

# ADAPTIVE TRANSMIT EIGENBEAMFORMING WITH ORTHOGONAL SPACE-TIME BLOCK CODING IN CORRELATED SPACE-TIME CHANNELS

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## ABSTRACT

The conventional space-time codes can provide a significant improvement in system performance only if the signal paths are spatially uncorrelated, a condition that is hardly met in practice. In this paper we mitigate this condition by combining a technique of eigenbeamforming, based on the channel correlation matrix, with orthogonal space-time block code (O-STBC) at the transmitter side of the link. No Feedback information from the receiver (the mobile station) is utilized in the proposed structure. Simulation results using 4-ary PSK signaling showed that this idea outperforms existing techniques in both uncorrelated and correlated channels in terms of bit-error rate and symbol-error rate.

## 1. INTRODUCTION

With help of advanced signal processing algorithms it has been shown in [1]-[3] that wireless communication systems employing multiple transmit and receive antennas can provide a considerable gain in both system capacity and error rate performance. Researchers in recent years have been focusing on formulating new designing criteria for optimizing system parameters [4]-[6]. However, common trade-offs in their systems often take place between the system performance and its computation complexity.

A remarkable milestone was reached in 1998 when Alamouti developed a space-time code and demonstrated in [7] that a significant diversity gain can be achieved with a little system complexity added. Later this scheme was generalized in [8] using orthogonal space-time block codes and including transmit diversity. However, the improvement introduced by these space-time codes is dependent on the assumption that the channel is sufficiently uncorrelated. In practice, different signal paths arrived at the receiving antennas are often correlated to some degree.

In this paper, we introduce a new transmit diversity scheme by jointly using transmit eigenbeamforming with O-STBC. This new scheme combines the capabilities of these two different transmission techniques in exploiting

propagation channel characteristics to achieve a considerable performance in wireless transmission in spatially correlated channel environment. The underlying concept of our proposed scheme is the transmission of data symbols that is encoded by the O-STBC in the direction of strongest signal path seen by the base station (BS). Transmit beamforming weights were generated by the decorrelation process of channel spatial covariance matrix formed by the reverse link path angles estimates. With this new transmission technique, it is possible to eliminate the need for feedback signalling of the forward link channel estimates from the mobile station (MS). Furthermore, the proposed scheme requires less computation and system complexity than that required by other schemes.

## 2. SYSTEM MODEL

As illustrated in Fig. 1, we consider a cellular communication system in a multiple input multiple output (MIMO) channel configuration comprising  $N_t$  transmit antennas at the BS and  $N_r$  receive antennas at the MS.

The BS transmitter is assumed to have no knowledge about the wireless channel coefficients but perfect knowledge of angle-of-arrival (AoA) of reverse link signals received at the BS. By utilizing this channel information, transmit antenna weights can be found by performing eigen-decomposition of channel correlation metric formed by this channel knowledge. In this paper, we propose the use of a transmitter consisting of a space-time encoder followed by an eigenbeamformer. First, a sequence of information bits  $\{b(n)\}_{n=0}^{N_b-1}$  is mapped into a set of  $N_s$  symbols  $\{s(n)\}_{n=0}^{N_s-1}$  then encoded into an  $p \times N_t$  matrix [9]:

$$C = \sum_{n=0}^{N_s-1} [s(n)A(n) + s(n)^* B(n)] \quad (1)$$

which are then split into a set of  $N_t$  parallel symbol sequences and linearly transformed by the BS antenna weights and transmitted during  $p$  time intervals.

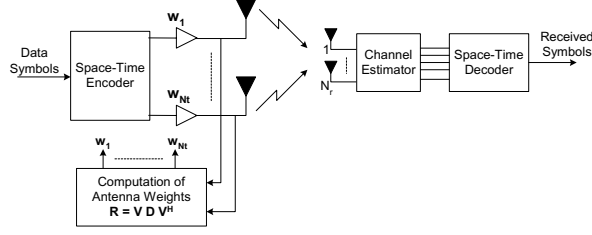


Fig. 1. Structure of transmit diversity scheme with transmit beamforming and space-time coding.

$\{A(n), B(n)\}$  are matrices that designed to satisfy the orthogonality condition that is well documented in both [8] and [9]:

$$CC^* = \sum_{n=0}^{N_s-1} |s(n)|^2 \cdot I \quad (2)$$

where  $(.)^*$  denotes the complex conjugate and  $I$  is the identity matrix.

The information-bearing signals are then transmitted over a wireless fading channel. The coherent bandwidth of the channel is assumed to be much larger than the signal bandwidth, hence, the individual channel between each transmit and receive antenna is modeled as frequency-nonselective in this paper. Finally, maximum-likelihood (ML) decoding is performed at the receiver in order to recover the transmitted data.

## 2.1 Correlated Fading Channel & Eigen-decomposition

We assume that a uniform linear array (ULA) configuration is used for the  $N_t$  transmitting antennas at the BS with a spacing of  $d$  meters between adjacent antenna elements. In the reverse link transmission over a multipath channel environment, let the  $n$ th path signal impinging on an ULA have an angle-of-arrival (AoA) of  $\phi_n$ . The array propagation vector can be found as follows:

$$a(\phi_n) = [1 \quad e^{(j\beta)} \quad e^{(j2\beta)} \quad \dots \quad e^{(j(N_t-1)\beta)}] \quad (3)$$

where  $\beta = \frac{2\pi}{\lambda} \cdot d \cdot \sin(\phi_n)$  and  $\lambda$  being the carrier frequency wavelength. In general, the spacing between the elements of ULA at the BS is not large due to the condition at the antenna site to have zero cross-correlation factor. Therefore, spatial correlation between antenna elements would be effective. This correlation is defined in [10] based on the geometric scenario characterized by AoA. According to [10], the spatial covariance matrix that specifies the spatial correlation between antenna elements is given by:

$$R = \frac{1}{L} \sum_{n=1}^L a(\phi_n) a^H(\phi_n) \quad (4)$$

where  $L$  denotes the number of dominant resolvable signal paths and  $H$  is Hermitian transposition notation. Let  $h_j$  denotes the channel vector between the  $j$ th mobile receive antenna and the BS transmit antennas. It has a form of  $h_j = \sqrt{R} \cdot u_j$  with  $u_j = [u_{1,j} \quad u_{2,j} \quad \dots \quad u_{N_t,j}]^T$ , where  $(.)^T$  denotes vector transpose operation,  $u_{i,j}$  is the channel-fading coefficient between the  $i$ th transmitting antenna (at the BS) and the  $j$ th receiving antenna (at the MS) and is modeled as a zero-mean complex-valued Gaussian process with Jakes power spectrum density.

To maximize the transmitted signal power along the dominant multi-paths, eigen-decomposition of the spatial covariance matrix should be performed, then we apply the resulting antenna weights given by the eigenvector that corresponds to the largest eigenvalue. The eigen-decomposition has the following form:

$$R = V D V^H \quad (5)$$

where  $D = \text{diag}(\mu_1 \quad \mu_2 \quad \dots \quad \mu_{N_t})$  is a diagonal matrix with ordered eigenvalues on the main diagonal and  $V$  is a unitary matrix composed of the corresponding eigenvectors. Hence, the transmit weight vector  $W = [w_1 \quad w_2 \quad \dots \quad w_{N_t}]^T$  can be found is the first column of  $V$  and the transmission on different eigenvectors will lead to uncorrelated channel fading.

## 2.2 Received Signal Model

Let  $H$  be the channel matrix of a dimension of  $N_t \times N_r$ , which is a superposition of the above vectors  $h_j$  as follows:  $H = [h_1 \quad h_2 \quad \dots \quad h_{N_r}]$ . If we transform the antenna weight vector  $W$  to a diagonal matrix,  $W = \text{diag}(w_1 \quad w_2 \quad \dots \quad w_{N_t})$ , and apply  $W^H$  to the space-time encoded codeword  $C$  prior to transmission by the BS antennas, the combined received signal at the MS can be expressed as:

$$Y = C W^H H + \eta \quad (6)$$

where  $\eta$  is the receiver noise matrix and its  $\eta_{i,j}$  denotes additive white Gaussian noise of the MS receive antenna  $j$  at time instance  $i$  with a zero mean and one-sided sample variance of  $N_0/2$ .

In this paper, we assume that the MS receiver has perfect channel knowledge. To decode the received signal matrix, the ML detector evaluate the decision matrix and decides in favor of the codeword that minimize  $\|Y - C W^H H\|_F^2$ , over all possible codeword  $\{C\}$ , where  $\|\cdot\|_F$  denotes the Forbenius norm.

### 3. ERROR PERFORMANCE ANALYSIS

In this section we analyze the error rate performance of the proposed transmission scheme for a time varying channel, where the receiver is assumed to have perfect knowledge of the channel. The system error probability is usually expressed as a function of SNR at the receiver. Therefore, we first find the SNR expression at the output of the maximum ratio combiner (MRC). With this result we then derive the system error probability.

#### 3.1 SNR Expression

We assume that the data sequence  $\{s(n)\}_{n=0}^{N_s-1}$  is phase shift keying (PSK) baseband modulated symbols, each having constant magnitude of  $\sqrt{\varepsilon_s}$ . The variance of the channel coefficients is arbitrarily set at  $\sigma_u^2 = 1$ . In [5] it is shown that the average received SNR at the output of MRC is expressed as  $\bar{\gamma} = E[\|Y\|_F^2] / p \cdot N_t \cdot N_r \cdot \sigma_n^2$ ,  $\sigma_n^2$  denotes the noise variance. Note that the transmit weight vector  $W = [w_1 \ w_2 \ \dots \ w_{N_t}]^T$  represents the dominant eigen-vector of the channel correlation matrix, hence, the product of  $W^H \sqrt{R} W$  equals  $\sqrt{\mu}$  where  $\mu$  is the largest eigen-value of the eigen-decomposition of matrix  $R$ . Following [11], we can express the average SNR as:

$$\begin{aligned} \bar{\gamma} &= \frac{E[\|CW^H H\|_F^2]}{p \cdot N_t \cdot N_r \cdot \sigma_n^2} \\ &= \frac{E[\|C\|_F^2] \cdot E[\|W^H \sqrt{R} W\|_F^2] \cdot E[\|U^H U\|_F^2]}{p \cdot N_t \cdot N_r \cdot \sigma_n^2} \\ &= \frac{\varepsilon_s \cdot \sqrt{\mu} \cdot N_r}{\sigma_n^2} \\ &= \frac{2 \cdot \varepsilon_s \cdot \sqrt{\mu} \cdot N_r}{N_0} \end{aligned} \quad (7)$$

#### 3.2 Probability of Error

If we rewrite the decision matrix at the receiver ML decoder as  $\hat{s}(n) = \arg \min_{\{s(n)\}} \|Y - CW^H H\|_F^2$  and follow the work in [9] and [12], the pairwise error probability (PEP) for the decoder to decide the symbol  $\hat{s}(n)$  in favor of  $s(n)$  can be given by:

$$P_r(s(n) \rightarrow \hat{s}(n)) = Q\left(\sqrt{\frac{|s(n) - \hat{s}(n)|^2}{2\varepsilon_s}} \bar{\gamma}\right) \quad (8)$$

where  $Q(\cdot)$  is the classical definition of the Gaussian  $Q$ -function. Assuming the output symbols  $\{s(n)\}$  to be taken from some 2-D constellation  $S$  and are modulated according to a Gray code, the BER for M-ray PSK modulated signals can be approximated by:

$$P_e \approx Q\left(\sqrt{\frac{2}{\log_2 M}} \bar{\gamma}\right) \quad (9)$$

It is clear in the above equations that as the SNR increases, the error probability decreases. This is also true for the number of receive antennas and the eigen-value,  $\mu$ . A slight variation in performance is observed between theoretical results and simulation. This is due to the fact that random realizations of channel coefficients in the numerical simulation are not always equal to the statistical averaged values used in theoretical analysis. Nevertheless, simulation provides comparable system performance curves.

### 4. NUMERICAL RESULTS AND DISCUSSION

In Fig. 2, we show simulation results of the new transmit diversity scheme that depicts both SER and BER performance with  $N_t = 4$  ULA having element spacing of  $0.5\lambda$  at the base station. Identity matrix  $I$  having a size of  $N_t$  was used for the channel correlation matrix to model a zero spatial correlation between signal paths. A Maximum likelihood decoder with perfect channel knowledge of fading coefficients was used at the MS receiver. 4-PSK for the baseband modulation of information bits is employed at the transmitter and OSTBC encoding and decoding algorithms used for  $N_t = 4$  is  $G_4$  as specified in [12]. The performance curves showed that the diversity improvement in having an additional receive antenna is at an average of 4.5 dB for both SER and BER over a single receive antenna. This is consistent with the simulation results in [11].

A similar performance curve is shown in Fig. 3, where the forward link channel is modeled as macrocell environment with geometrically based hyperbolically distributed scatterers (GBHDS). This channel model is recently proposed in [13]. In this case lower error rate probability is obtained. This proves that the decorrelation process (eigen-decomposition) gives better performance in an environment with a higher degree of spatial channel correlation. The diversity improvement of using two receive antennas over a single antenna is at an average of 5.5dB in the above GBHDS channel model.

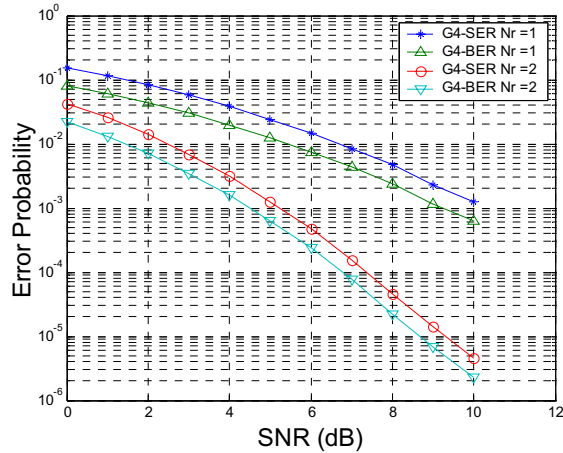


Fig. 2. Simulated error rate performance curves of the new transmit diversity system in zero correlation channel model with four transmitting and different number of receiving antennas.

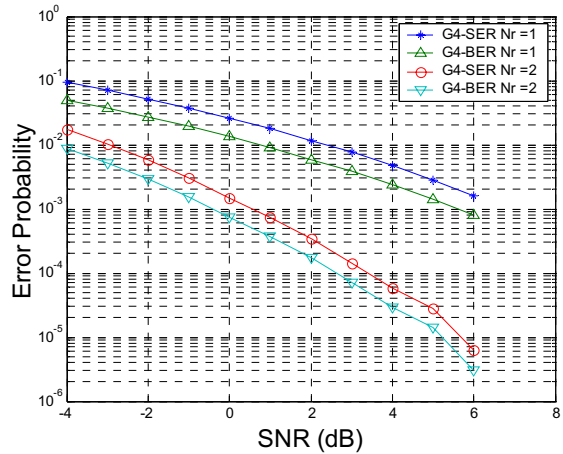


Fig. 3. Simulated error rate performance curves of the new transmit diversity system in GBHDS channel model with four transmitting and different numbers of receiving antennas.

More importantly, if we compare the error rate performance over these two figures, a significant gain in SNR of about 4dB at an error rate of  $10^{-4}$  for  $N_r = 2$  and 4.3dB at an error rate of  $10^{-2}$  for  $N_r = 1$  in both SER and BER is obtained. This means that the system employing the new transmission scheme gives approximately 4dB diversity improvement.

## 5. CONCLUSIONS

In this paper an adaptive transmit diversity scheme that combines eigenbeamforming with orthogonal space-time block coding is proposed. With an effective utilization of spatial signal correlation at the base station, the performance of a predetermined space-time code is improved. Simulation results demonstrated a significant

performance improvement in terms of symbol-error-rate and bit-error-rate over two channel models with different spatial correlations and different number of receiving antennas. Numerical analysis by means of pairwise error probability for the system performance is also presented.

The effects of channel parameters estimation error on the error rate performance of the proposed technique in the presence of Gaussian plus impulse noise are of our next interest. We are currently conducting research in this direction and the results would be presented in the future works.

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