# A NEW FREQUENCY OFFSET ESTIMATION TECHNIQUE FOR UP-LINK MC-CDMA SYSTEMS

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# ABSTRACT

We propose a new technique of frequency synchronization for up-link in an MC-CDMA system. The frequency offset estimation is done in the frequency domain, by estimating the phase difference between two consecutive MC-CDMA symbols of the user of interest. The phase alternation, caused by transmitted user symbols, is eliminated using the knowledge of the previous estimated frequency offset and the fact that the latter is very slowly varying. This eliminates the need of knowledge of user symbol and renders the technique blind.

## 1. INTRODUCTION

Multicarrier code division multiple access (MC-CDMA) is a new multiple access scheme [1]. It is a combination of orthogonal frequency division multiplexing (OFDM) and CDMA, and inherits the advantages of both such as the immunity to frequency selective fading, simplified equalization, and high spectral efficiency. Therefore, it is a potential candidate for the next generation broadband wireless and mobile communications systems.

Unfortunately, MC-CDMA also inherits the drawbacks of CDMA and OFDM. One of them is the sensitivity to the frequency offset. This requires us to estimate the frequency offset prior to the channel estimation and user symbol detection. In down-link, where the signals corresponding to different users are transmitted by the same base station, corrupted by the same fading channel and affected by the same frequency offset, the problem is the same as the frequency offset problem in single user OFDM systems. Therefore, some techniques developed for these system, such as some cyclic prefix based techniques [2],[3],[4], training based techniques [5],[6], and virtual carrier based techniques [7],[8], can be implemented in this case. On the other hand, the problem is much more difficult in the up-link case, because the signal received by the base station is a sum of different signals transmitted by different users, corrupted by different channels and affected by different frequency offsets. The aforementioned techniques cannot be applied in this case. A blind technique for up-link MC-CDMA has been proposed by Tureli et al. [9] which estimates the carrier offsets of multiple users. However, this technique uses unmodulated carriers.

In this paper, we propose a new method for estimating carrier frequency offset in up-link MC-CDMA. The method relies on the fact that received symbols contain phase information which is in function of the carrier frequency offset. The estimation is done in the frequency domain (post FFT), by estimating the phase difference between two consecutive MC-CDMA symbols of the user of interest. The phase alternation, caused by transmitted user symbols, is eliminated using the knowledge of the previous estimated frequency offset and the fact that the latter is very slowly varying. This eliminates the need of knowledge of user symbol and renders the technique blind.

This paper is organized as follows. The system model is specified in Section 2. In Section 3, we propose a new technique for frequency offset estimation in up-link of MC-CDMA systems. Section 4 describes how the frequency offset information is used in subspace based multiuser detections and associated channel estimation technique. Simulation results are presented in Section 5 and we finally conclude this paper in Section 6.

## 2. SYSTEM MODEL

We consider a quasi-synchronous up-link MC-CDMA using N subcarriers shared by K users. In such a system, the discrete-time signal transmitted by the users are generated using IFFT.

The signal  $q_{k,l}(i)$  corresponding to the *l*th sub-carrier of the signal  $x_{k,n}(i)$ , at the *i*th symbol period is given by

$$q_{k,l}(i) = \sqrt{\mathcal{E}_k} b_k(i) s_{k,l}, \quad \text{for } l = 0, \dots, N-1, \tag{1}$$

where  $\mathcal{E}_k > 0$ ,  $b_k$ , and  $s_{k,l}$  (with k = 1, ..., K, and  $|s_{k,l}|^2 = \frac{1}{N}$ ), denote the power, symbol, and spreading code of the *k*th user, respectively.

After an N-IFFT operation is performed to the signal, a guard interval of  $N_{cp}$  in form of a cyclic prefix is added, resulting in a symbol frame of  $N + N_{cp}$  length. The symbol frame elements are transmitted one after another, producing a sequence of transmitted signal which is then up-converted to a transmittable frequency using an up-conversion mixing signal.

At the receiver, a *P*-element array of antennas to create diversity. This means that a signal transmitted by a user will travel through different multipath channels to the antenna elements. Each antenna element will then receive a combination of signals transmitted by different users and corrupted by different multipath channels.

The received signals are down-converted using a single mixing signal. The frequency mismatch between this signal and different up-conversion mixing signals at the transmitter results in different frequency offsets experienced by signal components corresponding to different users. We assume that the frequency offsets due to the Doppler effect are negligable. There-

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fore, the frequency offset associated to one user will affect multipath signal components corresponding to the user the same way.

The demodulation is done by placing the FFT window on the sequence of samples such that there is no ISI. The demodulated received signal vector  $r^p(i)$  received by the *p*th array element at the *i*th symbol period can be written as

$$\boldsymbol{r}^{p}(i) = \sum_{k=1}^{K} \sqrt{\mathcal{E}_{k}} \tilde{b}_{k}(i) \boldsymbol{W}_{k} \{ \boldsymbol{s}_{k} \odot \boldsymbol{g}_{k}^{p} \} + \boldsymbol{z}^{p}(i), \quad (2)$$

where

$$\begin{split} \tilde{b}_{k}(i) &= b_{k}(i) \exp(j2\pi\epsilon_{k}i\frac{N+N_{cp}}{N}), \\ \boldsymbol{r}^{p}(i) &= [r_{0}^{p}(i), \cdots, r_{N-1}^{p}(i)]^{T}, \\ \boldsymbol{s}_{k} &= [s_{k,0}, \cdots, s_{k,N-1}]^{T}, \\ \boldsymbol{g}_{k}^{p} &= [g_{k,0}^{p}, \cdots, g_{k,N-1}^{p}]^{T}, \\ \boldsymbol{z}^{p}(i) &= [z_{0}^{p}(i), \cdots, z_{N-1}^{p}(i)]^{T}, \end{split}$$

the symbol  $\odot$  denotes the Hadamard product, and  $W_k$  is the  $N \times N$  matrix of complex weighting factors due to the carrier frequency offset of the *k*th user, with its elements

$$\boldsymbol{W}_{k}(l,m) = \frac{1}{N} \sum_{n=0}^{N-1} \exp(j\frac{2\pi n(m-l+\epsilon_{k})}{N}), \qquad (3)$$

with  $\epsilon_k$  and  $g_{k,l}^p$  being the frequency offset and fading coefficient of the kth user on the lth subcarrier. The whole demodulated received signal vector  $\mathbf{r}(i)$  is constructed as  $\mathbf{r}(i) = [(\mathbf{r}^1)^T(i), \cdots, (\mathbf{r}^P)^T(i)]^T$ .

# 3. A FREQUENCY OFFSET ESTIMATION TECHNIQUE

Let us consider that the user of interest is user k = 1. From Eq. (2) we can see that  $\tilde{b}_1(i)$  contains phase information which is in function of frequency offset. The frequency offset can be obtained from two consecutive symbols using

$$\epsilon_1 = \frac{1}{2\pi} \frac{N}{N + N_{cp}} \frac{b_1(i+1)}{b_1(i)} \angle \frac{b_1(i+1)}{\tilde{b}_1(i)}.$$
 (4)

In real situations  $\tilde{b}_1(i)$  is not available. Therefore we have to replace it with the soft estimate  $\check{b}_1(i)$  which is interfered by other users signals and noise. Assume that the channel information and the frequency offset are constant during several symbol periods or are varying at such a very low rate that we can estimate the current user symbol using the previous channel and frequency offset information. The soft estimate  $\check{b}_1(i)$ can be obtained by multiplying the demodulated received vector r(i) with a detection weighting vector  $w_1(i)$ .

In a single user detection  $w_1(i)$  is replaced by  $\tilde{s}_1(i)$  where

$$\tilde{\boldsymbol{s}}_1 = \boldsymbol{W}_1 \left\{ (\boldsymbol{s}_1 \odot \boldsymbol{g}_1^1)^T, \cdots, (\boldsymbol{s}_1 \odot \boldsymbol{g}_1^P)^T \right\}^T.$$
(5)

Therefore

$$\check{b}_1(i) = \tilde{s}_1^H \boldsymbol{r}(i) \tag{6}$$

$$=\tilde{\boldsymbol{s}}_{1}^{H}\left(\sum_{k=1}^{K}\sqrt{\mathcal{E}_{k}}\tilde{b}_{k}(i)\tilde{\boldsymbol{s}}_{k}+\boldsymbol{z}(i)\right)$$
(7)

$$= \sqrt{\mathcal{E}_1} \tilde{b}_1 \tilde{s}_1^H \tilde{s}_1 + \sum_{k=2} \sqrt{\mathcal{E}_k} \tilde{b}_k(i) \tilde{s}_1^H \tilde{s}_k + \tilde{s}_1^H \boldsymbol{z}(i) \quad (8)$$

$$= \kappa b_1(i) + \zeta_{MAI} + \zeta_{noise},\tag{9}$$

where  $\kappa$  is a positive number,  $\zeta_{MAI}$  and  $\zeta_{noise}$  are terms due to the MAI and noise, respectively.

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In multiuser detection,  $w_1(i)$  can be replaced by decorrelating detector weighting vector  $d_1(i)$  or MMSE detector weighting vector  $m_1(i)$ . In both cases the MAI is eliminated and the soft estimate can be written as

$$\dot{b}_1(i) = \kappa \dot{b}_1(i) + \zeta_{noise}.$$
 (10)

With the assumption that the MAI and noise are complex with normally distributed real and imaginary parts,  $\check{b}_1(i)$  can be written as

$$\check{b}_1(i) = \kappa \exp(j\phi(i)) + \alpha(i)\exp(j\beta(i)),$$
 (11)

where  $\phi(i)$  is the phase corresponding to the *i*th symbol,  $\alpha(i)$  is Rayleigh distributed with variance  $\sigma_{\alpha}^2$  whose value depends on the detection used, and  $\beta(i)$  is uniformly distributed over  $[-\pi, \pi)$ .

The frequency offset we are after is the expectation of the difference between the phases of two consecutive soft estimates.

$$\hat{\epsilon}_{1} = \frac{1}{2\pi} \frac{N}{N + N_{cp}} E \left\{ \frac{b_{1}(i+1)}{b_{1}(i)} \angle \frac{\check{b}_{1}(i+1)}{\check{b}_{1}(i)} \right\}$$
(12)

$$= \frac{1}{2\pi} \frac{N}{N+N_{cp}} E\left\{\frac{b_1(i+1)}{b_1(i)} (\angle \check{b}_1(i+1) - \angle \check{b}_1(i))\right\}.$$
(13)

The expectation term in Eq. (13) can take any value in the interval  $[-\pi, \pi)$ . This means that the proposed method will only be able to estimate a frequency offset in the interval of  $[-\frac{N}{2(N+N_{cp})}, \frac{N}{2(N+N_{cp})})$ . However, initialized by a much wider range frequency offset acquisition, tracking will allow estimation beyond this interval.

According to Eq. (13),  $\bar{b}_1(i)$  should be known for two consecutive symbol periods, i and i + 1, in order to allow us to estimate the frequency offset. This means that we have to send a training sequence which will decrease the throughput of the system. In order to alleviate this drawback, we propose a technique to eliminate the need of sending training sequences.

nique to eliminate the need of sending training sequences. Because  $b_k(i) \in \{-1, +1\}, \frac{b_1(i+1)}{b_1(i)}$  will take two different values, which are +1 when two consecutive symbols are equal and -1 when two consecutive symbols are different. Since the frequency offset is constant during a sufficiently large number of symbol periods, the phase difference distribution function will be a sum of two distribution functions with two different means  $\pi$  apart from each other. The one corresponds to the frequency offset and the other is the dephased version. Fig. 1.a depicts two normal distributions with two different means  $\pi$  apart. Fig. 1.b depicts the sum of the distribution functions. Since angle is a circular variable the real phase distribution will



Fig. 1. Phase difference distribution function.

look more like in Fig. 1.c. Knowing the previous frequency offset value we can bring the dephased phase difference distribution to the correct position and obtain a distribution function with a single mean which corresponds to our frequency offset estimate as depicted by Fig. 1.d.

### 4. FREQUENCY OFFSET COMPENSATION

It is clear that, according to Equation (2), frequency offset introduces inter-carrier interference (ICI). But, since in MC-CDMA only a single user symbol is spread over subcarriers, frequency offset will simply produce a new spreading vector. If we know  $\tilde{s}_k$  for  $k = 1, \dots, K$ , *i.e.* we know the fading and frequency offset of all users, then we will be able to better recover the user symbols using multiuser detection techniques. The effect of channel fading and frequency offset should be minimized if we consider  $\tilde{s}_k$  instead of  $s_k$ . A weighting vector for the decorrelating detector for the first user  $d_1$  is given by [10]

$$\boldsymbol{d}_1 = \sum_{k=1}^{K} [\boldsymbol{R}^{-1}]_{1,k} \tilde{\boldsymbol{s}}_k \tag{14}$$

where  $\mathbf{R} = \tilde{\mathbf{s}}^H \tilde{\mathbf{s}}$ , with  $\tilde{\mathbf{s}} = [\tilde{\mathbf{s}}_1, \cdots, \tilde{\mathbf{s}}_K]$ . The decision on  $b_1$  is then

$$\hat{b}_1 = \operatorname{sgn}\{\Re(\boldsymbol{d}_1^H \boldsymbol{r})\}.$$
(15)

While a weighting vector for the MMSE detector  $m_1$  is the vector that minimizes [10]

$$MSE(\boldsymbol{m}_1) = E\left\{ (\sqrt{\mathcal{E}_1} b_1 - \boldsymbol{m}_1^H \boldsymbol{r})^2 \right\} \text{ s.t. } \boldsymbol{m}_1^H \tilde{\boldsymbol{s}}_1 = 1, \quad (16)$$

and the decision on  $b_1$  is

$$\hat{b}_1 = \operatorname{sgn}\{\Re(\boldsymbol{m}_1^H \boldsymbol{r})\}.$$
(17)

Wang and Poor [11] proposed subspace-based detectors for CDMA which eliminate the need of other users spreading codes than the user of interest. In [12], we extended these detectors for multiple receive antenna based MC-CDMA systems and proposed a new subspace technique for estimation of channel parameters. The new detectors and associated channel estimation technique are presented next.

Following [11],  $d_1$  and  $m_1$  can be written in terms of the signal subspace parameters as

$$\boldsymbol{d}_{1} = \frac{\boldsymbol{U}_{s}(\boldsymbol{\Lambda}_{s} - \sigma^{2}\boldsymbol{I}_{K})^{-1}\boldsymbol{U}_{s}^{H}\boldsymbol{\tilde{s}}_{1}}{\boldsymbol{\tilde{s}}_{1}^{H}\boldsymbol{U}_{s}(\boldsymbol{\Lambda}_{s} - \sigma^{2}\boldsymbol{I}_{K})^{-1}\boldsymbol{U}_{s}^{H}\boldsymbol{\tilde{s}}_{1}}$$
(18)

and

$$\boldsymbol{m}_1 = \frac{\boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} \boldsymbol{U}_s^H \tilde{\boldsymbol{s}}_1}{\tilde{\boldsymbol{s}}_1^H \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} \boldsymbol{U}_s^H \tilde{\boldsymbol{s}}_1}.$$
(19)

The signal subspace  $U_s$  and the diagonal matrix of corresponding eigenvalues  $\Lambda_s$  are obtained from the eigen-decomposition of the autocorrelation matrix C, with the assumption that the signature waveforms of the K users are linearly independent and that the users' signals are mutually uncorrelated, as follows:

$$\boldsymbol{C} = \boldsymbol{E}[\boldsymbol{r}\boldsymbol{r}^{H}] = \sum_{k=1}^{K} \mathcal{E}_{k} \tilde{\boldsymbol{s}}_{k} \tilde{\boldsymbol{s}}_{k}^{H} + \sigma^{2} \boldsymbol{I}_{NP}$$
(20)

$$= \tilde{\boldsymbol{S}}\boldsymbol{E}\tilde{\boldsymbol{S}}^{H} + \sigma^{2}\boldsymbol{I}_{NP} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^{H}$$
(21)

$$= \begin{bmatrix} U_s & U_n \end{bmatrix} \begin{bmatrix} \Lambda_s & \mathbf{0} \\ \mathbf{0} & \Lambda_n \end{bmatrix} \begin{bmatrix} U_s^H \\ U_n^H \end{bmatrix} \quad (22)$$

where  $\boldsymbol{E} = \operatorname{diag}(\mathcal{E}_1, \dots, \mathcal{E}_K)$ ,  $\boldsymbol{\Lambda}_s = \operatorname{diag}(\lambda_1, \dots, \lambda_K)$ , with  $\lambda_1 \ge \lambda_2 \ge \dots \ge \lambda_K > \sigma^2$ , and  $\boldsymbol{\Lambda}_n = \sigma^2 \boldsymbol{I}_{NP-K}$ , with  $\boldsymbol{I}_N$  being the identity matrix of size  $N \times N$ .

We use the fact that the fading coefficients  $g_1^p$  at subcarriers are discrete Fourier transform of the multipath coefficient  $h_1^p = [h_{1,0}^p, \dots, h_{1,L-1}^p]^T$ . We prefer the estimation of  $h_1^p$  to  $g_1^p$  as the number of unknown coefficients on the former case is much smaller than the latter [13]. Thus, the vector  $g_1 = [(g_1^1)^T, \dots, (g_1^p)^T]^T$  can be written as [12]

$$\boldsymbol{g}_1 = (\boldsymbol{I}_P \otimes \boldsymbol{F}_L)\boldsymbol{h}_1, \tag{23}$$

where  $h_1 = [(h_1^1)^T, \dots, (h_1^P)^T]^T$ ,  $\otimes$  denotes the Kronecker product, and  $F_L$  is the truncated Fourier transform matrix containing the *L* first columns. Thus, we can rewrite  $\tilde{s}_1$  as

$$\tilde{\boldsymbol{s}}_1 = (\boldsymbol{I}_P \otimes \boldsymbol{W}_1) (\boldsymbol{I}_P \otimes \boldsymbol{S}_1) ((\boldsymbol{I}_P \otimes \boldsymbol{F}_L) \boldsymbol{h}_1), \qquad (24)$$

where  $S_1 = \text{diag}(s_1)$  is the diagonal matrix of the first user spreading codes. Exploiting the orthogonality between the noise and signal subspace, we can estimate  $h_1$  using the following approach:

$$\hat{\boldsymbol{h}}_{1} = \arg \max_{||\boldsymbol{h}||=1} ||\boldsymbol{U}_{s}^{H}(\boldsymbol{I}_{P} \otimes \boldsymbol{W}_{1})(\boldsymbol{I}_{P} \otimes \boldsymbol{S}_{1})(\boldsymbol{I}_{P} \otimes \boldsymbol{F}_{L})\boldsymbol{h}||^{2} (25)$$
$$= \arg \max_{||\boldsymbol{h}||=1} \boldsymbol{h}^{H} \boldsymbol{Q} \boldsymbol{h}, \qquad (26)$$





where

$$Q = (I_P \otimes F^H)(I_P \otimes S_1)(I_P \otimes W_1^H)U_s$$
  
$$.U_s^H(I_P \otimes W_1)(I_P \otimes S_1)(I_P \otimes F_L). (27)$$

Therefore, we can estimate  $\hat{h}_1$  as the principal eigenvector of Q.

### 5. SIMULATION RESULTS

Fig. 2 shows a result of frequency tracking simulation at SNR = 10dB. The number of subcarriers is N = 32 and the number of users is K = 16. The cyclic prefix length is  $N_{cp} = 8$  and the channels are L = 4 path channels with coefficients are zero mean complex Gaussian number with equal variance. The actual normalized frequency offset is 20% and the initial estimated frequency offset is set to 15% and tracked for M = 10000 symbol periods.

Fig. 3 shows the performance of subspace based decorrelating detector with frequency offset compensation (solid lines), compared to the same detector without compensation (dotted line). We can see that the detector alleviates effectively the frequency offset problem.

### 6. CONCLUSIONS

We proposed a new technique of frequency synchronization for an up-link MC-CDMA system. The frequency offset estimation is done in the frequency domain, by estimating the phase difference between two consecutive MC-CDMA symbols of the user of interest. The phase alternation, caused by transmitted user symbols, is eliminated using the knowledge of the previous estimated frequency offset and the fact that the latter is very slowly varying, eliminating the need of knowledge of user symbol which renders the technique blind. Simulation results show that the new technique allow us to track the frequency offset and alleviate the frequency offset problem.

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Fig. 3. Performance of Detectors with Frequency offset compensation.

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