CYCLOSTATIONARY JOINT PHASE AND TIMING ESTIMATION FOR STAGGERED MODULATIONS

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ABSTRACT

This paper presents a cyclostationary approach to the Non-Data-Aided (NDA) phase and timing estimation problem for staggered modulations. The problem is addressed under the low-SNR Unconditional Maximum Likelihood (UML) framework, and modulations such as Offset Quadrature Amplitude Modulation (OQAM) and Minimum Shift Keying (MSK) are considered. In this sense, it is found that not only the timing parameter but also the phase, can be jointly obtained from the asymptotic UML cost function based on the spectral line generation after a second-order non-linearity.

1. INTRODUCTION

Staggered or *offset* modulations belong to the class of bandwidth efficient modulations, providing a good ratio between the transmitted data rate and the required bandwidth. The main difference with respect to classical linear modulations is that the in-phase and the quadrature data streams are not time aligned. In particular, staggered modulations usually introduce a delay in the quadrature data stream of half the symbol period, which results in smoother phase transitions that restrict the envelope variations. This fact solves most of the problems encountered by modulations when passing through non-linear devices such as Travelling Wave Tubes (TWT), which generate out-of-band interference when the incoming signal envelope collapses to zero (see [1]-[2] and the references therein).

In this paper, we investigate in a systematic way the Non-Data-Aided (NDA) joint phase and timing estimation problem for the case of staggered modulations. In this sense, we consider staggered modulations such as Offset Quadrature Amplitude Modulation (OQAM) or Minimum Shift Keying (MSK), the latter being equivalent to an staggered modulation with a sinusoidal pulse shape [2]. The study presented herein is closely related to the one presented in [3], with the difference that here, the exploitation of the cyclostationary property leads us to an estimator which is asymptotically based on the spectral line generation from a second-order non-linearity. Therefore, this scheme can be thought as an extension for staggered modulations of the well-known Square Timing recovery method by Oerder & Meyr (O&M) [4].

The paper is structured as follows: Section 2 introduces the signal model and the problem statement in terms of the UML framework. Next, Section 3 derives the likelihood cost function for the phase and timing estimation by exploiting the cyclostationary properties of the received signal, and presents the joint phase and timing estimator. Section 4 presents some simulation results, and finally, conclusions are drawn in Section 5.

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2. SIGNAL MODEL AND PROBLEM STATEMENT

The staggered modulations considered in this paper include the time misalignment between the in-phase and the quadrature data streams by an offset equal to half the symbol period. Although some other offset values can be applied, this is the commonly adopted in practice due to its optimum performance in terms of phase jitter inmunity in the presence of Gaussian noise [5]. Thus, the discrete-time signal model for a generic baseband staggered modulation can be expressed as follows:

$$\begin{aligned} r(k) \;=\; & \sum_{n=-\infty}^{+\infty} \Big[x_n^{\mathcal{R}} g\left(k - nN_{ss} - \tau\right) \\ & + j x_n^{\mathcal{I}} g\left(k - nN_{ss} - \frac{N_{ss}}{2} - \tau\right) \Big] e^{j\theta} + w\left(k\right) \end{aligned} \tag{1}$$

where $\left\{x_n^{\mathcal{R}}, x_n^{\mathcal{I}}\right\}$ stand for the in-phase and the quadrature symbol streams, both assumed to be i.i.d and part of a real and finite alphabet. The discrete-time pulse shape filter is denoted by $g(k)$, and is assumed to be band-limited. The discrete-time symbol period is indicated by N_{ss} samples per symbol, and finally, $w\left(k\right)$ are the complex additive white Gaussian noise (AWGN) samples with zero mean and σ_w^2 variance, $\mathcal{N}\left(0, \sigma_w^2\right)$. Regarding the synchronization parameters, they comprise the discrete-time symbol timing error τ constrained within $[-N_{ss}/2, +N_{ss}/2)$[1], and the carrier phase error θ comprised within $[-\pi, +\pi)$. Taking into consideration an observation interval comprising L transmitted symbols, it is possible to express the received signal in terms of a vector comprising a total of $M = N_{ss}L$ samples as follows:

$$\mathbf{r} = \mathbf{A}\left(\boldsymbol{\Theta}\right)\mathbf{x}_{\mathcal{R}} + j\mathbf{J}\mathbf{A}\left(\boldsymbol{\Theta}\right)\mathbf{x}_{\mathcal{I}} + \mathbf{w} \tag{2}$$

where $\mathbf{r}$ is the $(M \times 1)$ vector of received samples, $\mathbf{A}\left(\boldsymbol{\Theta}\right)$ is the $(M \times L)$ signal shaping matrix given by (3), and $\boldsymbol{\Theta} = [\tau, \theta]^T$. The $(L \times 1)$ vectors $\{\mathbf{x}_{\mathcal{R}}, \mathbf{x}_{\mathcal{I}}\}$ correspond to the in-phase and quadrature transmitted symbols respectively, and the matrix $\mathbf{J}$ is an $(M \times M)$ shift-matrix for modelling the $N_{ss}/2$ offset in the quadrature component. Finally, $\mathbf{w}$ is the $(M \times 1)$ AWGN vector.

$$\left[\mathbf{A}\left(\boldsymbol{\Theta}\right)\right]_{p,q} = g\left(p - qN_{ss} - \tau\right) e^{j\theta} \tag{3}$$

$$\left[\mathbf{J}\right]_{p,q} = \begin{cases} 1, & p - q = N_{ss}/2 \\ 0, & p - q \neq N_{ss}/2 \end{cases} \tag{4}$$

In the sequel, the notation $(\,\cdot\,)^T$ and $(\,\cdot\,)^H$ will be used for indicating the transpose and conjugate transpose operation.

From the signal model in (2), the NDA parameter estimation problem is here attempted under the *Stochastic* or Unconditional

[1]The equivalent continuous-time symbol period is $T = N_{ss}T_s$, with T_s the sampling period. Thus, the continuous-time symbol timing error is denoted by τ_c, with $\tau_c \in [-T/2, +T/2)$.

Maximum Likelihood (UML) framework [6]. This approach assumes the transmitted symbols to be all random and yields asymptotically efficient and unbiased estimators. In particular, the Maximum Likelihood (ML) estimation problem for the AWGN channel is based on maximizing the following Likelihood function:

$$\Lambda(\mathbf{r}|\mathbf{\Theta};\mathbf{x}) = C_1 \exp\left(-\frac{1}{\sigma_w^2}\|\mathbf{r} - \mathbf{A}(\mathbf{\Theta})\mathbf{x}_{\mathcal{R}} - j\mathbf{J}\mathbf{A}(\mathbf{\Theta})\mathbf{x}_{\mathcal{I}}\|^2\right) \tag{5}$$

with $\mathbf{x} = \{\mathbf{x}_{\mathcal{R}}, \mathbf{x}_{\mathcal{I}}\}$, and C_1 an irrelevant constant. Expanding the above expression, and taking into consideration just those terms with truly dependence on the unknown parameters, we have:

$$\Lambda(\mathbf{r}|\mathbf{\Theta};\mathbf{x}) = C_1 \exp\left(-\frac{2}{\sigma_w^2}\chi(\mathbf{r};\mathbf{\Theta};\mathbf{x})\right) \tag{6}$$

$$\chi(\mathbf{r};\mathbf{\Theta};\mathbf{x}) = \mathrm{Re}\left[\mathbf{x}_{\mathcal{R}}{}^T\mathbf{A}^H\mathbf{r}\right] + \mathrm{Im}\left[\mathbf{x}_{\mathcal{I}}{}^T\mathbf{A}^H\mathbf{J}^T\mathbf{r}\right] \tag{7}$$

where, for the sake of simplicity, the dependence of $\mathbf{A}$ with the vector of unknown parameters $\mathbf{\Theta}$ has been omitted in the notation.

The main drawback that we encounter is the dependence of the Likelihood function on the transmitted symbols. In order to overcome this problem, a common practice in the literature is the assumption of a low-SNR scenario. This low-SNR approximation is reasonable in nowadays communication systems (e.g. the range of application for Turbo Codes), but in addition, it provides a robust performance of the estimators derived hereafter, as it assumes a worst-case scenario. The low-SNR approach enables us to expand the Likelihood function in (6) into a Taylor series, in a way that the expectation with respect to the transmitted symbols can easily be performed [6]. Taking into consideration the expansion up to the quadratic term, we have:

$$\Lambda(\mathbf{r}|\mathbf{\Theta};\mathbf{x}) \approx C_1\left[1 - \frac{2}{\sigma_w^2}\chi(\mathbf{r};\mathbf{\Theta};\mathbf{x}) + \frac{2}{\sigma_w^4}\chi^2(\mathbf{r};\mathbf{\Theta};\mathbf{x})\right] \tag{8}$$

which is the basis for the subsequent derivations.

3. CYCLOSTATIONARITY EXPLOITATION FOR JOINT PHASE AND TIMING ESTIMATION

The Likelihood function introduced in Section 2 can be further simplified by exploiting the cyclostationary statistics (CSS) of the received signal. This fact enables us to express the UML cost function in (8) in terms of the received signal Cyclic Autocorrelation Function (CAF), providing a simple structure that can easily be optimized with respect to the unknown synchronization parameters [7]. Indeed, this is not a surprising fact, as several CSS-based estimators found in the literature can also be interpreted as asymptotic ML estimators [8].

3.1. Cyclostationary Approach to the UML Cost Function

The exploitation of CSS for the problem under study requires us to distinguish between the complex conjugate and the non-complex conjugate CAF. In this sense, let $R_{x*}^{\alpha}(m)$ be the complex conjugate CAF, and $R_x^{\alpha}(m)$ the non-complex conjugate CAF of signal $x(k)$, evaluated at the cycle-frequency α and time-lag m. Similarly to [9], we define:

$$R_{x*}^{\alpha}(m) = \lim_{N\to\infty}\frac{1}{N}\sum_{k=0}^{N-1} x^*(k)\,x(k+m)\,e^{-j2\pi\alpha k} \tag{9}$$

$$R_{x}^{\alpha}(m) = \lim_{N\to\infty}\frac{1}{N}\sum_{k=0}^{N-1} x(k)\,x(k+m)\,e^{-j2\pi\alpha k} \tag{10}$$

with $(\cdot)^*$ the complex conjugate operator.

Taking into consideration this distinction, we can recover the Likelihood function in (8) to show the application of the above definitions. First of all, the expectation with respect to the transmitted symbols must be performed in (8) so as to obtain a purely non-data-aided cost function, namely $\Lambda(\mathbf{r}|\mathbf{\Theta}) = \mathrm{E}_{\mathbf{x}}[\Lambda(\mathbf{r}|\mathbf{\Theta};\mathbf{x})]$. When performing this expectation, we find that $\mathrm{E}_{\mathbf{x}}[\chi(\mathbf{r};\mathbf{\Theta};\mathbf{x})] = 0$, and thus:

$$\Lambda(\mathbf{r}|\mathbf{\Theta}) \propto \mathrm{E}_{\mathbf{x}}\left[\chi^2(\mathbf{r};\mathbf{\Theta};\mathbf{x})\right] \tag{11}$$

By noting that the covariance matrix for the real-valued transmitted symbols is $E\left[\mathbf{x}_{\mathcal{R}}\mathbf{x}_{\mathcal{R}}^T\right] = E\left[\mathbf{x}_{\mathcal{I}}\mathbf{x}_{\mathcal{I}}^T\right] = \sigma_x^2\mathbf{I}$, with $\sigma_x^2 = 0.5$, we have:

$$\begin{aligned}\mathrm{E}_{\mathbf{x}}\left[\chi^2(\mathbf{r};\mathbf{\Theta};\mathbf{x})\right] &= \frac{1}{4}\mathrm{Re}\left[\mathbf{r}^T\mathbf{A}^*\mathbf{A}^H\mathbf{r} - \mathbf{r}^T\mathbf{J}\mathbf{A}^*\mathbf{A}^H\mathbf{J}^T\mathbf{r}\right] \\ &\quad + \frac{1}{4}\left(\mathbf{r}^H\mathbf{A}\mathbf{A}^H\mathbf{r} + \mathbf{r}^H\mathbf{J}\mathbf{A}\mathbf{A}^H\mathbf{J}^T\mathbf{r}\right)\end{aligned} \tag{12}$$

The structure provided by the outer product $\mathbf{A}\mathbf{A}^H$ has already been investigated in [7] for a more general case including carrier frequency uncertainty. There, it was shown that the terms along the diagonals of matrix $\mathbf{A}\mathbf{A}^H$ can be asymptotically ($M \to \infty$) seen as a Fourier series expansion whose coefficients are given by the transmitted pulse shape CAF. For the case under study, where the carrier frequency error is not considered and the pulse shape is band-limited, the m-th diagonal terms are then given by:

$$\left[\mathbf{A}\mathbf{A}^H\right]_{k+m,k} = \frac{M}{N_{ss}}\sum_{n=-1}^{+1} R_{g*}^{\frac{1}{N_{ss}}n}(m)\, e^{j\frac{2\pi}{N_{ss}}(k-\tau)n} \tag{13}$$

In the case of non band-limited pulses (such as the rectangular pulse), more spectral lines should be considered. However, by taking just the first spectral line at the frequency-lag $\alpha = \frac{1}{N_{ss}}$, we often collect the most important contribution, and hence, it provides a reasonable approximation which leads to a simple implementation. Following this approach, the quadratic terms in the form $\mathbf{r}^H\mathbf{A}\mathbf{A}^H\mathbf{r}$ are found to be given, except for some negligible constant factor, by [7]:

$$\mathbf{r}^H\mathbf{A}\mathbf{A}^H\mathbf{r} \propto \mathrm{Re}\left[e^{-j\frac{2\pi}{T}\tau_c}\sum_{m=-M}^{+M}\left[R_{g*}^{\frac{1}{N_{ss}}}(m)\right]^* R_{r*}^{\frac{1}{N_{ss}}}(m)\right] \tag{14}$$

Now, let $y_1(k)$ and $y_2(k)$ be the following matched filter outputs:

$$y_1(k) = r(k) * g(-k) \tag{15}$$

$$y_2(k) = r(k) * g(-k + N_{ss}/2) \tag{16}$$

with "$*$" the convolution operator. Then, assuming a long enough observation interval ($M \to \infty$), it is found that (14) and the remaining terms in (12) can asymptotically be expressed for synchronization purposes as:

$$\mathbf{r}^H\mathbf{A}\mathbf{A}^H\mathbf{r} = C_2\mathrm{Re}\left[e^{-j\frac{2\pi}{T}\tau_c}R_{y_1*}^{\frac{1}{N_{ss}}}(0)\right] \tag{17}$$

$$\mathbf{r}^H\mathbf{J}\mathbf{A}\mathbf{A}^H\mathbf{J}^T\mathbf{r} = C_2\mathrm{Re}\left[e^{-j\frac{2\pi}{T}\tau_c}R_{y_2*}^{\frac{1}{N_{ss}}}(0)\right] \tag{18}$$

$$\begin{aligned}\mathbf{r}^T\mathbf{A}^*\mathbf{A}^H\mathbf{r} &= C_3 e^{-j2\theta}\Big[e^{-j\frac{2\pi}{T}\tau_c}R_{y_1}^{\frac{1}{N_{ss}}}(0) \\ &\quad + e^{j\frac{2\pi}{T}\tau_c}R_{y_1}^{-\frac{1}{N_{ss}}}(0)\Big]\end{aligned} \tag{19}$$

$$\begin{aligned}\mathbf{r}^T\mathbf{J}\mathbf{A}^*\mathbf{A}^H\mathbf{J}^T\mathbf{r} &= C_3 e^{-j2\theta}\Big[e^{-j\frac{2\pi}{T}\tau_c}R_{y_2}^{\frac{1}{N_{ss}}}(0) \\ &\quad + e^{j\frac{2\pi}{T}\tau_c}R_{y_2}^{-\frac{1}{N_{ss}}}(0)\Big]\end{aligned} \tag{20}$$

with C_2 and C_3 some irrelevant constants. It is important to remark that the zero frequency-lag is not included in (17)-(20), as it does not provide any information on the unknown synchronization parameters. Moreover, due the $N_{ss}/2$ time-offset of signal $y_2(k)$ with respect to $y_1(k)$, a phase shift equal to $e^{-j\frac{2\pi}{N_{ss}}\left(\frac{N_{ss}}{2}\right)}$ is introduced in the CAFs corresponding to $y_2(k)$. In particular, we have:

$$R_{y_2}^{\frac{1}{N_{ss}}}(0) = -R_{y_1}^{\frac{1}{N_{ss}}}(0) \quad (21)$$

$$R_{y_2}^{-\frac{1}{N_{ss}}}(0) = -R_{y_1}^{-\frac{1}{N_{ss}}}(0) \quad (22)$$

$$R_{y_2*}^{\frac{1}{N_{ss}}}(0) = -R_{y_1*}^{\frac{1}{N_{ss}}}(0) \quad (23)$$

Hence, with (21)-(23), and ignoring the zero frequency-lag of the CAF, we can state in (12) that:

$$\mathbf{r}^H\mathbf{A}\mathbf{A}^H\mathbf{r} + \mathbf{r}^H\mathbf{J}\mathbf{A}\mathbf{A}^H\mathbf{J}^T\mathbf{r} = 0 \quad (24)$$

and

$$\mathbf{r}^T\mathbf{A}^*\mathbf{A}^H\mathbf{r} - \mathbf{r}^T\mathbf{J}\mathbf{A}^*\mathbf{A}^H\mathbf{J}^T\mathbf{r} = \quad (25)$$
$$e^{-j\left(2\theta+\frac{2\pi}{T}\tau_c\right)}R_{y_1}^{\frac{1}{N_{ss}}}(0) + e^{-j\left(2\theta-\frac{2\pi}{T}\tau_c\right)}R_{y_1}^{-\frac{1}{N_{ss}}}(0)$$

Then, we just have to substitute (24)-(25) into (11)-(12) for obtaining that the asymptotic low-SNR UML cost function is given by:

$$\Lambda(\mathbf{r}|\mathbf{\Theta}) \propto \quad (26)$$
$$\mathrm{Re}\left[e^{-j\left(2\theta+\frac{2\pi}{T}\tau_c\right)}R_{y_1}^{\frac{1}{N_{ss}}}(0) + e^{-j\left(2\theta-\frac{2\pi}{T}\tau_c\right)}R_{y_1}^{-\frac{1}{N_{ss}}}(0)\right]$$

3.2. Asymptotic Joint Phase & Timing Estimation

There are some remarks to be made about the asymptotic Likelihood cost function derived in (26). Firstly, this Likelihood function does not depend on the complex-conjugate CAF as usual (e.g. as in the O&M algorithm), but on its non-complex conjugate version. Secondly, the maximization of the Likelihood function must be achieved by solving the following system equation:

$$2\theta + \frac{2\pi}{T}\tau_c = arg\left\{R_{y_1}^{\frac{1}{N_{ss}}}(0)\right\} \quad (27)$$

$$2\theta - \frac{2\pi}{T}\tau_c = arg\left\{R_{y_1}^{-\frac{1}{N_{ss}}}(0)\right\} \quad (28)$$

By doing so, we easily find that the asymptotic UML joint phase and timing estimator is then given by:

$$\theta = \frac{1}{4}\left[arg\left\{R_{y_1}^{\frac{1}{N_{ss}}}(0)\right\} + arg\left\{R_{y_1}^{-\frac{1}{N_{ss}}}(0)\right\}\right] \quad (29)$$

$$\tau = \frac{T}{4\pi}\left[arg\left\{R_{y_1}^{\frac{1}{N_{ss}}}(0)\right\} - arg\left\{R_{y_1}^{-\frac{1}{N_{ss}}}(0)\right\}\right] \quad (30)$$

However, an asymptotically large number of received samples is not available in practice. Thus, we must resort to estimating the asymptotic CAF in (29)-(30) from a finite set of M received samples. In this sense, a consistent and efficient estimator for the non-complex conjugate CAF is given by [10]:

$$\widehat{R}_x^{\alpha}(m) = \frac{1}{M}\sum_{k=0}^{M-1} x(k)\,x(k+m)\,e^{-j2\pi\alpha k} \quad (31)$$

Thus, we can estimate $R_{y1}^{\alpha}(0)$ in (29)-(30), namely $\widehat{R}_{y1}^{\alpha}(0)$, by applying (32) at the cycle-frequencies $\alpha = \pm\frac{1}{N_{ss}}$.

$$\widehat{R}_{y1}^{\alpha}(0) = \frac{1}{M}\sum_{k=0}^{M-1}\Big(r(k) * g(-k)\Big)^2 e^{-j2\pi\alpha k} \quad (32)$$

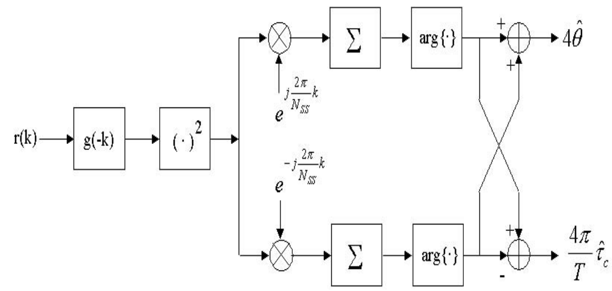

Fig. 1. Block diagram for the low-SNR UML joint phase and timing estimator for staggered modulations.

The block diagram of the resulting joint phase and timing estimator is finally presented in Fig. 1, whose structure is found to be similar to the estimator presented by [3]. In fact, the phase and timing estimates exhibit an ambiguity due to the modulation format of staggered modulations. As a result, the timing estimates are ambiguous by multiples of $T/2$, and the phase estimates by multiples of π (see [3] for details). However, the main difference can be found in the fact that the phase and timing estimates are now easily obtained in (29)-(30) from a simple spectral line generation by using a square-law non-linearity. No other prefiltering apart from the matched filter is needed. Hence, the proposed joint phase and timing estimator can asymptotically be thought as an extension of the well-known O&M Square Timing recovery method for the case of staggered modulations.

4. SIMULATION RESULTS

Computer simulations have been carried out for evaluating the performance of the joint phase and timing estimator presented in Section 3.2. Different types of pulse shape filters have been employed for encompassing a wide range of staggered modulations. In this sense, 16-OQAM with both rectangular (RECT) and overlapped square-root raised cosine (SQRRC) pulses have been simulated. In addition, a sinusoidal (SIN) pulse shape filter is also considered for including the case of binary-MSK modulation. It is important to remark that the proposed joint phase and timing estimator was derived in section 3 under a band-limited assumption, in the sense that just the first spectral line at the symbol rate would be retrieved from the squared matched filter output. In particular, neither the RECT nor the SIN pulse are band-limited pulses, so the timing performance provided by the proposed estimator is not expected to be optimum in these cases.

When selecting the different pulse shape filters to simulate, a similar main-lobe time duration has been maintained so as to deal with similar mean-square bandwidths (see Fig. 2). Hence, approximately twice the symbol rate is achieved by the overlapped SQRRC with respect to the RECT or SIN pulses. The simulation results have been obtained by setting $N_{ss} = 4$ for the overlapped SQRRC and $N_{ss} = 8$ for the RECT and SIN pulses. Finally, we assume a common observation interval of $L = 100$ symbols.

Figure 3 shows the phase and timing variance as a function of the working E_s/N_0. It has been employed 16-OQAM with a SQRRC pulse and roll-off factors RF=$\{0.35, 0.50, 0.75, 1.00\}$. The Modified Cramer-Rao Bound (MCRB) is also included for the case of roll-off factor 1.0, which is a lower bound on the SQRRC timing variance [11].

Figure 4 depicts the phase and timing variance as a function of the working E_s/N_0. Now, both OQPSK with a RECT pulse and MSK modulation are considered. Note that for the timing variance

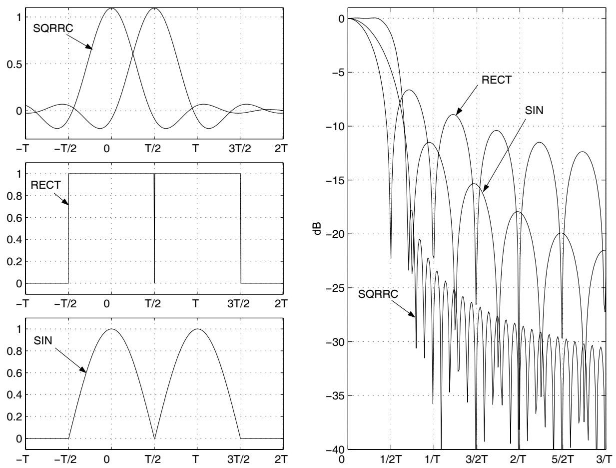

Fig. 2. Representation of the symbol transmission $\{+1, +1\}$ with the SQRRC, RECT and SIN pulses (left) and the pulses frequency response (right).

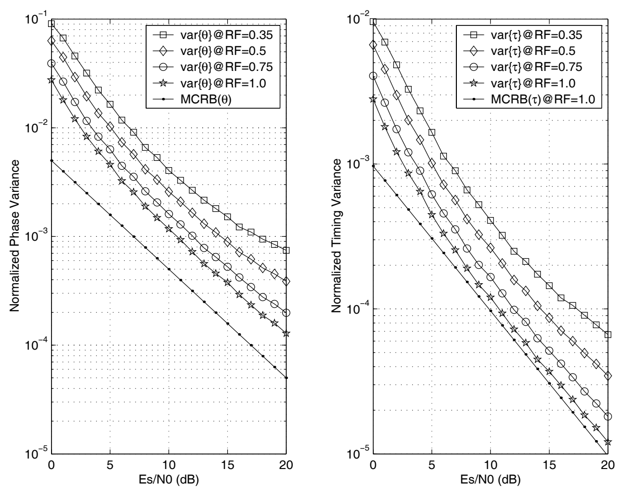

Fig. 3. Phase variance (left) and timing variance (right) as a function of E_s/N_0 for 16-OQAM with SQRCC pulse.

in MSK, the estimator performance is closer to the MCRB than in the case of OQPSK. This is due to the higher effective bandwidth of the RECT pulse with respect to the SIN pulse, which makes the former more sensible to the band-limited approximation.

5. CONCLUSIONS

An UML joint phase and timing estimator has been proposed in this paper for the case of staggered modulations. The exploitation of the cyclostationary property of the received signal allows us to derive a quadratic NDA estimator for both phase and timing, irrespective of the symbol constellation. The core of this estimator is based on the spectral line generation from a square-law nonlinearity, which can be thought as an extension of the well-known Square Timing recovery method by O&M for the case of staggered modulations.

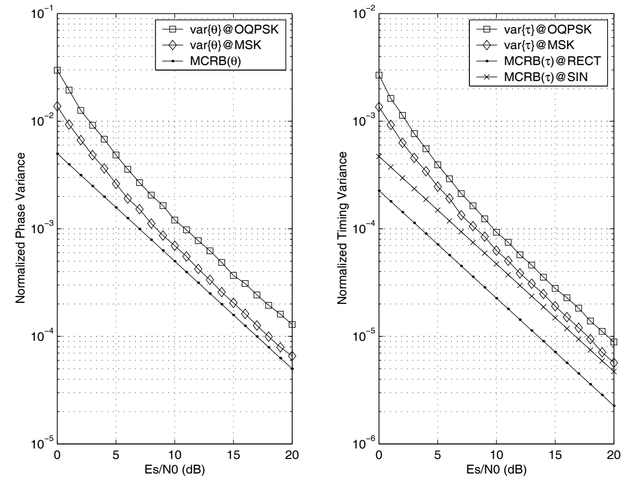

Fig. 4. Phase variance (left) and timing variance (right) as a function of E_s/N_0 for OQPSK with RECT pulse, and MSK.

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