GROUP MAP BLAST DETECTOR

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ABSTRACT

The Bell-Labs Layered Space-time (BLAST) architecture is a simple and efficient multi-antenna coding structure that can achieve high-spectral efficiency [1]. Many BLAST detectors require more receiver antennas than transmitter antennas. We propose a novel turbo-processing BLAST detector based on a group detection strategy that can operate in systems with fewer receiver antennas than transmitter antennas. A maximum *a posteriori* (MAP) decision is made using a group of transmitted symbols and the remaining signal contribution is treated as interference. The interference is characterized as non-zero mean colored noise source that is whitened before a decision is made. The proposed detector, the Group MAP (GMAP) detector, is a generalization of both the MAP detector and the turbo-processing Minimum Mean Squared Error (MMSE) detector in [2, 3]. A novel grouping algorithm is proposed for the GMAP detector. Simulation is used to compare the GMAP detector with the MAP detector and MMSE detector.

1. INTRODUCTION

The BLAST architecture is a simple and efficient coding structure that can take advantage of the multiple-input multiple output (MIMO) channel capacity [1]. The original detector proposed in [1] uses an Interference Cancellation and Nulling Algorithm (ICNA). An ICNA detector cannot however be applied to systems that have more transmitter antennas than receiver antennas. Such systems can exist in the downlink of a cellular systems where it is often infeasible to have a mobile station with many antennas due to size limitations. A similar scenario can exist when there are more than one transmitters and a single receiver, thus the total number of transmitter antennas can easily exceed the number of receiver antennas.

There are several detection strategies that can be applied to systems that have an excess number of transmitter antennas. An optimal solution is the maximum likelihood (ML) detector, which unfortunately has exponential complexity. Suboptimal ML detectors have been applied to BLAST systems using tree-search algorithms [4] and group detection strategies [5, 6].

Turbo processing receivers have also been applied to systems with an excess number of transmitter antennas. The optimal turbo-BLAST detector is the MAP detector that has high complexity. A more computational feasible MMSE BLAST detector [2, 3] uses *a prior* information to partially cancel interference and an instantaneous MMSE filter to suppress residual interference. With successive iterations, the performance of the MMSE detector improves as more interference is cancelled, but falls short of the MAP detector performance.

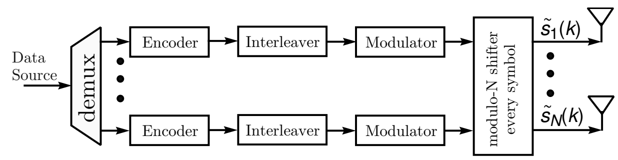

Fig. 1. D-BLAST transmitter

In this paper, we a propose a novel BLAST detector, termed the Group MAP (GMAP) detector, based on a group detection strategy. This detector bridges the performance gap between MAP and MMSE detectors in systems with an excess number of transmitter antenna. The GMAP detector divides the transmitted symbol vector into a set of disjoint groups of equal size. A decision is made for all the bits in a group by treating the remaining signal contribution as an interfering noise source, which is whitened by applying an appropriate filter. The prior probabilities for interfering symbols are used to determine the mean of the interfering noise source. A novel grouping algorithm is proposed to form groups for the GMAP detector. The size of each group N_G is an adjustable parameter that determines the complexity of GMAP detector. Through the choice of this parameter, the GMAP detector is a generalization of both MAP detector and MMSE detector in [2, 3]. Our group detection strategy is different from that in [5] as the solution in [5] does not use a noise whitening filter and different from the solution in [6] because we incorporate prior information in the whitening filter.

The remainder of this paper is organized as follows. Section 2 provides a system model that includes the D-BLAST transmitter, channel model and turbo-processing receiver structure. Section 3 describes the GMAP detector design. A complexity analysis and BER comparison is contained in Section 4, followed by a summary and concluding remarks in Section 5.

2. SYSTEM MODEL

Consider a D-BLAST transmitter [1] with N antennas having a structure shown in Figure 1. Binary data is demultiplexed into N layers that are independently encoded, interleaved and modulated, then passed through a modulo-N shifter. We only consider QPSK modulation such that the modulated output for the n^{th} layer is given by $x_n(k) = \{2b_n(2k)-1\}+\sqrt{-1}\{2b_n(2k+1)-1\}$, where

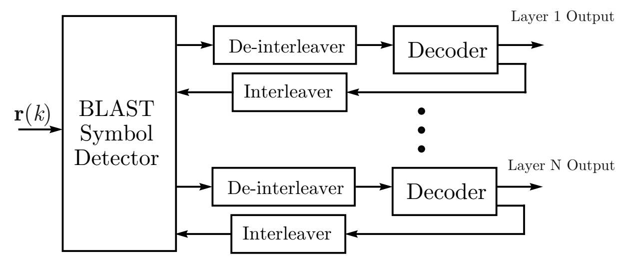

Fig. 2. Turbo Processing BLAST Receiver

$\{b_n(l)\}$ is the coded binary $\{0,1\}$ bitstream. The transmitted symbol on antenna n is given by $\tilde{s}_n(k) = \tilde{x}_\alpha(k), \quad \alpha = (n-k) \mod N$. Assuming a flat fading channel model, the vector channel output can be expressed as

$$\tilde{\mathbf{r}}(k) = \tilde{\mathbf{H}}\tilde{\mathbf{s}}(k) + \tilde{\mathbf{v}}(k) \tag{1}$$

where $\tilde{\mathbf{H}}$ is an $M \times N$ complex channel matrix, $\tilde{\mathbf{r}}(k)$ is the channel output, $\tilde{\mathbf{v}}(k)$ is a Gaussian noise source of variance σ^2, and M is the number of receiver antennas. It is convenient to transform the complex channel equation in (1) into real matrix equation $\mathbf{r}(k) = \mathbf{H}\mathbf{s}(k) + \mathbf{v}(k)$ where $\mathbf{r}(k) = [\ \Re\{\tilde{\mathbf{r}}^{\mathbf{T}}(k)\} \quad \Im\{\tilde{\mathbf{r}}^{\mathbf{T}}(k)\}\ ]$, $\mathbf{s}(k) = [\ \Re\{\tilde{\mathbf{s}}^{\mathbf{T}}(k)\} \quad \Im\{\tilde{\mathbf{s}}^{\mathbf{T}}(k)\}\ ]$, $\mathbf{v}(k) = [\ \Re\{\tilde{\mathbf{v}}^{\mathbf{T}}(k)\} \quad \Im\{\tilde{\mathbf{v}}^{\mathbf{T}}(k)\}\ ]$ and

$$\mathbf{H} = \begin{bmatrix} \Re\{\tilde{\mathbf{H}}\} & -\Im\{\tilde{\mathbf{H}}\} \\ \Im\{\tilde{\mathbf{H}}\} & \Re\{\tilde{\mathbf{H}}\} \end{bmatrix} \tag{2}$$

is the $M' \times N'$ real channel matrix with $M' = 2M$ and $N' = 2N$

The block diagram for the turbo processing BLAST receiver is shown in Figure 2. The receiver consists of a BLAST symbol detector, a set of N channel decoders, and a interleaver and deinterleaver between each decoder and the detector. There are modulo-N shifters at the input and output of the detector that have been omitted from Figure 2 for clarity. In each iteration, the decoders produce a set of log domain prior probabilities $\lambda_2^p[b_n(l)] = log\{P(b_n(l)=1)/P(b_n(l)=0)\}$ that are used by the detector, which in turn produces a log domain *a posteriori* probability as

$$\Lambda_1[b_n(k)] = \log\frac{P(b_n(k)=1|\mathbf{r}(k))}{P(b_n(k)=0|\mathbf{r}(k))} \equiv \lambda_1[b_n(l)] + \lambda_2^p[b_n(l)] \tag{3}$$

where $\lambda_1[b_n(l)]$ is the *extrinsic* information that is fed to the channel decoder for the n^{th} layer and $\lambda_2^p[b_n(l)]$ is the *a priori* information provided by the n^{th} channel decoder. The channel decoders can be efficient implemented using SISO APP module [7].

3. DETECTOR DESIGN

The optimal MAP detector evaluates (3) directly, by summing over the $2^{N'}$ possible signal vectors $\mathbf{s}(k)$. The MAP detector becomes computationally prohibitive for a system with a moderate to large number of transmitter antennas. The GMAP detector forms an MAP decision for a group of symbols in $\mathbf{s}(k)$, by treating the remaining symbols as interference. Let N_G be the group size and with no loss of generality, we assume the signal vector $\mathbf{s}(k)$ can be divided in N_Ψ disjunct sets $\Psi = \{G_1, \ldots, G_{N_\Psi}\}$, such that the members of G_i are the indices of the elements of $\mathbf{s}$ belonging to the i^{th} set. For an arbitrary group $G = G_i$, define a complimentary set of interfering symbols $\bar{G} = \{\beta_1, \ldots, \beta_{N_{\bar{G}}}\}$ of $N_{\bar{G}} = N' - N_G$ integers such that $G \bigcap \bar{G} = \emptyset$ and $G \bigcup \bar{G} = \{1, \ldots, N'\}$. For a particular G, the channel output can be expressed as

$$\mathbf{r} = \mathbf{H}_G\mathbf{s}_G + \mathbf{H}_{\bar{G}}\mathbf{s}_{\bar{G}} + \mathbf{v} \tag{4}$$

here $\mathbf{s}_G = [s_{\alpha_1}, \ldots, s_{\alpha_{N_G}}]^T$ is the reduced dimension signal vector, $\mathbf{s}_{\bar{G}} = [s_{\beta_1}, \ldots, s_{\beta_{N_{\bar{G}}}}]^T$ is the interference vector, $\mathbf{H}_G = [\mathbf{h}_{\alpha_1}, \ldots, \mathbf{h}_{\alpha_{N_G}}]$, $\mathbf{H}_{\bar{G}} = [\mathbf{h}_{\beta_1}, \ldots, \mathbf{h}_{\beta_{N_{|\bar{G}|}}}]$, and $\mathbf{h}_i$ is the i^{th} column of $\mathbf{H}$. The contribution of the interference and Gaussian noise can be treated as a colored noise source. Let $\mathbf{w} = \mathbf{H}_{\bar{G}}\mathbf{s}_{\bar{G}} + \mathbf{v}$ be the colored noise source whose mean is $\bar{\mathbf{w}} = \mathrm{E}[\mathbf{w}] = \mathbf{H}_{\bar{G}}\hat{\mathbf{s}}_{\bar{G}}$, where $\hat{\mathbf{s}}_{\bar{G}} = [\hat{s}_{\bar{G}1}, \ldots, \hat{s}_{\bar{G}N_{\bar{G}}}]$ and $\hat{s}_{\bar{G}i}$ is evaluated using the prior probabilities from the channel decoders as

$$\hat{s}_{\bar{G}i} = \sum_{s_{\bar{G}i} \in \{+1,-1\}} s_{\bar{G}i} P(s_{\bar{G}i}) = \tanh\left(\frac{\lambda_2^p[b]}{2}\right) \tag{5}$$

where b is the bit that determines the symbol $s_{\bar{G}i} = 2b - 1$. The covariance of $\mathbf{w}$ is given by

$$\mathbf{R}_w = \mathrm{E}[(\mathbf{w} - \bar{\mathbf{w}})(\mathbf{w} - \bar{\mathbf{w}})^T] = \mathbf{H}_{\bar{G}}\mathbf{\Omega}\mathbf{H}_{\bar{G}}^H + \mathbf{I}\sigma^2 \tag{6}$$

where $\Omega = \mathrm{diag}(\omega_1, \ldots, \omega_{|\bar{G}|})$ and $\omega_i = \mathrm{E}[|s_{\bar{G}i} - \hat{s}_{\bar{G}i}|^2] = 1 - \hat{s}_{\bar{G}i}^2$. We have assumed perfect interleaving in (6) such that the elements of $\hat{\mathbf{s}}_{\bar{G}}$ are independent. The noise $\mathbf{w}$ can be whitened by first removing the mean $\bar{\mathbf{w}}$ and then applying an appropriate noise whitening filter $\mathbf{F} = \mathbf{\Sigma}^{-1/2}\mathbf{Q}^H$, where $\mathbf{\Sigma}$ is a diagonal matrix and $\mathbf{Q}$ is an orthogonal matrix, both obtain from the eigenvalue decomposition of $\mathbf{R}_w = \mathbf{Q}\mathbf{\Sigma}\mathbf{Q}^H, \mathbf{Q}\mathbf{Q}^H = \mathbf{I}$. The whitened channel observation is given by

$$\mathbf{y} = \mathbf{F}(\mathbf{r} - \bar{\mathbf{w}}) \tag{7}$$

The APP for the bit b_i corresponding to the i^{th} symbol in $\mathbf{s}_G$ can be evaluated as

$$\Lambda_1[b_i] = \log\frac{P(b_i=1|\mathbf{y})}{P(b_i=0|\mathbf{y})} \tag{8}$$

$$= \log\frac{\sum_{s_{Gi}=+1} P(\mathbf{s}_G)\exp\left(\frac{-||\mathbf{y}-\mathbf{F}\mathbf{H}_G\mathbf{s}_G||^2}{2}\right)}{\sum_{s_{Gi}=-1} P(\mathbf{s}_G)\exp\left(\frac{-||\mathbf{y}-\mathbf{F}\mathbf{H}_G\mathbf{s}_G||^2}{2}\right)} \tag{9}$$

where s_{Gi} is the i^{th} element in $\mathbf{s}_G$ and the summations in (9) are over the set of possible $\mathbf{s}_G$. The GMAP detector evaluates (9) for each bit in G.

What remains in the development of the GMAP detector is an algorithm to choose the groups in Ψ. It is advantageous to group symbols together that have a high correlation at the channel output, since suppressing interference from a highly correlated symbol can give rise to significant noise enhancement in the filtering process. In order to quantify correlation, we use the normalized correlation matrix $\mathbf{R}$ whose entry in row i and column j is given by

$$r_{ij} = \frac{|\mathbf{h}_i^T\mathbf{h}_j|}{\sqrt{\|\mathbf{h}_i\|^2\|\mathbf{h}_i\|^2}} \tag{10}$$

The element r_{ij} is the normalized correlation between the symbols s_i and s_j at the channel output. Given a normalized correlation matrix $\mathbf{R}$, consider forming Ψ using the following objective function

$$\Theta = \arg \max_{G_1 \dots G_{N_\Psi}} \sum_{k=1}^{N_\Psi} \arg \max_{j \in G_k} r_{ij} \qquad (11)$$

which is equivalent to maximizing the maximum pairwise correlation amongst the members of each group, averaged over all groups. We use the maximum pairwise correlation criteria, instead of for example the average pairwise correlation, since the former relates to the minimum distance between columns of $\mathbf{H}$. At a high SNR, the bit error rate (BER) is usually largely influenced by the minimum distance between constellation points instead of average distances. An approximate solution to (11) can be found using the following greedy algorithm

1 $\mathbf{R} \leftarrow \mathbf{R} - \mathbf{I}$;
2 $\Psi \leftarrow \{G_1, \dots, G_{N'}\}, \quad G_i = \{i\}$;
3 **while** $|\Psi| > N_\Psi$
4   **if** $\sum_{i=1}^{|\Psi|} \mathrm{I}(|G_i| > 1) < N_\Psi$
5     $\delta_{ij} = I(|G_i| + |G_j| \leq N_G)$;
6   **else**
7     $\delta_{ij} = I(2 < |G_i| + |G_j| \leq N_G)$;
8   **end if**
9   $r_{kl} = \arg\max_{ij} r_{ij}\delta_{ij}$;
10   $G_k \leftarrow G_k \cup G_l$;
11   $\Psi \leftarrow \Psi \setminus \{G_l\}$;
12   **for** $n = 1 \dots |\Psi|, \ n \neq k$;
13     $r_{nk} = r_{kn} = \max\{r_{nk}, r_{nl}\}$;
14   **end for**
15   Remove k^{th} row and column from $\mathbf{R}$
16 **end while**

where $I(x)$ is an indicator function that evaluates to 1 if x is true and 0 otherwise. The first two lines are initialization, placing zeros on the diagonal of $\mathbf{R}$ and forming Ψ as a set of N groups with one element in each group. The remained of the program is loop that reduces the size of Ψ by merging two groups each iteration. Lines 4-8 determine if a possible merger is feasible based on the number of groups with more than one element. The two groups having maximum correlation and a feasible merger are found in line 9 and these groups are merged in the remainder of the program. The preceding algorithm is greedy as it attempts to form highly correlate groups first. Although this may not optimally satisfy the objective function in (11), it is advantageous to form highly correlated groups first, since the BER is more strongly influenced by groups with a high pairwise correlation, instead of the average pairwise correlation across all groups.

The GMAP detector is equivalent to the MAP in the limiting case of $N_G = N'$ and equivalent to the MMSE detector [2, 3] detector in the limiting case of $N_G = 1$. For $N_G = N'$, $\mathbf{H}_G = \mathbf{H}\mathbf{P}^T$, where $\mathbf{P}$ is a permutation matrix. Since $\mathbf{P}$ does not affect the MAP decision, the GMAP detector in this case is equivalent to the MAP. The MMSE detector forms a decision according to

$$\lambda_1[b_i] = \frac{2\mathbf{h}_i^T \mathbf{R}_i^{-1}(\mathbf{r} - \bar{\mathbf{w}})}{1 - \mathbf{h}_i^T \mathbf{R}_i^{-1} \mathbf{h}_i} \qquad (12)$$

where $2\mathbf{h}_i^T \mathbf{R}_i^{-1}$ is the MMSE filter, $1 - \mathbf{h}_i^T \mathbf{R}_i^{-1} \mathbf{h}_i$ is an estimate of the noise vairance at the filter output, $\mathbf{R}_i = [\mathbf{H}\boldsymbol{\Omega}_i \mathbf{H}^T + \sigma^2 \mathbf{I}]$, $\boldsymbol{\Omega}_i = \mathrm{diag}(\omega_1, \dots, \omega_{i-1}, 1, \omega_{i+1}, \omega_N)$ and $\omega_j = \mathrm{E}[|s_i - \hat{s}_i|^2] = 1 - \hat{s}_i^2$. For $N_G = 1$, the extrinsic component $\lambda_1[b_i]$ of the GMAP decision in (9) can be simplified as

$$\lambda_1[b_i] = 2\mathbf{h}_i \mathbf{R}_w^{-1}(\mathbf{r} - \bar{\mathbf{w}}) \qquad (13)$$

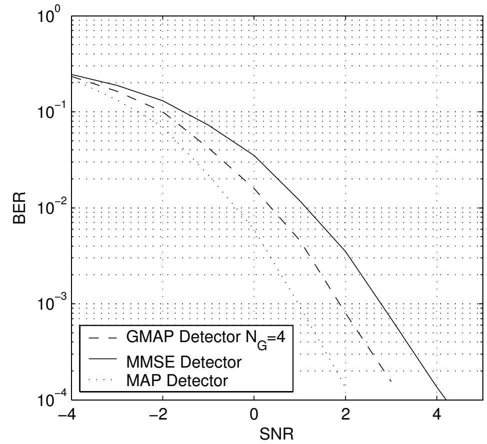

Fig. 3. BER Comparison $N = 6, M = 3$ for GMAP, MMSE, and MAP detectors

Substituting $\mathbf{R}_w = \mathbf{R}_i + \mathbf{h}_i \mathbf{h}_i^T$ in (13) and using the matrix inversion lemma [1] with $\mathbf{A} = \mathbf{R}_i$, $\mathbf{B} = -1$ and $\mathbf{X} = \mathbf{h}_i$ gives the MMSE decision in (12) after some simplification. Thus the GMAP detector is equivalent to the MMSE detection in [2, 3] for $N_G = 1$.

4. SIMULATED RESULTS

This section analyzes the BER performance and complexity of the GMAP detector. The MAP detector and MMSE detector in [2, 3] are used for comparison in terms of both complexity and performance. Simulations were preformed using bursts of 100 symbols and each layer was encoded using a rate 1/2 convolutional code with generating polynomial $(7, 5)$. A random interleaver and deinterleaver was used. Estimates for uncoded bits were produced after 10 turbo iterations. An independent Rayleigh fading model was used to determine the channel matrix $\mathbf{H}$ and perfect channel knowledge was assumed at the receiver.

A complexity analysis for the GMAP detector is shown in Table 1 along with the complexity of the MAP and MMSE [2, 3] detectors for reference. All operations are shown for a single coded bit decision. The number of multiplications (mult) and additions (add) is approximate since only the highest polynomial term of M', N', N_G etc. is shown for clarity and lower power terms are omitted. The number of elementary operations involve in a matrix inverse (inv) and eigenvalue decomposition (eig) is difficult to evaluate, thus the complexity is expressed in $O(n)$ notation. It was assume the noise free channel outputs $\mathbf{r} = \mathbf{H}\mathbf{s}$ were precomputed for the MAP detector, but produced online for the GMAP detectors. If the group size N_G is chosen to be moderately small, the complexity of GMAP detector is polynomial with respect to N', M', and is quite comparable to that of the MMSE detector.

We consider two simulated examples. The first example is a system with $N = 6$ transmitting and $M = 3$ receiving antennas. The BER curves for the GMAP, MMSE, and MAP detectors are shown in Figure 3. For the real signal vector of di-

[1] $(\mathbf{A} + \mathbf{X}\mathbf{B}\mathbf{X}^T)^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{X}(\mathbf{B}^{-1} + \mathbf{X}^T \mathbf{A}^{-1} \mathbf{X})^{-1} \mathbf{X}^T \mathbf{A}^{-1}$

mension $N' = 12$, the group size for GMAP detector was set to $N_G = 4$. The GMAP detector preformed approximately $1dB$ better that the MMSE detector at nominal BER of 10^{-3}. In the second example, we consider a system with $N = 10$ transmitting antennas and $M = 4$ receiving antennas. The BER curves for the GMAP detector are shown in Figure 4 for different groups sizes $N_G = 1, 2, 4, 5$, where the $N_G = 1$ case corresponds to the MMSE detector. The MAP detector performance has been omitted as each decision requires on order of 2^{20} operations. The GMAP detector performance improves with increasing N_G, with a performance improvement of approximately $1.5dB$ over the MMSE detector for $N_G = 5$ case.

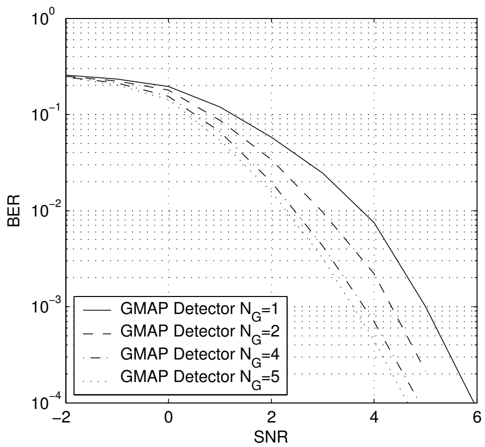

Fig. 4. BER for $N = 10, M = 4$ system using GMAP detector for $N_G = 1, 2, 4, 5$

5. SUMMARY AND CONCLUSIONS

In this paper, we proposed a novel GMAP group detector that operates within a turbo processing BLAST receiver. This detector can be applied to systems having fewer receiver antennas than transmitter antenna. The GMAP detector allows a tradeoff between complexity and performance through the MAP group size and includes as special cases, both the MAP detector and MMSE detector in [2, 3]. A novel grouping algorithm is developed for the GMAP detector. For systems with an excess number transmitter antennas, the proposed detector has a significant performance improvement over the MMSE detector in [2, 3] with a relatively small MAP group.

	MAP	MMSE	GMAP
mult	$2^{N'}(M'+N')$	M'^2N'	$M'^2\frac{N_{\bar{G}}}{N_G} + 2^{N_G}M'$
add	$2^{N'}M'$	M'^2N'	$M'^2\frac{N_{\bar{G}}}{N_G} + 2^{N_G}M'$
inv		$O(M'^3)$	
eig			$O(M'^3)/N_G$

Table 1. Approximate complexity of MAP, MMSE [2, 3], and GMAP detectors for each coded bit decision

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