A Deterministic Multiuser Code-Timing Estimator for Long-Code Bandlimited CDMA Systems

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***Abstract*— In this paper, we present a *deterministic* multiuser code-timing estimator for asynchronous direct-sequence (DS) code-division multiple-access (CDMA) systems with *aperiodic long* spreading codes and *bandlimited* chip waveforms. A key feature of the proposed estimator is that it captures and capitalizes a deterministic structure of the overall interference, namely multi-access interference (MAI) and inter-symbol interference (ISI), in the frequency domain. This allows complete interference elimination in a deterministic manner, which is in general more effective and data-efficient than stochastic approaches. Numerical results show that the proposed estimator can achieve fast acquisition; it is also near-far resistant, providing accurate code acquisition for even *overloaded* systems (i.e., systems with more users than the processing gain) in multipath fading environments.**

I. Introduction

Multiuser code-timing estimation is a challenging task in CDMA systems. Traditional techniques, typically based on matched-filter (MF) estimation [1, ch. 5] that ignores the inherent structure of the MAI, are found inadequate in multiuser environments. Recently, a variety of multiuser acquisition techniques have been proposed by exploiting the structure of the MAI (e.g., [2]–[5] and references therein). However, most of these schemes assume *short* (symbol-periodic) spreading codes. In contrast, most practical CDMA systems, including the IS-95 standard and the majority of 3G CDMA-based wireless networks, make use of *long* (symbol-aperiodic) spreading codes to randomize the interference.

Among limited studies on multiuser acquisition for long-code CDMA, Buzzi and Poor [6] proposed a centralized acquisition scheme for use at the base station (i.e, in the uplink). However, their method assumes *rectangular* chip waveform, which is not bandlimited. Meanwhile, most practical systems utilize bandlimited chip waveforms, such as the square-root raised-cosine pulse. As admitted by the authors of [6], it is non-trivial to extend their scheme to the bandlimited case. The purpose of this letter is to introduce an alternative centralized multiuser acquisition scheme that can deal with both long spreading codes and bandlimited chip waveforms.

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Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; superscripts $(\cdot)^T$, and $(\cdot)^H$ denote transpose and conjugate transpose, respectively; $\star$ denotes convolution; $\mathrm{diag}\{\cdot\}$ denotes a diagonal matrix; $\mathbf{0}$ denotes an all-zero matrix or vector; finally, $\odot$ denotes the elementwise Hadamard product.

II. Data Model and Problem Formulation

Consider a baseband asynchronous (uplink) K-user DS-CDMA system with *long* spreading codes. Let $p(t)$ be the *bandlimited* chip waveform. The transmitted signal for user k is

$$x'_k(t) = \sum_{m=0}^{M-1} d_k(m) s'_{k,m}(t - mT_s),$$

where M is the number of symbols used for acquisition, T_s the symbol period, $d_k(m)$ the mth symbol of user k, and $s'_{k,m}(t)$ the spreading waveform: $s'_{k,m}(t) = \sum_{n=0}^{N-1} c_{k,m}(n)p(t-nT_c)$, with $T_c = T_s/N$ denoting the chip interval, N the *processing gain*, and $\{c_{k,m}(n)\}_{n=0}^{N-1}$ the spreading codes for the mth symbol of user k. Signal $x'_k(t)$ passes through a frequency-selective channel with L_k paths. The signal received at the base station, after chip-matched filtering, is

$$y(t) = \sum_{k=1}^{K} \sum_{l=1}^{L_k} \alpha_{k,l} x_k(t - \tau_{k,l}) + w(t), \tag{1}$$

where $\alpha_{k,l}$ and $\tau_{k,l}$ are, respectively, the attenuation and *code-timing* for the lth path of user k, $w(t)$ the channel noise, and $x_k(t)$ the output of the chip-matched filter given input $x'_k(t)$:

$$x_k(t) \triangleq x'_k(t) \star p(T_c - t) = \sum_{m=0}^{M-1} d_k(m) s_{k,m}(t - mT_s),$$

where $s_{k,m}(t) \triangleq s'_{k,m}(t) \star p(T_c - t)$.

The problem is to estimate the multiuser multipath code-timings $\tau_{k,l}$, $\forall k, \forall l$, from $y(t)$. Once we have the code-timing estimates, $\alpha_{k,l}$ can be estimated by least-squares. Similar to earlier works (e.g.,[2]–[6]), we assume $\max_{k,l} \tau_{k,l} < T_s$; extension to larger delays is possible but skipped for space limitation. Also similar to the centralized scheme of [6], we assume that the data symbols and spreading codes for all users are

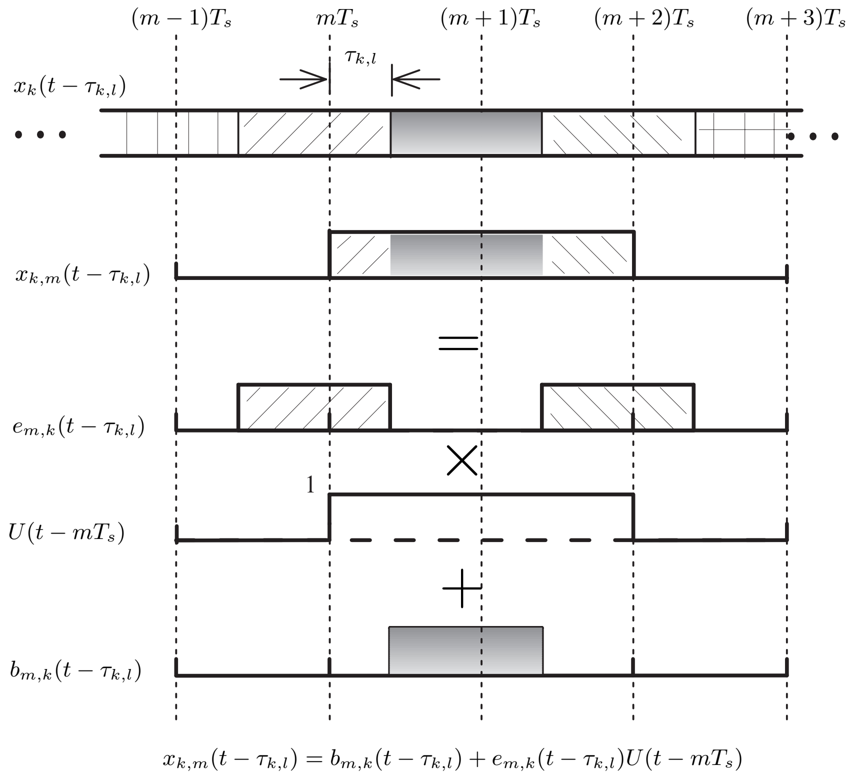

Fig. 1. Graphical illustration of the decomposition of (2).

known. This may correspond an asynchronous CDMA packet network with all users jointly transmitting known preambles before real communication starts [6].

III. Proposed Scheme

First, we split $y(t)$ into overlapping blocks, $y_m(t)$, $m = 0, \ldots, M-1$, each with duration $2T_s$ and every two adjacent blocks are offset by T_s seconds. Specifically, let $U(t)$ be a rectangular window function with duration $2T_s$: $U(t) = 1, 0 \leq t < 2T_s$, and zero elsewhere. Then, we have $y_m(t) \triangleq y(t)U(t-mT_s)$. Likewise, let $x_{k,m}(t-\tau_{k,l}) \triangleq x_k(t-\tau_{k,l})U(t-mT_s)$. Clearly, $y_m(t)$ is a superposition of $x_{k,m}(t-\tau_{k,l}), \forall k, \forall l$, plus the channel noise.

To quantify the overall interference deterministically, we need find an analytical expression for $x_{k,m}(t-\tau_{k,l})$. To do so, let

$$\begin{aligned} b_{m,k}(t-\tau_{k,l}) &\triangleq d_k(m)s_{k,m}(t-mT_s-\tau_{k,l}), \\ e_{m,k}(t-\tau_{k,l}) &\triangleq d_k(m-1)s_{k,m-1}(t-(m-1)T_s-\tau_{k,l}) \\ &\quad + d_k(m+1)s_{k,m+1}(t-(m+1)T_s-\tau_{k,l}), \end{aligned}$$

and note $e_{m,k}(t-\tau_{k,l})$ denotes the inter-symbol interference (ISI) observed in $y_m(t)$. As illustrated in Fig. 1, $x_{k,m}(t-\tau_{k,l})$ can be analytically expressed as

$$x_{k,m}(t-\tau_{k,l}) = b_{m,k}(t-\tau_{k,l}) + e_{m,k}(t-\tau_{k,l})U(t-mT_s). \quad (2)$$

Let $w_m(t) \triangleq w(t)U(t-mT_s)$. It follows from (2) that

$$\begin{aligned} y_m(t) = &\sum_{k=1}^{K}\sum_{l=1}^{L_k} \alpha_{k,l}[e_{m,k}(t-\tau_{k,l})U(t-mT_s) \\ &+ b_{m,k}(t-\tau_{k,l})] + w_m(t). \end{aligned} \quad (3)$$

For digital processing, $y_m(t)$ is sampled with a sampling interval $T_i = T_c/Q$: $y_m(n) = y_m(t)|_{t=mT_s+nT_i}$, $n = 0, \ldots, 2NQ-1$, where Q is an integer; typically, $Q = 1$ or $Q = 2$. Let

$$\bar{\mathbf{y}}_m \triangleq [\mathbf{0}_{1\times NQ}, y_m(0), \cdots, y_m(2NQ-1), \mathbf{0}_{1\times NQ}]^T,$$

viz., it is formed by samples of $y_m(t)$ with NQ zeros padded at the beginning and end. Likewise, let $\bar{\mathbf{b}}_{m,k}(\tau_{k,l})$, $\bar{\mathbf{e}}_{m,k}(\tau_{k,l})$, $\bar{\mathbf{u}}$, and $\bar{\mathbf{w}}_m$ be $4NQ \times 1$ vectors formed from samples of $b_{m,k}(t-\tau_{k,l})$, $e_{m,k}(t-\tau_{k,l})$, $U(t-mT_s)$ and $w_m(t)$, respectively, all treated as signals of duration $4T_s$ with possibly zeros padded at the beginning and end (cf. Fig. 1). Then, the discrete-time form of (3) is

$$\bar{\mathbf{y}}_m = \sum_{k=1}^{K}\sum_{l=1}^{L_k} \alpha_{k,l}\left[\bar{\mathbf{b}}_{m,k}(\tau_{k,l}) + \bar{\mathbf{e}}_{m,k}(\tau_{k,l}) \odot \bar{\mathbf{u}}\right] + \bar{\mathbf{w}}_m. \quad (4)$$

Next, we convert the signals to the frequency domain. Let $\mathbf{y}_m \triangleq \mathcal{F}\{\bar{\mathbf{y}}_m\}$, $\mathbf{u} \triangleq \mathcal{F}\{\bar{\mathbf{u}}\}$, and $\mathbf{w}_m \triangleq \mathcal{F}\{\bar{\mathbf{w}}_m\}$, where $\mathcal{F}\{\cdot\}$

denotes the $4NQ$-point DFT operator. By the time-shifting property of Fourier transform, we have[1]

$$\begin{aligned}\mathbf{y}_m &= \sum_{k=1}^{K}\sum_{l=1}^{L_k}\alpha_{k,l}\left[\operatorname{diag}(\mathbf{b}_{m,k}) + \mathbf{U}\operatorname{diag}(\mathbf{e}_{m,k})\right]\boldsymbol{\phi}(\tau_{k,l}) + \mathbf{w}_m \\ &\triangleq \sum_{k=1}^{K}\mathbf{G}_{m,k}\boldsymbol{\beta}_k(\boldsymbol{\alpha}_k,\boldsymbol{\tau}_k) + \mathbf{w}_m,\end{aligned} \tag{5}$$

where $\mathbf{U}$ is a $4NQ \times 4NQ$ circular matrix with the first row given by $\mathbf{u}$, $\boldsymbol{\phi}(\tau_{k,l}) \triangleq \left[1, \phi^{\tau_{k,l}}, \ldots, \phi^{(4NQ-1)\tau_{k,l}}\right]^T$, with $\phi = e^{-j\frac{2\pi}{4NQ}}$, and $\mathbf{b}_{m,k}$ and $\mathbf{e}_{m,k}$ are $4NQ \times 1$ vectors formed by the $4NQ$-point DFT of samples of $b_{m,k}(t)$ and $e_{m,k}(t)$ with zero delays. In the second equality of (5), $\boldsymbol{\alpha}_k$ and $\boldsymbol{\tau}_k$ are $L_k \times 1$ vectors of the unknown attenuations and code-timings, and

$$\begin{aligned}\boldsymbol{\beta}_k(\boldsymbol{\tau}_k,\boldsymbol{\alpha}_k) &\triangleq \sum_{l=1}^{L_k}\boldsymbol{\phi}(\tau_{k,l})\alpha_{k,l}, \\ \mathbf{G}_{m,k} &\triangleq \operatorname{diag}(\mathbf{b}_{m,k}) + \mathbf{U}\operatorname{diag}(\mathbf{e}_{m,k}).\end{aligned}$$

Note that $\mathbf{G}_{m,k}$ is known and independent of the unknown $\boldsymbol{\alpha}_k$ and $\boldsymbol{\tau}_k$.

Remark: Equation (5) is instrumental to our proposed scheme. It captures the *deterministic* structure of the overall interference, including the ISI due to $e_{m,k}(t-\tau_{k,l})$, whereas many earlier acquisition schemes for short-code systems (e.g.,[2]–[5] and references therein) often treat all or part of the interference as a stochastic process. Equation (5) enables deterministic and complete elimination of both ISI and MAI, which is in general more effective than stochastic approaches. It also leads to faster code acquisition due to no need for second-order statistics estimation that is vital to stochastic approaches and which typically converges slowly.

Based on (5), we have a two-step estimator that first obtains an *unstructured* linear estimate, $\hat{\boldsymbol{\beta}}_k$ of $\boldsymbol{\beta}_k$, followed by imposing the structure of $\boldsymbol{\beta}_k$ on $\hat{\boldsymbol{\beta}}_k$ to produce estimates of the parameters of interest. Specifically, let $\mathbf{y} \triangleq \left[\mathbf{y}_0^T, \cdots, \mathbf{y}_{M-1}^T\right]^T$, $\boldsymbol{\beta} \triangleq \left[\boldsymbol{\beta}_1^T, \cdots, \boldsymbol{\beta}_K^T\right]^T$, and $\mathbf{w} \triangleq \left[\mathbf{w}_0^T, \cdots, \mathbf{w}_{M-1}^T\right]^T$. Then, we have $\mathbf{y} = \mathbf{G}\boldsymbol{\beta} + \mathbf{w}$, where $\mathbf{G}$ is a $4MNQ \times 4KNQ$ matrix with the mkth block given by $\mathbf{G}_{m,k}$, $m = 0, \ldots, M-1$, $k = 1, \ldots, K$. Assuming $\mathbf{G}$ is tall and has full rank, we have

$$\hat{\boldsymbol{\beta}} = \left(\mathbf{G}^H\mathbf{G}\right)^{-1}\mathbf{G}^H\mathbf{y}, \tag{6}$$

from which we can obtain $\hat{\boldsymbol{\beta}}_k$. Next, observe that $\hat{\boldsymbol{\beta}}_k$ is composed of L_k complex sinusoids:

$$\hat{\beta}_k(n) = \sum_{l=1}^{L_k}\alpha_{k,l}e^{-j\frac{2\pi\tau_{k,l}n}{4NQ}} + v_k(n), \quad n = 0, 1, \ldots, 4NQ-1, \tag{7}$$

where $\hat{\beta}_k(n)$ is the nth element of $\hat{\boldsymbol{\beta}}_k$, and $v_k(n)$ denotes the estimation error incurred in (6). Hence, the problem reduces to a sinusoidal parameter estimation problem, and a wealth of good methods can be used to estimate the code-timing $\tau_{k,l}$ and attenuation $\alpha_{k,l}$. See, e.g., [8, ch. 4], for details.

We need $M \geq K$ so that $\mathbf{G}$ is tall. This is a necessary but not sufficient condition for $\mathbf{G}$ to be full-rank. However, since $\mathbf{G}$ is known at the base station, we can check its rank condition. Should it be rank deficient, we can discard certain rows of $\mathbf{G}$ (likewise the corresponding elements of $\mathbf{y}$) to ensure invertability. We mention that the block structure of $\mathbf{G}$ can be exploited for efficient computation. We skip the details for space limitation. Finally, since $v_k(n)$ in (7) is free of any residual interference, the proposed estimator is near-far resistant.

IV. Simulation Results

We consider a K-user asynchronous CDMA system using BPSK modulation and randomly generated long spreading codes with processing gain $N = 16$. The chip waveform is a square-root raised-cosine pulse with $Q = 2$. We focus on a near-far environment whereby the total (from all paths) average power for the desired user is scaled to unity, whereas the $K-1$ interfering users transmit at a mean power P dB higher than the desired user. The near-far ratio (NFR) is defined as P in dB. The channel attenuations, which are zero-mean complex Gaussian random variables, code-timings, uniformly distributed in $[0, T_s)$, data bits, and channel noise are changed independently from trial to trial. Two performance measures are considered: one is the probability of acquisition (PA), defined as the probability of the event that the code-timing estimate is within $T_c/2$ of the true value; the other is the root mean-squared error (RMSE) of the code-timing estimate, given correct acquisition. For the multipath case, we evaluate the performance measures for each path, and present the average results averaged over all path estimates.

We compare the proposed estimator with the conventional MF scheme [1, ch. 5]. Note that we do not compare with the centralized scheme of [6] sine it cannot deal with bandlimited chip pulses. For the proposed estimator, we use the root-MUSIC algorithm [8, pp. 158–160] to solve the sinusoidal parameter estimation problem (7). Fig. 2 depicts the *user capacity* of both schemes when $M = 100$, $L_k = 2$, $\forall k$, SNR $= 15$ dB, and NFR $= 10$ dB. Also shown in the lower half of Fig. 2 is the Cramér-Rao bound (CRB) which provides a lower bound on the RMSE for all unbiased estimators. It is seen that the proposed estimator is insensitive to the number of users in the

[1] Equation (5) holds only approximately because of the aliasing caused by the windowing $U(t)$. The aliasing is in general negligible compared to the interference and noise induced estimation error [7].

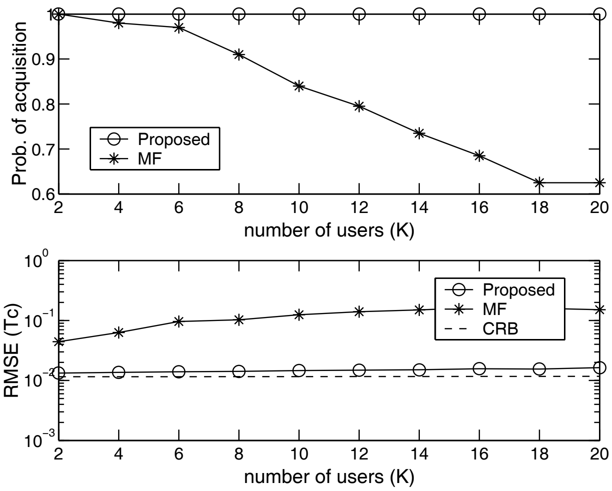

Fig. 2. Performance versus user capacity K when $N = 16$, $M = 100$, SNR $= 15$ dB, and NFR $= 10$ dB in multipath channels.

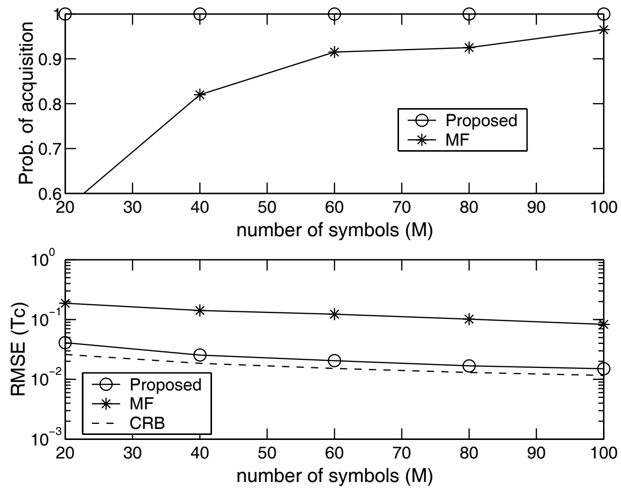

Fig. 3. Performance versus observation time M when $N = 16$, $K = 5$, SNR $= 15$ dB, and NFR $= 10$ dB in multipath channels.

system. Remarkably, it even can support *overloaded* systems with $K > N$, while experiencing little degradation in terms of both PA and RMSE. Fig. 3 illustrates the *observation time* that is needed for both methods, when $K = 5$, M is varying and the other parameters are identical to the previous example. We can see that the proposed scheme achieves much faster acquisition than the MF estimator. In particular, it is seen that the former can produce good acquisition even with $M = 20$ bits. Finally, in Fig. 4, we present the performance of both methods versus SNR by fixing $M = 100$ and varying the SNR from -10dB to 30dB, where the remaining parameters are kept the same as shown in the Fig. 3. From Fig. 4, one can see that the proposed scheme has a very low SNR threshold and approaches the CRB consistently, while the MF has shown irreducible estimation errors indicating that it is sensitive to the MAI.

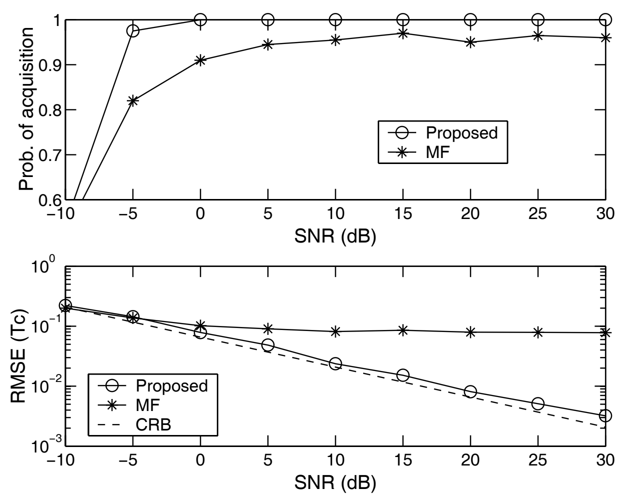

Fig. 4. Performance versus SNR when $M = 100$, $N = 16$, $K = 5$, and NFR $= 10$ dB in multipath channels.

V. Conclusions

In this paper, we present a *deterministic* multiuser code-timing estimator for asynchronous DS-CDMA systems with *long* spreading codes and *bandlimited* chip waveforms. Since the proposed scheme captures and models a deterministic structure of the overall interference, namely MAI and ISI, in the frequency domain, it can provide complete interference elimination in a deterministic manner, which is in general more effective and data-efficient than stochastic approaches. Numerical results show that the proposed estimator is near-far resistant, and can support even overloaded users with fast acquisition.

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