A Low Complexity VBLAST OFDM Detection Algorithm

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Abstract— **In this paper, we propose a low complexity VBLAST OFDM detection algorithm. In this method, we partition the subcarriers into a number of groups. The group size is determined by the frequency correlation between different subcarriers. Within each group, we use conventional VBLAST detection on the centre subcarrier and record the detection order. For the rest of the subcarriers in the group, we use QR decomposition technique according to the recorded detection order. We show that this algorithm significantly reduces the complexity of VBLAST OFDM detection. The performance degradation of the proposed algorithm is small compared to conventional detection.**

I. Introduction

It was shown in [1] that large capacity can be exploited in rich scattering wireless channels by employing multiple antennas at both the transmitter and the receiver. The BLAST (Bell LAb's Layered Space Time)[1] structure was proposed to realize such systems. However, the complexity of the diagonal BLAST structure makes it difficult to implement. A simpler version, the vertical BLAST (VBLAST) [2] [3] was proposed and achieved spectral efficiency of 20 to 40 bps/Hz at average SNR's between 24 to 34dB.

The conventional VBLAST detection employs an ordered serial nulling plus cancellation technique [3]. The optimal detection order is determined to maximize the minimum post-detection SNR of all data streams. This algorithm is computation intensive as a number of pseudo inverse operations need to be performed.

VBLAST can be combined with OFDM to achieve high data rate transmission in frequency selective fading channels [4]. As OFDM effectively divides the frequency selective channel into a number of flat fading subchannels, the VBLAST OFDM system comprises a number of narrow band VBLAST systems on different subcarriers. Therefore, the detection of an $M \times N$ VBLAST OFDM system requires taking N pseudo-inverses on each subcarrier, which is even more complex. To reduce the complexity of VBLAST detection, in [6], the authors porposed Gram Schmidt Orthogonalization (GSO) algorithm with a sub-optimal detection ordering. This scheme reduces the complexity of VBLAST detection with small degradation to the performance. However, it still involves taking one pseudo-inverse at each subcarrier to determine the sub-optimal detection order. In [7], a simplified detection scheme for VBLAST OFDM system was presented. In this method, it is assumed that the nulling vectors change only slightly within a small number of subcarriers. Hence, the same nulling vector can be used for all these subcarriers. However, the degradation in BER performance of this method is rather high. In addition, the authors did not provide specific rule in determining the group size.

In this paper, we propose a new low complexity detection algorithm for VBLAST OFDM system. In this algorithm, the subcarriers are partitioned into a number of groups such that the correlation among subcarriers within each group is above a threshold value referred as *threshold correlation* (TC). For each group, conventional VBLAST detection is first performed on the center subcarrier and the detection order is recorded. Following the recorded order, the other subcarriers in the same group are detected using QR decomposition technique as in [6]. We show that this algorithm reduces the complexity of VBLAST OFDM detection significantly. Moreover, the performance degradation of the proposed algorithm is small.

In this paper, we use the following notations: vectors and matrices are denoted as bold lower and upper case letters respectively. All vectors are column vectors. We use the superscripts ^{H} and * to denote matrix Hermitian and conjugation. The elements of vectors/matrices are denoted by letters with subscripted indices.

II. VBLAST OFDM detection by QR decomposition

A VBLAST OFDM system with N transmit and M receive antennas effectively consists of one narrow band VBLAST system on each subcarrier. Therefore, the received signal vector on a particular subcarrier can be written as

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{H}$ is a $M \times N$ matrix with $H_{i,j}$ representing the channel response between transmit antenna j and receive antenna i on that subcarrier. Vector $\mathbf{x}$ is the transmitted signal and $\mathbf{n}$, the AWGN noise vector. As same detection algorithm is used on each subcarrier, we omit the subcarrier index in the above formulation.

Assume the optimal detection order for the narrow band VBLAST system is $\mathbf{p} = [p_1, p_2, \cdots, p_N]$, which is a permutation of $[1, 2, \cdots, N]$, we can rewrite (1) as

$$\mathbf{r} = \mathbf{H}_\mathrm{p}\mathbf{x}_\mathrm{p} + \mathbf{n}_\mathrm{p}, \tag{2}$$

where the subscript p denotes permutation with

$$\mathbf{H}_\mathrm{p} = [\mathbf{h}_{p_N}, \mathbf{h}_{p_{N-1}}, \cdots, \mathbf{h}_{p_2}, \mathbf{h}_{p_1}],$$

and

$$\mathbf{x}_\mathrm{p} = [x_{p_N}, x_{p_{N-1}}, \cdots, x_{p_2}, x_{p_1}]^\mathrm{T}.$$

TABLE I

COMPUTATION COMPLEXITY FOR CALCULATION OF QR DECOMPOSITION AND PSEUDO-INVERSE OF AN $M \times N$ MATRIX.

operations	QR decomposition	Pseudo Inverse
add	$MN^2-\frac{N^2}{2}-\frac{N}{2}$	$3MN^2+2M^2N+\frac{8}{3}N^3+5N^2-4MN-\frac{8}{3}N-7$
mul	MN^2	$5MN^2+2M^2N+\frac{14}{3}N^3+23N^2+4MN-\frac{44}{3}N-4M-21$
div	MN	$MN+4N^2+3N-7$
sqrt	N	$8N^2-4N-4$

Perform QR decomposition on $\mathbf{H}_\mathrm{p}$, we have $\mathbf{H}_\mathrm{p} = \mathbf{Q} \times \mathbf{R}$, where $\mathbf{Q}$ is an unitary matrix and $\mathbf{R}$ is an upper triangular matrix. Pre-multiply $\mathbf{r}$ by $\mathbf{Q}^\mathrm{H}$, we get

$$\begin{aligned}\mathbf{d} = \mathbf{Q}^\mathrm{H}\mathbf{r} &= \mathbf{Q}^\mathrm{H}\mathbf{H}_\mathrm{p}\mathbf{x}_\mathrm{p} + \mathbf{Q}^\mathrm{H}\mathbf{n}_\mathrm{p} \\ &= \mathbf{R}\mathbf{x}_\mathrm{p} + \mathbf{n}'. \end{aligned} \quad (5)$$

As $\mathbf{Q}$ is a unitary matrix, $\mathbf{n}'$ is still AWGN. The detection of individual elements of $\mathbf{x}$ can be performed using standard procedures. It is noted that once the detection order $\mathbf{p}$ is determined, we only need to perform QR decomposition once on each subcarrier rather than taking pseudo-inverse N times in conventional detection. Moreover, using the same detection order, the BER using QR decomposition technique is the same as conventional VBLAST detection.

Table I shows the number of operations required to calculate the QR decomposition of an $M \times N$ matrix in comparison with operation counts for pseudo-inverse calculation. We use Modified Gram-Schmidt method [5], which is numerically more stable than classical GSO, to compute QR decomposition. To compute the pseudo inverse of $\mathbf{H}$, we first perform singular value decomposition on $\mathbf{H}$ using Golub-Reinsch algorithm [5] [10], such that $\mathbf{H}=\mathbf{U}\mathbf{\Lambda}\mathbf{V}^\mathrm{H}$. The pseudo inverse of $\mathbf{H}$ is then calculated as $\mathbf{H}^+ = \mathbf{V}\mathbf{\Lambda}^+\mathbf{U}^\mathrm{H}$, where $\mathbf{\Lambda}^+$ is formed by inverting all the nonzero values in $\mathbf{\Lambda}$.

It is clearly shown in Table I that QR decomposition incurs much less complexity compared to pseudo-inverse. Moreover, as we only need to calculate QR decomposition once rather than taking pseudo-inverse N times, this results in another large saving in complexity [6].

III. LOW COMPLEXITY VBLAST OFDM DETECTION ALGORITHM

We have shown in Section II that once the detection order is given, the QR decomposition technique reduces the complexity of VBLAST OFDM detection significantly. However, determining the optimal detection order requires taking pseudo-inverse N times. In this section, we present a method to determine a sub-optimal detection order, making use of the correlation among different subcarriers.

For VBLAST OFDM system, the channels between different transmit and receive antennas are statistically the same. Therefore, we only need to study one channel and the same properties apply to all the other channels. The impulse response of multipath channel can be written as $\mathbf{h} = [h_1, h_2, \cdots, h_{L+1}]^T$, where L is the channel order. The frequency response of the channel for a K subcarrier OFDM system is given by

$$\mathbf{h}_\mathcal{F} = \mathbf{F}\mathbf{h}, \quad (6)$$

where $\mathbf{F}$ is a $K \times (L+1)$ matrix [1] with $F_{i,j} = \frac{1}{\sqrt{K}} \exp\left(-j2\pi\frac{(i-1)(j-1)}{K}\right)$.

The correlation coefficient of the frequency response at subcarrier m and n can be worked out as

$$\rho_{m,n} = \frac{\mathbf{f}_m^H \mathrm{E}(\mathbf{h}\mathbf{h}^H)\mathbf{f}_n}{\sqrt{\mathbf{f}_m^H \mathrm{E}(\mathbf{h}\mathbf{h}^H)\mathbf{f}_m \mathbf{f}_n^H \mathrm{E}(\mathbf{h}\mathbf{h}^H)\mathbf{f}_n}}, \quad (7)$$

where $\mathbf{f}_m^H$ denotes the mth row of $\mathbf{F}$ and $\mathrm{E}(x)$ denotes statistical expectation. For uncorrelated scattering channel, different paths are uncorrelated, (7) can be further simplified to

$$\rho_{m,n} = \frac{K \sum_{i=1}^{L+1} \sigma_i^2 F_{m,i} F_{n,i}^*}{\sum_{i=1}^{L+1} \sigma_i^2}, \quad (8)$$

where $\sigma_i^2 = \mathrm{E}\left(h_i h_i^*\right)$. Due to the symmetrical properties of the DFT matrix, it can be shown that the correlation coefficient $\rho_{m,n}$ depends only on the absolute values of $m - n$.

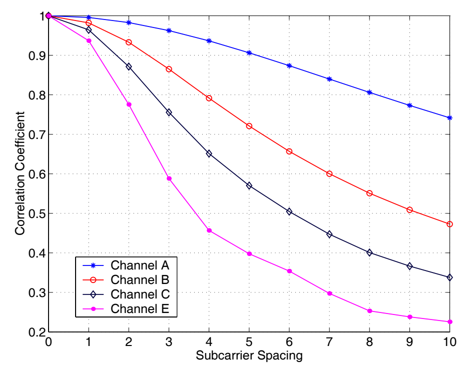

Fig. 1. Correlation coefficient between neighboring subcarriers.

Figure 1 shows the correlation coefficients among neighbouring subcarriers for non line of sight (NLOS) HiperLAN/2 channel models A, B, C and E given in [8]. The power delay profiles of the channels are exponential, with root mean square (RMS) delay spread ranging from $50ns$ (channel A) to $250ns$ (channel E). The maximum delay spread for all the channels is $800ns$, which corresponds to 16 channel taps for a 20MHz bandwidth sample spaced channel. Here, We use a 64-subcarrier OFDM system with 16-tap cyclic prefix. From Figure 1, we can see that the correlation is high among neighbouring subcarriers for all the channels. This implies that the channel changes only slightly within a number of subcarriers. Hence, it is reasonable to assume the optimal detection order does not change within a group of subcarriers, among which the correlation is above TC. Once TC is set,

[1] $\mathbf{F}$ is actually the first $L+1$ columns of the Discrete Fourier Transform (DFT) matrix of size K.

from Figure 1, we can determine the size of the group, in which the same detection order is used. Denoting the group size as k, we propose the following low complexity detection algorithms for VBLAST OFDM system:

1) Pre-determine the value of TC, and hence the group size k for the channel of interest, based on the statistical knowledge of the channel correlation;
2) Partition the subcarriers into groups of size k;
3) Perform conventional VBLAST detection on the center subcarrier within a group and record the optimal detection order $\mathbf{p}$;
4) Perform detection on subcarriers in the same group using QR decomposition technique according to detection order $\mathbf{p}$;
5) Repeat steps 3-4 until all the groups are detected.

Since QR decomposition technique requires much less complexity compared to conventional VBLAST detection, large reduction in complexity can be achieved using the proposed algorithm.

It is possible that the optimal detection order changes within the group. This leads to degradation in the performance of the proposed algorithm compared to conventional VBLAST OFDM detection. Therefore, there is a tradeoff on the choice of TC. The larger TC is, the less probable that the optimal detection order changes within the group. The degradation in performance is thus, smaller. On the other hand, larger TC results in smaller group size k, hence, less reduction in complexity.

IV. Computation complexity comparison

In this section, we compare the amount of arithmetic operations needed to determine the nulling vectors and detection order for the conventional detection and the proposed low complexity algorithm for an $M \times N$ VBLAST OFDM system. We listed the complexity reduction in different complex arithmetic operations. The reduction in total amount of floating point operations (flops) is also given. As we are dealing with complex numbers here, addition is counted as 2 flops, multiplication and division are counted as 6 flops each and square root is counted as 10 flops. We will also show some numerical results on the reduction in complexity for two specific VBLAST OFDM systems with 4×4 and 6×4 antenna configurations.

We denote the total operation counts for conventional VBLAST detection on one subcarrier as $\mathcal{N}$. We also define the operation counts by using QR decomposition technique on one subcarrier as $\mathcal{Q}$. The total number of operations required for conventional VBLAST OFDM detection for a group of k subcarriers is equal to $k \times \mathcal{N}$. By using the low complexity algorithm we proposed, the total number of operation counts is equal to $\mathcal{N} + (k-1) \times \mathcal{Q}$. Based on this calculation, we listed in Table II the complexity reduction to detect a group of k subcarriers using the proposed method compared with conventional VBLAST OFDM detection. We use channel A as the channel model. Setting $\mathrm{TC} = 0.9$ results in group size of $k = 6$ and $\mathrm{TC} = 0.8$ results in $k = 9$.

From Table II, we can see that the complexity can be reduced by about 80% for all the cases for channel A. The reduction is less for less correlated channels. However, according to our calculation, even for the least correlated channel E with TC=0.9, we can still achieve complexity reduction of 47.10% for 4×4 systems and 46.59% for 6×4 systems.

Recently, a number of MIMO channel models were proposed in 802.11n documents [12]. The frequency correlation for the proposed MIMO channels is higher compared to the channel A to E we studied above, due to smaller RMS delay spread. Therefore, for those channel, the proposed detection scheme is able to achieve larger reduction in complexity.

V. Simulation results

Simulations are carried out to study the BER of the proposed algorithm. The transmit power is evenly distributed on each transmit antenna with total power equal to 1. We assume the channels between different antennas are uncorrelated. To estimate the channel, we send N orthogonal preambles from N transmit antennas before the transmission of real data. The preamble is designed according to [9] and least square method is used for channel estimation. We compare the performance of the conventional VBLAST OFDM detection with the proposed low complexity algorithm for $\mathrm{TC} = 0.9$ and $\mathrm{TC} = 0.8$ respectively. From the simulations using different channel models, we observed the performance degradation depends only on the value of TC, not on the channel models used. Therefore, we use channel A in all the simulations in this paper.

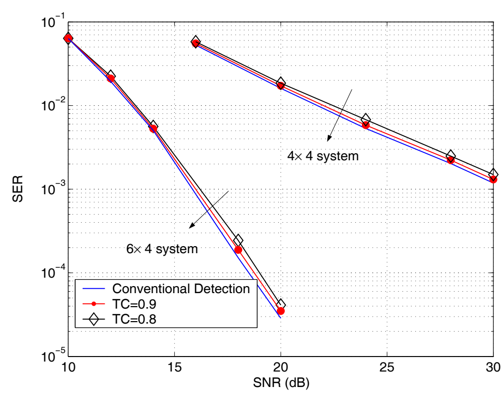

Fig. 2. Symbol error rate comparison of conventional VBLAST OFDM detection with low complexity algorithm. (QPSK modulation)

For the uncoded case, we performed simulations for 4×4 and 6×4 VBLAST OFDM systems with QPSK modulation. Figure 2 shows the performance comparison in SER for the conventional VBLAST OFDM detection and the proposed low complexity algorithm. For a 4×4 system, the degradation using the low complexity method is only 0.5dB for $\mathrm{TC} = 0.9$. The degradation for $\mathrm{TC} = 0.8$ is 1dB. For a 6×4 system, the degradation for TC of 0.9 is 0.5 dB at $\mathrm{SER} = 10^{-4}$.

TABLE II

COMPLEXITY REDUCTION USING THE PROPOSED ALGORITHM COMPARED TO CONVENTIONAL VBLAST OFDM DETECTION

	$\mathcal{N}$	$\mathcal{Q}$	**Reduction** ($k=6$)		**Reduction** ($k=9$)	
			4×4	6×4	4×4	6×4
mul	$5MN^3+2M^2N^2+\frac{14}{3}N^4+23N^3 +4MN^2-\frac{44}{3}N^2-4MN-21N$	MN^3	78.41 %	77.73 %	83.64 %	82.91 %
add	$3MN^3+2M^2N^2+\frac{8}{3}N^4+5N^3 -4MN^2-\frac{8}{3}N^2-7N$	$MN^3-\frac{1}{2}N^3-\frac{1}{2}N^2$	74.13 %	73.28 %	79.07 %	78.17 %
div	$MN^2+4N^3+3N^2-7N$	MN	79.41 %	77.96 %	84.71 %	83.15 %
sqrt	$8N^3-4N^2-4N$	N	82.56%	82.56 %	88.07 %	88.07 %
total	$36MN^3+16M^2N^3+\frac{100}{3}N^4+252N^3 +22MN^2-\frac{346}{3}N^2-24MN-222N$	$6MN^3+2MN^3-N^3 -N^2+6MN+10N$	78.50 %	77.64 %	83.73 %	82.82 %

The degradation for TC of 0.8 is only 0.8 dB. Therefore, we can claim that for both cases, the degradation of the proposed algorithm is very small.

We performed simulations for coded VBLAST OFDM systems as well. According to [2], the error control coding is applied to each MIMO transmit branch independently and the decoding is also performed in an independent manner. The error control coding used in the simulation is a half-rate convolutional code with constraint length of 7 and generator polynomials $g_0 = 133_8$ and $g_1 = 171_8$. Interleaver design follows IEEE802.11a standard [11]. We use hard decision Viterbi algorithm to decode the received data.

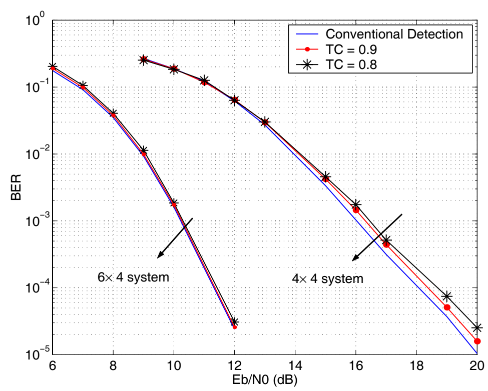

Fig. 3. BER comparison of conventional VBLAST OFDM detection with low complexity algorithm. (QPSK modulation, 1/2 convolutional code)

Figure 3 shows the BER comparison for the conventional VBLAST OFDM detection and the proposed low complexity algorithm for a 4×4 and a 6×4 system respectively. We can see that with the help of error control coding, we can further reduce the performance degradation. For both systems, the degradation in BER using the proposed method is only marginal compared to the conventional method.

In practise, it is not necessary to obtain the explicit values of the correlation at the receiver. We can always use a pessimistic estimate of the group size, i.e. smaller group size, for the chosen TC. Although smaller group size results in less reduction in complexity, it guarantees good performance of the system.

VI. CONCLUSION

We present a low complexity VBLAST OFDM detection algorithm, which determines a common detection order for a group of subcarriers by employing conventional VBLAST detection on the center subcarrier in the group. The detection on the other subcarriers is carried out using QR decomposition technique. This algorithm significantly reduces the complexity of the VBLAST OFDM detection with only marginal degradation to the system performance.

REFERENCES

[1] G. J. Foschini, "Layered space-time architecture for wireless communications in a fading environment when using multi-element antennas," *Bell Labs Technical Journal*, vol.1, No. 2, pp.41-59, Aut. 1996.

[2] G. J. Foschini, G. D. Golden, R. A. Valenzuela, and P. W. Wolnianski, "Simplified processing for high spectral efficiency wireless communication employing multi-element arrays," *IEEE Journal on Selected Areas of Communications*, vol. 17, No.11, pp. 1841-1852, Nov. 1999.

[3] P.W. Wolniansky, G. J. Foschini,G.D. Golden, and R.A. Valenzuela, "V-BLAST: an architecture for realizing very high data rates over the rich-scattering wireless channel," *1998 URSI International Symposium on Signals, Systems, and Electronics*, pp. 295-300, Oct. 1998.

[4] N. Boubaker, K.B. Letaief, and R. D. Murch, "A layered space-time coded wideband OFDM architecture for dispersive wireless link," *Proceedings Sixth IEEE Symposium on Computers and Communications*, pp. 518-523, 2001.

[5] G.H. Golub, and C.F. Van Loan, "Matrix computations," *The John Hopkins University Press*, 3rd Edition, 1996.

[6] Wong Kwan Wai, Chi-Ying Tsui, and R. S. Cheng, "A low complexity architecture of the V-BLAST system," *Proceedings IEEE Wireless Communications and Networking Conference*, vol. 1, pp. 310-314, 2000.

[7] N. Boubaker, K.B. Letaief, and R. D. Murch, "A low complexity multi-carrier BLAST architecture for realizing high data rates over dispersive fading channels," *Proceedings IEEE Vehicular Technology Conference Spring*, vol. 2, pp. 800-804, 2001.

[8] J. Medbo, H. Hallenberg, J.-E. Berg, "Propagation characteristics at 5 GHz in typical radio-LAN scenarios", *Proceedings IEEE Vehicular Technology Conference Spring*, vol. 1, pp. 185-189, May 1999.

[9] Sumei Sun, Ingo Wiemer, C.K. Ho, and T. T. Tjhung, "Training sequence assisted channel estimation for MIMO OFDM," *Proceedings IEEE Wireless Communications and Networking Conference 2003*, vol. 1, pp. 38-43, 2003.

[10] T. F. Chan, "An improved Algorithm for computing the singular value decomposition," *ACM Trasactions on Mathematical Software*, vol. 8, pp. 72-83, 1982.

[11] "Part11: wireless LAN medium access control (MAC) and physical layer (PHY) specifications: High-speed physical layer in the 5GHz band," *IEEE std 802.11a-1999:Supplement to IEEE std 802.11-1999*, Sept, 1999.

[12] V. Erceg, L. Schumacher and *et. al.*, "Indoor MIMO WLAN channel models", *IEEE 802.11 work group document*, No. IEEE 802.11-03/161R1, Jul. 2003.