

# DIVERSITY INTERFERENCE CANCELLATION USING PREFILTERING AND REDUCED-STATE MIMO EQUALIZATION

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## ABSTRACT

We consider a joint detection approach for cancellation of co-channel interference in time-division multiple access (TDMA) mobile communications systems like GSM/EDGE (Enhanced Data Rates for GSM Evolution). Concepts from reduced-state equalization of frequency-selective multiple-input multiple-output (MIMO) channels are applied together with prefiltering. A novel efficient prefilter computation algorithm is presented. Simulation results demonstrate the high performance of the proposed receiver for GSM/EDGE applications.

## 1. INTRODUCTION

With co-channel interference cancellation, the system capacity of GSM/EDGE (Enhanced Data Rates for GSM Evolution) can be significantly increased, cf. [1]. Therefore, interference cancellation receiver algorithms are of high current interest. In this paper, we consider a multiuser (joint) detection approach for cancellation of co-channel interference in receivers with antenna diversity, which is suited for synchronized networks where the channel impulse responses (CIRs) of both the desired signal and the interferers can be estimated. Synchronized networks are already in service and are expected to play a dominant role in the future. If all CIRs are available at the receiver, multiple-input multiple-output (MIMO) equalizers are applicable which may be viewed as multiuser detectors and yield a better performance than simple linear suppression of interference. For example, MIMO decision-feedback equalization (DFE) schemes [2, 3] may be employed. The optimum maximum-likelihood receiver for MIMO systems [4] in general has a significantly better performance than a DFE but is too complex for implementation. As in the single-input single-output (SISO) case (no interferers), reduced-state trellis-based equalization schemes like MIMO delayed decision-feedback sequence estimation (DDFSE) [5] may close the gap between DFE and the optimum receiver at a quite low complexity. For SISO reduced-state equalization, it is well

known that appropriate prefiltering is required for high performance. For the MIMO case, prefiltering has been considered in [6, 5]. The feedforward filter of a MIMO minimum mean-squared error (MMSE) DFE is well suited for prefiltering and can be computed after channel estimation via the solution of a system of equations. However, a high computational complexity results for this approach because the matrix to be inverted is not well structured.

In the following, we first state some properties of MIMO minimum- and maximum-phase systems and show their relation to reduced-state equalization. Based on these results, a novel algorithm for MIMO prefilter computation is given which has a lower complexity than the MMSE-DFE approach and is well suited for an implementation. Finally, simulation results for GSM/EDGE for the practically most interesting case of one dominant co-channel interferer demonstrate the high performance of a MIMO DDFSE with the proposed prefiltering approach.

## 2. TRANSMISSION MODEL, MINIMUM- AND MAXIMUM-PHASE EQUIVALENT SYSTEM

The discrete-time equivalent complex baseband representation of a transmission over a frequency-selective MIMO channel according to Fig. 1 is considered. We assume a

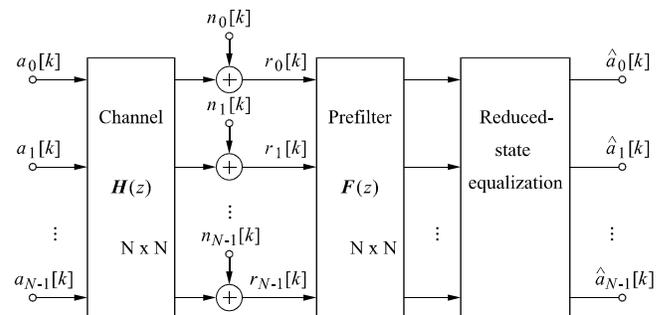


Fig. 1. Transmission Model.

square system with  $N > 1$  i.i.d. and mutually independent input signals  $a_i[k]$ ,  $i \in \{0, 1, \dots, N-1\}$ , each with variance  $\sigma_a^2$  corresponding to one desired signal and several co-channel interferers from neighboring cells (each signal corresponds to one transmit antenna).  $N$  output signals  $r_i[k]$ ,  $i \in \{0, 1, \dots, N-1\}$ , corresponding to  $N$  receive antennas are available. The given system model results e.g. when a receiver with antenna diversity has to process the superpositions of a number of signals originating from transmitters at different locations, where each transmitter is equipped with a single antenna. Therefore, the proposed receiver is mainly interesting for an uplink transmission (mobile stations to base station). In the downlink, receive diversity is usually not available due to cost and size limitations. Collecting all received signals in a vector  $\mathbf{r}[k] = [r_0[k] \ r_1[k] \ \dots \ r_{N-1}[k]]^T$  ( $(\cdot)^T$ : transposition), the input–output relation of the MIMO channel reads

$$\mathbf{r}[k] = \sum_{\kappa=0}^{q_h} \mathbf{H}[\kappa] \mathbf{a}[k - \kappa] + \mathbf{n}[k], \quad (1)$$

with  $\mathbf{a}[k] = [a_0[k] \ a_1[k] \ \dots \ a_{N-1}[k]]^T$  and  $\mathbf{n}[k] = [n_0[k] \ n_1[k] \ \dots \ n_{N-1}[k]]^T$ .  $\mathbf{H}[\cdot]$  denotes the causal FIR impulse response of order  $q_h$  of the overall channel including continuous–time transmit and receive filtering, and  $n_i[\cdot]$  is the additive Gaussian noise process of the  $i$ th receive antenna with variance  $\sigma_n^2$ . The noise processes are assumed to be spatially and temporally white,  $E\{\mathbf{n}[k + \kappa] \mathbf{n}^H[k]\} = \sigma_n^2 \mathbf{I}_N \delta[\kappa]$  ( $E\{\cdot\}$ : expectation,  $(\cdot)^H$ : Hermitian transposition,  $\mathbf{I}_N$ :  $N \times N$  identity matrix,  $\delta[\kappa]$ : unit pulse sequence). The transfer function representing the discrete–time overall MIMO channel is denoted by  $\mathbf{H}(z) = \sum_{\kappa=0}^{q_h} \mathbf{H}[\kappa] z^{-\kappa}$ . Reduced–state, trellis–based MIMO equalization using, for example, a MIMO DDFSE [5] is employed in conjunction with a suitable front–end MIMO prefilter  $\mathbf{F}(z)$ .

We now define the *minimum–phase equivalent*  $\mathbf{H}_{min}(z)$  of  $\mathbf{H}(z)$  via

$$\mathbf{H}^H(1/z^*) \mathbf{H}(z) = \mathbf{H}_{min}^H(1/z^*) \mathbf{H}_{min}(z), \quad (2)$$

where  $\det(\mathbf{H}_{min}(z))$  ( $\det(\cdot)$ : determinant of a matrix) has roots only inside the unit circle. Hence, the causal and stable transfer matrix  $\mathbf{H}_{min}(z)$  may be obtained from spectral factorization of  $\mathbf{H}^H(1/z^*) \mathbf{H}(z)$ , which exists for most cases of practical interest [7]. It should be noted that the solution is unique only up to a unitary matrix factor. MIMO spectral factorization algorithms are in general characterized by a huge computational complexity [7].  $\mathbf{H}_{min}^{-1}(z) = 1/\det(\mathbf{H}_{min}(z)) \cdot \text{adj}(\mathbf{H}_{min}(z))$  ( $\text{adj}(\cdot)$ : adjoint matrix) has poles only inside the unit circle and can be realized with a causal and stable filter, in contrast to  $\mathbf{H}^{-1}(z)$ .

Consider now the dual spectral factorization

$$\mathbf{H}^H(1/z^*) \mathbf{H}(z) = \mathbf{H}_{max}^H(1/z^*) \mathbf{H}_{max}(z), \quad (3)$$

where the *maximum–phase equivalent*  $\mathbf{H}_{max}(z)$  is anticausal and stable and  $\det(\mathbf{H}_{max}(z))$  has roots only outside

the unit circle.  $\mathbf{H}_{max}(z)$  can be obtained from a factorization of  $\mathbf{H}^T(z) \mathbf{H}^*(1/z^*)$  for a minimum–phase equivalent  $\overline{\mathbf{H}}_{min}(z)$ ,  $\mathbf{H}_{max}(z) = \overline{\mathbf{H}}_{min}^*(1/z^*)$ . Due to the noncommutativity of matrix multiplication,  $\overline{\mathbf{H}}_{min}(z) \neq \mathbf{H}_{min}(z)$ <sup>1</sup> (unlike the SISO case), cf. also [8].

Similar to the SISO case, energy concentration properties hold for MIMO minimum– and maximum–phase transfer functions. For any causal and stable  $\mathbf{H}(z)$ , it can be shown that

$$\sum_{\kappa=0}^k \|\mathbf{H}_{min}[\kappa]\|_F^2 \geq \sum_{\kappa=0}^k \|\mathbf{H}[\kappa]\|_F^2, \quad 0 \leq k \leq q_h, \quad (4)$$

( $\|\cdot\|_F$ : Frobenius norm of a matrix), i.e.,  $\mathbf{H}_{min}[\cdot]$  has a better energy concentration in the front part than  $\mathbf{H}[\cdot]$ . Similarly,  $\mathbf{H}_{max}[\cdot]$  has a better energy concentration in the back part than  $\mathbf{H}[\cdot]$ . Due to these energy concentration properties, prefiltering for an overall transfer function  $\mathbf{H}_{min}(z)$  is desirable for reduced–state equalization in the forward time direction because in this case, only the front part of the impulse response is used for trellis definition. Furthermore, prefiltering for an overall transfer function  $\mathbf{H}_{max}(z)$  is desirable for backward equalization (in reverse time direction, starting e.g. from a training sequence located in the middle of a transmission burst (midamble)).

### 3. MIMO PREFILTER COMPUTATION

In order to develop a prefilter computation algorithm with low complexity, we propose for prefiltering a cascade of a channel matched filter  $\mathbf{H}^H(1/z^*)$  which immediately results if the CIR matrix is known and a one–step prediction–error filter  $\mathbf{P}(z)$  for the noise at the output of the matched filter.

#### A. Backward prediction–error filter

For the case that  $\mathbf{P}(z)$  is chosen as a backward prediction–error filter, the  $z$ –transform of the autocorrelation sequence of the prediction error  $e_1[k]$  (output signal of  $\mathbf{P}(z)$ ) is written in the form

$$\begin{aligned} \Phi_{e_1 e_1}(z) &= \sigma_n^2 \mathbf{P}(z) (\mathbf{H}_{min}^H(1/z^*) \mathbf{H}_{min}^{-H}[0]) \mathbf{H}_{min}^H[0] \\ &\quad \cdot \mathbf{H}_{min}[0] (\mathbf{H}_{min}^{-1}[0] \mathbf{H}_{min}(z)) \mathbf{P}^H(1/z^*), \end{aligned} \quad (5)$$

where (2) has been used. Exploiting the fact that the optimum backward prediction–error filter is an anticausal and monic whitening filter, its transfer matrix can be obtained from (5) as

$$\mathbf{P}_b(z) = \mathbf{H}_{min}^H[0] \mathbf{H}_{min}^{-H}(1/z^*), \quad (6)$$

resulting in an overall prefilter transfer function

$$\mathbf{F}(z) = \mathbf{H}_{min}^H[0] \mathbf{H}_{min}^{-H}(1/z^*) \mathbf{H}^H(1/z^*) \quad (7)$$

$$= \mathbf{H}_{min}^H[0] \mathbf{H}_{min}(z) \mathbf{H}^{-1}(z), \quad (8)$$

<sup>1</sup>Both matrices differ in more than a unitary factor.

where from (7) to (8) again (2) has been used. Hence, for an infinite order of the prediction–error filter, the optimum prefilter results (up to a constant matrix factor) transforming the channel into its minimum–phase equivalent. For finite filter orders, a reasonable FIR approximation thereof is obtained.

### B. Forward prediction–error filter

In case of a forward prediction–error filter, we use the representation

$$\Phi_{e_1 e_1}(z) = \sigma_n^2 \mathbf{P}(z) (\mathbf{H}_{max}^H(1/z^*) \mathbf{H}_{max}^{-H}[0] \mathbf{H}_{max}^H[0] \cdot \mathbf{H}_{max}[0] (\mathbf{H}_{max}^{-1}[0] \mathbf{H}_{max}(z)) \mathbf{P}^H(1/z^*)) \quad (9)$$

for  $\Phi_{e_1 e_1}(z)$ , where (3) has been used. Therefore, the optimum causal and monic forward prediction–error filter is

$$\mathbf{P}_f(z) = \mathbf{H}_{max}^H[0] \mathbf{H}_{max}^{-H}(1/z^*), \quad (10)$$

resulting in an overall prefilter transfer function

$$\mathbf{F}(z) = \mathbf{H}_{max}^H[0] \mathbf{H}_{max}^{-H}(1/z^*) \mathbf{H}^H(1/z^*) \quad (11)$$

$$= \mathbf{H}_{max}^H[0] \mathbf{H}_{max}(z) \mathbf{H}^{-1}(z). \quad (12)$$

Once the optimum prediction–error filters (or reasonably accurate approximations thereof) have been obtained, the minimum– and maximum–phase equivalent systems can be calculated up to constant matrix factors without spectral factorization procedures. From (6) we obtain

$$\mathbf{H}_{min}(z) = \mathbf{H}_{min}[0] \mathbf{P}_b^{-H}(1/z^*), \quad (13)$$

and from (10)

$$\mathbf{H}_{max}(z) = \mathbf{H}_{max}[0] \mathbf{P}_f^{-H}(1/z^*). \quad (14)$$

The obtained results for infinite–length filters suggest the following approach for FIR prefilter computation with low complexity. An FIR prediction–error filter is designed for the noise at the output of the matched filter. Because the optimum prefilter results in the limiting case of infinite filter length, a reasonable approximation to the optimum prefilter can be expected if the order of the FIR filter is selected sufficiently high. FIR prediction–error filters may be realized in a lattice structure, cf. Fig. 2, whose coefficient matrices can be calculated with the MIMO Levinson algorithm [8]. For this, only the autocorrelation sequence of the matched filter output noise is required which can be immediately calculated after channel estimation. In Fig. 2,  $\mathbf{P}_f^i(z)$  and  $\mathbf{P}_b^i(z)$  denote the transfer functions of forward and backward prediction–error filter of order  $i$ , respectively, which can be calculated recursively according to

$$\begin{bmatrix} \mathbf{P}_f^{i+1}(z) \\ \mathbf{P}_b^{i+1}(z) \end{bmatrix} = \begin{bmatrix} \mathbf{I}_N & -\mathbf{K}_{i+1}^r \\ -\mathbf{K}_{i+1}^e & \mathbf{I}_N \end{bmatrix} \begin{bmatrix} \mathbf{P}_f^i(z) \\ z^{-1} \mathbf{P}_b^i(z) \end{bmatrix} \quad (15)$$

where the recursions for the reflection coefficient matrices  $\mathbf{K}_i^r$  and  $\mathbf{K}_i^e$ <sup>2</sup> are given in [8].

<sup>2</sup>Here, the notation of [8] has been used.

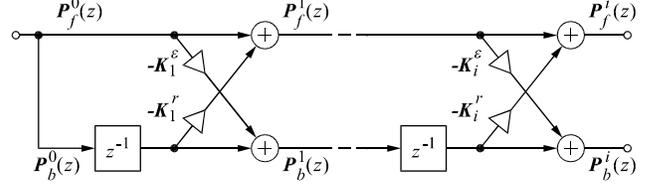


Fig. 2. Lattice prediction–error filter.

The application of the MIMO Levinson algorithm enables an adaptive selection of the prefilter order according to the given channel because the algorithm calculates the forward and backward prediction–error zero–lag autocorrelation matrices  $\mathbf{R}_i^e$  and  $\mathbf{R}_i^r$ , respectively, for each considered order  $i$ . Hence, the trace of  $\mathbf{R}_i^e$  or  $\mathbf{R}_i^r$  (depending on the selected time direction of prediction) is monitored, and the algorithm may be stopped if it changes only slightly from one iteration to the next.

The unknown factors  $\mathbf{H}_{min}^H[0]$  and  $\mathbf{H}_{max}^H[0]$  in (8) and (12) can be obtained (up to unitary matrices) from an eigen-decomposition of  $\mathbf{R}_{q_p}^r$  and  $\mathbf{R}_{q_p}^e$ , respectively ( $q_p$ : finally selected predictor order). Hence, the prefiltered signals may be multiplied with the corresponding inverse matrices before reduced–state equalization in order to remove spatial correlations of the noise.

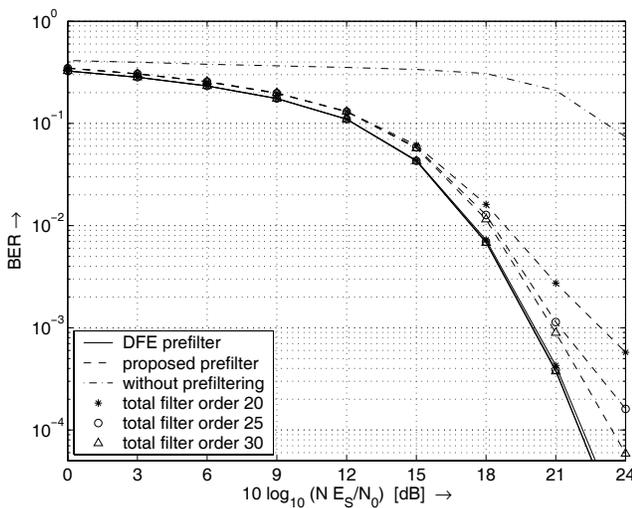
## 4. NUMERICAL RESULTS

For numerical results, we consider a GSM/EDGE uplink transmission with one desired signal and one interferer. For EDGE, 8–ary phase–shift keying (8PSK) and Gaussian minimum–shift keying (GMSK) modulation, respectively, have been adopted. The latter can be modeled as filtered binary phase–shift keying (BPSK) modulation. Several channel power delay profiles have been specified for performance tests. First, the equalizer test (EQ) profile is selected, which may be viewed as a worst case scenario for equalization. Fig. 3 shows the bit error rate (BER) of the desired signal versus  $10 \log_{10}(N E_s/N_0)$  ( $E_s$ : average received energy per symbol and antenna,  $N_0$ : single–sided power spectral density) for 8PSK modulation and a MIMO DDFSE with 64 states for multiuser detection. Desired signal and interferer have the same average energy (carrier–to–interference ratio (CIR):  $10 \log_{10}(\text{CIR}) = 0$  dB), and ideal knowledge of the subchannel impulse responses at the receiver has been assumed, which are mutually statistically independent. For prefiltering, an FIR MIMO MMSE–DFE feedforward filter and the proposed FIR prefilter, respectively, with various total orders have been employed. It can be observed that the performance of the proposed prefilter tends to that of the DFE feedforward filter for moderate–to–high filter orders. Also, results are given for a DDFSE without prefiltering which indicate that prefiltering is mandatory.

The typical urban (TU) channel profile is more realistic for practical applications. Fig. 4 shows BER for TU and GMSK modulation. The cases  $10 \log_{10}(\text{CIR}) = 0$  dB

and  $10 \log_{10}(\text{CIR}) = 15 \text{ dB}$ , respectively, are considered. Again, prefiltering using a DFE feedforward filter is compared to the proposed prefiltering approach (total filter order in both cases: 30). Performance of a MIMO DDFSE with 4 states is only slightly worse if the proposed prefilter is adopted instead of the DFE feedforward filter, while complexity of prefilter calculation is significantly reduced. Using DDFSE in conjunction with the proposed prefilter, the performance of the optimum maximum-likelihood (ML) receiver (also depicted in Fig. 4) is approached up to 1 dB.

It should be noted that complexity of equalization can be further reduced without sacrificing performance if reduced-state sequence estimation (RSSE) is used instead of DDFSE, cf. [5, 9].



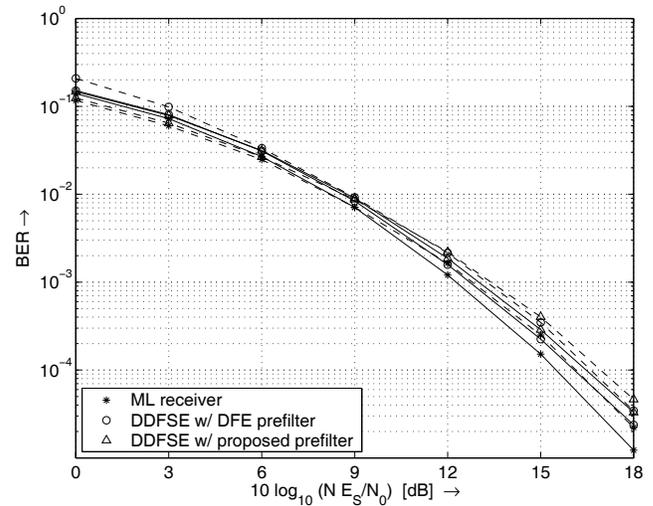
**Fig. 3.** BER vs.  $10 \log_{10}(N E_s/N_0)$  for 8PSK transmission over EQ channel. DDFSE with 64 states,  $10 \log_{10}(\text{CIR}) = 0 \text{ dB}$ .

## 5. CONCLUSIONS

For joint detection of a desired signal together with co-channel interferers in time-division multiple access (TDMA) systems like GSM/EDGE, MIMO DDFSE combined with a prefilter computed with a novel algorithm has been considered. The approach can be employed whenever antenna diversity is available at the receiver (especially in the uplink) and when the impulse responses of all subchannels can be estimated which is the case in synchronized networks.

## 6. REFERENCES

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**Fig. 4.** BER vs.  $10 \log_{10}(N E_s/N_0)$  for GMSK transmission over TU channel. DDFSE with 4 states and ML receiver,  $10 \log_{10}(\text{CIR}) = 0 \text{ dB}$  (solid lines),  $15 \text{ dB}$  (dashed lines).

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