tau_k = a\tau_{k-1} + u_k \tag{2}$$

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where 0 < a < 1, and  $u_k \sim \mathcal{N}(0, \sigma_u^2)$ . The values of a and  $\sigma_u^2$  depend on the transmitter and receiver timing jitter. It is usually acceptable to set a close to one and a very small  $\sigma_u^2$  [4].

Similarly, the variation of the fading coefficient,  $h_k$ , is modeled by a second order AR process driven by a complex white Gaussian process [5],

$$h_k = \gamma_1 h_{k-1} + \gamma_2 h_{k-2} + e_k \tag{3}$$

where the values of the coefficients,  $\gamma_1$  and  $\gamma_2$ , and the variance of the zero-mean complex white Gaussian noise  $e_k$  are functions of the fading rate of the channel.

Using equations (1), (2) and (3), and assuming zero frequency offset, we obtain the dynamic model

$$\tau_{k} = a\tau_{k-1} + u_{k} \mathbf{h}_{k} = \mathbf{A}\mathbf{h}_{k-1} + \mathbf{f}e_{k} \mathbf{s}_{k} = \mathbf{S}\mathbf{s}_{k-1} + \mathbf{d}_{k}$$
 state eq. (4)

$$y_k = h_k \mathbf{s}_k^\top \mathbf{g}(\tau_k) + v_k$$
 observation eq. (5)

where  $\mathbf{h}_k = [h_k, h_{k-1}]^{\top}$ ,  $\mathbf{f} = \begin{bmatrix} 1 & 0 \end{bmatrix}^{\top}$ ,  $\mathbf{A} = \begin{bmatrix} \gamma_1 & \gamma_2 \\ 1 & 0 \end{bmatrix}$ ,  $\mathbf{S}$  is an  $(L+1) \times (L+1)$  shifting matrix that verifies

$$\mathbf{S}[s_{k-L},\ldots,s_{k-1},s_k]^{\top} = [s_{k-L+1},\ldots,s_k,0]^{\top},$$

 $\mathbf{d}_k = [0, \dots, 0, s_k]^\top$  is an  $(L + 1) \times 1$  perturbation vector that contains the new symbol, and  $\mathbf{g}(\tau_k) = \mathbf{g}(\tau_k, \omega_k = 0)$ . The system state at time k is given by  $(\mathbf{s}_k, \mathbf{h}_k, \tau_k)$ , while the parameters  $a, \sigma_u^2, \sigma_v^2, \gamma_1, \gamma_2, \sigma_e^2$  and L are assumed fixed and known. We focus on the joint estimation of the symbols,  $s_{0:M-1} = \{s_0, \dots, s_{M-1}\}$ , and the delays,  $\tau_{0:M-1} = \{\tau_0, \dots, \tau_{M-1}\}$  from the available observations,  $y_{0:M-1} = \{y_0, \dots, y_{M-1}\}$ . Note that the channel complex fading process  $h_{0:M-1} = \{h_0, \dots, h_{M-1}\}$  is a nuisance process, and we will try to avoid its explicit estimation.

### 3. A PARTICLE FILTER FOR JOINT DATA DETECTION AND TIMING RECOVERY

#### 3.1. Particle filtering

We are interested in the sequential estimation of the transmitted data and the symbol timing. From Bayesian perspective, all information is contained in the *a posteriori* probability distribution function (PDF),  $p(\mathbf{s}_{0:k}, \tau_{0:k}|y_{0:k})$ , which is, unfortunately, analytically intractable. Hence, we propose to use particle filtering, which is an emerging signal processing tool for numerically computing Bayesian estimates. A particle filter approximates a desired PDF by means of a discrete measure with a random support. Specifically, the distribution  $p(\mathbf{s}_{0:k}, \tau_{0:k}|y_{0:k})$  can be approximated by *N* sample trajectories (particles) as

$$\hat{p}(\mathbf{s}_{0:k},\tau_{0:k}|y_{0:k}) = \sum_{n=1}^{N} \delta^{(n)}(\mathbf{s}_{0:k},\tau_{0:k}) w_{k}^{(n)},$$

where  $\mathbf{s}_{0:k}$  and  $\tau_{0:k}$  are sample trajectories of the data and the delays, respectively, while  $w_k^{(n)}$  are weights associated to the particles, and  $\delta(\cdot)$  denotes the Dirac's delta function,

$$\delta^{(n)}(\mathbf{s}_{0:k}, \tau_{0:k}) = \begin{cases} 1, & \text{if } \mathbf{s}_{0:k} = \mathbf{s}_{0:k}^{(n)}, \tau_{0:k} = \tau_{0:k}^{(n)} \\ 0, & \text{otherwise} \end{cases}$$

Using the approximated posterior PDF, Bayesian estimates can easily be computed. For online processing, we usually wish to work with the marginal Minimum Mean Square Error (MMSE) estimate of the delay,

$$\hat{\tau}_k^{mmse} = \sum_{n=1}^N \tau_k^{(n)} w_k^{(n)}, \tag{6}$$

and the marginal Maximum A Posteriori (MAP) estimate of the data,

$$\hat{s}_{k}^{map} = \arg\max_{s_{k}} \left\{ \sum_{n=1}^{N} \delta(s_{k} - s_{k}^{(n)}) w_{k}^{(n)} \right\}.$$

#### 3.2. Sequential Importance Sampling

Most particle filtering methods rely upon the principle of Importance Sampling (IS) [3] for building an empirical approximation of a desired PDF (say p(x)) by drawing samples from a different distribution, known as *importance function* or *proposal PDF* (denoted  $\pi(x)$ ). These samples are then assigned appropriate normalized *importance* weights, i.e.,

$$\begin{aligned} x^{(n)} &\sim & \pi(x) \\ w^{(n)} &\propto & \frac{p(x^{(n)})}{\pi(x^{(n)})} \end{aligned}$$

where  $\sum_{n=1}^{N} w^{(n)} = 1$ . For the problem at hand, we wish to approximate  $p(\mathbf{s}_{0:k}, \tau_{0:k}|y_{0:k})$ , hence we need an importance function of the form  $\pi(\mathbf{s}_{0:k}, \tau_{0:k}|y_{0:k})$ .

One of the most appealing features of the particle filtering approach is its potential for online processing. Indeed, the IS principle can be sequentially applied by exploiting the recursive decomposition of the posterior distribution

$$p(s_{0:k}, \tau_{0:k}|y_{0:k}) \propto p(y_k|s_{0:k}, \tau_{0:k}, y_{0:k-1})p(\tau_k|\tau_{k-1}) \times p(s_{0:k-1}, \tau_{0:k-1}|y_{0:k-1}),$$
(7)

which is easily derived by taking into account the *a priori* uniform PDF of the symbols, and an adequate importance function that can be factored as

$$\pi(\mathbf{s}_{0:k}, \tau_{0:k} | y_{0:k}) = \pi(\mathbf{s}_k, \tau_k | \mathbf{s}_{k-1}, \tau_{k-1}, y_k) \\ \times \pi(\mathbf{s}_{0:k-1}, \tau_{0:k-1} | y_{0:k-1}).$$
(8)

The recursive algorithm that combines the IS principle and the decompositions (7) and (8) to build a particle filter approximation of the posterior PDF is called Sequential Importance Sampling (SIS) [3]. Let  $\Omega_k = \left\{ (s_{0:k}, \tau_{0:k})^{(n)}, w_k^{(n)} \right\}_{n=1}^N$  denote the particle filter at time k. When a new observation is collected at time k + 1, the SIS algorithm proceeds as follows to recursively compute  $\Omega_{k+1}$ :

1. Importance sampling:

$$(\mathbf{s}_k, \tau_k)^{(n)} \sim \pi(\mathbf{s}_k, \tau_k | \mathbf{s}_{k-1}^{(n)}, \tau_{k-1}^{(n)}, y_k)$$

2. Weight update:

$$\tilde{w}_{k}^{(n)} = w_{k-1}^{(n)} \frac{p(y_{k}|\mathbf{s}_{0:k}^{(n)}, \tau_{0:k}^{(n)}, y_{0:k-1})p(\tau_{k}^{(n)}|\tau_{k-1}^{(n)})}{\pi(\mathbf{s}_{k}^{(n)}, \tau_{k}^{(n)}|\mathbf{s}_{k-1}^{(n)}, \tau_{k-1}^{(n)}, y_{k})}$$

#### 3. Weight normalization:

$$w_k^{(n)} = \frac{\tilde{w}_k^{(n)}}{\sum_{i=1}^N \tilde{w}_k^{(i)}}$$

It can be shown that the particle filter computed with the SIS algorithm converges to the desired posterior PDF for a sufficiently large number of particles [6], i.e.,

$$\hat{p}(\mathbf{s}_{0:k},\tau_{0:k}|y_{0:k}) \xrightarrow{N \to \infty} p(\mathbf{s}_{0:k},\tau_{0:k}|y_{0:k}).$$

#### 3.3. Computation

The proposed SIS algorithm requires the numerical evaluation of the likelihood function in the weight update equation. It is straightforward to show that

$$p(y_{k}|\mathbf{s}_{0:k}, \tau_{0:k}, y_{0:k-1}) = \int_{\mathbf{h}_{k}} p(y_{k}|\mathbf{s}_{k}, \tau_{k}, \mathbf{h}_{k}) \\ \times p(\mathbf{h}_{k}|\mathbf{s}_{0:k-1}, \tau_{0:k-1}, y_{0:k-1}) d\mathbf{h}_{k},$$
(9)

where

$$p(y_k|\mathbf{s}_k, \tau_k, \mathbf{h}_k) = \mathcal{N}(y_k; h_k \mathbf{g}^{\top}(\tau_k)\mathbf{s}_k, \sigma_v^2)$$

and

$$p(\mathbf{h}_k | \mathbf{s}_{0:k-1}, \tau_{0:k-1}, y_{0:k-1}) = \mathcal{N}(\mathbf{h}_k; \boldsymbol{\mu}_{k|k-1}, \boldsymbol{\Sigma}_{k|k-1})$$

are Gaussian PDFs. The predictive channel mean,

$$\boldsymbol{\mu}_{k|k-1} = E_{p(\mathbf{h}_k|\mathbf{s}_{0:k-1},\tau_{0:k-1},y_{0:k-1})}\mathbf{h}_k,$$

and its predictive covariance matrix,  $\Sigma_{k|k-1}$ , can be computed using a Kalman filter. Therefore, the integral in (9) can be solved to yield the expression in eq. (10) (shown at the top of next page), where  $\mu_{k|k-1}$  is the first component of  $\mu_{k|k-1}$  and  $\Sigma_{k|k-1}$  is the element from the first row and first column of  $\Sigma_{k|k-1}$ .

As for the importance function, at time k we choose

$$(s_{k}^{(i)}, \tau_{k}^{(i)}) \sim \pi_{k}(s_{k}, \tau_{k}) = p(s_{k} | \mathbf{s}_{0:k-1}^{(i)}, \tau_{k}, \tau_{0:k-1}^{(i)}, y_{0:k}) \times p(\tau_{k} | \tau_{k-1}^{(i)}), \quad (11)$$

which can be sampled in two steps. First, we obtain a new delay particle according to

$$\tau_k^{(i)} = \mathcal{N}(a\tau_{k-1}^{(i)}, \sigma_u^2).$$

Then, a sample of the transmitted symbol is obtained from the first density on the right hand side of (11). This is feasible because we can rewrite  $p(s_k|\mathbf{s}_{0:k-1}^{(i)}, \tau_{0:k}^{(i)}, y_{0:k})$  as

$$p\left(s_{k} = S|\mathbf{s}_{0:k-1}^{(i)}, \tau_{0:k}^{(i)}, y_{0:k}\right) \propto p\left(y_{k}|s_{k} = S, \mathbf{s}_{0:k-1}^{(i)}, \tau_{0:k}^{(i)}, y_{0:k-1}\right)$$
(12)

where  $S \in \mathcal{A}$  is a symbol in the modulation alphabet,  $\mathcal{A}$ . Notice that the likelihood on the right hand side of (12) can be evaluated using equation (10). The resulting importance weights of the new particles are shown in equation (13), where  $\mathbf{s}_{k,j}^{(i)} = [s_{k-L}^{(i)}, \dots, s_{k-1}^{(i)}, s_k = S_j]^{\top}$ , and  $\boldsymbol{\mu}_{k|k-1}^{(i)}$  and  $\boldsymbol{\Sigma}_{k|k-1}^{(i)}$  are the predictive channel mean and covariance matrix, respectively, obtained by Kalman filtering from the observations and the *i*-th state trajectory,  $(\mathbf{s}_{0:k-1}, \tau_{0:k-1})^{(i)}$ .

It is important to remark that the implementation of the proposed SIS algorithm requires a bank of Kalman filters (one for each particle) in order to compute the fading process statistics that are needed for the importance PDF and the weight update equation. The combination of the SIS algorithm and Kalman filtering has already been applied to other communication problems and is sometimes termed Mixture Kalman Filter (MKF) [7].

#### 3.4. Resampling

One major problem in the practical implementation of the SIS algorithm is that, after few time steps most of the trajectories have importance weights with negligible values (very close to zero). The common solution to this problem is to *resample* the particles. Resampling is algorithmic step that stochastically discards particles with insignificant weight while replicating the ones with significant weight. In its simplest form, the scheme generates N new trajectories  $\{s_{0:k}, \tau_{0:k}\}_{i=1}^{N}$  having weights equal to 1/N by drawing samples from the trajectories with probability  $w_k^{(i)}$ .

#### 4. BLIND RECEIVER

The SIS algorithm described above simply obtains the joint estimate of the transmitted symbols and their relative delays. The symbol error rate (SER) of the proposed scheme can be significantly improved if ISI is removed from the observed signal using the delay estimates provided by the particle filter. This can be achieved by considering the closed-loop receiver architecture depicted in Figure 1.



Fig. 1. Closed-loop architecture

The SMC block represents the SIS algorithm with resampling described in the previous section. In the closed-loop receiver, the (asymptotically optimal) MMSE estimate of the relative symbol delay at time k,  $\hat{\tau}_k^{mmse}$ , obtained according to (6), is fed back and used to adjust the sampling epoch of the next observation. Therefore, instead of sampling the received signal uniformly, to obtain  $y_k = y(kT)$ , the sampling time is adaptively adjusted according to the most recent estimate of the relative symbol delay, to yield  $y_k = y(kT - \tilde{\tau}_k)$  where  $\tilde{\tau}_k = a\hat{\tau}_{k-1}^{mmse}$  is the MMSE prediction of  $\tau_k$  according to the observations up to time k - 1.

The observation collected in this way has the form

$$y_k = h_k \mathbf{s}_k^{\mathsf{T}} \mathbf{g}(\tau_k - \tilde{\tau}_k) + v_k.$$

If  $\tilde{\tau}_k \simeq \tau_k$ , the resulting observation,  $y_k$ , is free of ISI and the corresponding symbol estimates attain (asymptotically) minimal SER.

$$p(y_k|\mathbf{s}_{0:k},\tau_{0:k},y_{0:k-1}) = \mathcal{N}\left(y_k;\mathbf{g}^{\top}(\tau_k)\mathbf{s}_k\mu_{k|k-1},\sigma_v^2 + \left(\mathbf{g}^{\top}(\tau_k)\mathbf{s}_k\right)^2\Sigma_{k|k-1}\right)$$
(10)

$$w_{k}^{(i)} \propto w_{k-1}^{(i)} \sum_{S_{j} \in \mathcal{A}} \mathcal{N}\left(y_{k}; \mathbf{g}^{\top}(\tau_{k}^{(i)}) \mathbf{s}_{k,j}^{(i)} \mu_{k|k-1}^{(i)}, \sigma_{v}^{2} + \left(\mathbf{g}^{\top}(\tau_{k}^{(i)}) \mathbf{s}_{k,j}^{(i)}\right)^{2} \Sigma_{k|k-1}^{(i)}\right)$$
(13)

#### 5. SIMULATION RESULTS

We now present computer simulations that illustrate the validity of our approach. We have considered a differential encoded binary modulation with symbol alphabet {±1}, and a flat fading channel with a fading rate 0.022 which corresponds to vehicle velocity of v = 75 miles per hour, carrier frequency of  $f_c = 2$  GHZ and symbol period of  $T = 10^{-4}$ . The delay was modeled as a first order autoregressive process with parameter a = 0.999 and noise variance  $\sigma_u^2 = 3 \times 10^{-4}$ . A time-limited causal raisedcosine pulse with a roll-off factor  $\alpha = 0.7$  yielding an ISI spread of L + 1 = 3 symbols was used. The algorithms were run for N = 300 particles.



Fig. 2. SER as a function of SNR.

Figure 2 depicts the Symbol Error Rate (SER) for several values of the Signal-to-Noise Ratio (SNR). The proposed algorithm is compared with two genie-aided particle filters, as well as with the optimal detector (perfect timing and known channel). Figure 3 illustrates the ability of the proposed algorithm to track the time varying delay,  $\tau_k$ , in a single simulation with SNR= 25 dB.

# 6. CONCLUSIONS

We have introduced a novel algorithm for blind timing recovery and data detection in fast fading, frequency non-selective wireless channels. The resulting receiver numerically performs the (asymptotically optimal) Bayesian estimation of the transmitted symbols and their delays using the particle filtering methodology. The algorithm is fit to a closed-loop structure that allows to adaptively adjust the timing epoch in order to suppress the ISI.



Fig. 3. Actual and estimated delay,  $\tau$ , for SNR = 25 dB.

Computer simulations are presented that illustrate the new receiver performance in terms of timing recovery and SER.

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