ON THE EQUALIZATION OF ASYNCHRONOUS MULTIUSER OFDM SIGNALS IN FADING CHANNELS

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ABSTRACT

In this paper we study asynchronous multiuser orthogonal frequency division multiplexing (OFDM) systems over fading channels. Multiuser multiplexing is essential to design a spectrally efficient communication system. On the other hand, a synchronous mode of communication may not be a practically realizable solution considering the demand for high-speed services with portability. Therefore, we develop the model of asynchronous multiuser OFDM signals, and propose several minimum mean squared error (MMSE) based equalization methods to suppress the asynchronous interference effectively. The proposed space-time domain MMSE filter mitigates the inter-carrier interference (ICI) and the inter-block interference (IBI) induced by the interfering user's relative delay. Furthermore, additional performance gains can be achieved via a two-step procedure, which applies the space-time MMSE filter twice, first to estimate the interference and secondly to equalize the signal after interference cancellation.

1. INTRODUCTION

Multicarrier systems, especially OFDM, are considered as an attractive solution to combat multipath fading and accordingly facilitate high speed wireless communications. On the other hand, the efforts to use limited spectral resources efficiently result in communication systems multiplexing several users' signals on the same time-frequency slot and employing multiple receive antennas to suppress the interference. In the multiuser environment, however, the synchronization of all users' signals is not always feasible. Therefore, techniques to suppress asynchronous interference need to be developed.

Space-time alignment and partial equalization algorithms for cyclic prefix based systems were proposed for asynchronous interference suppression [1]. In addition, there is an attempt to describe cyclic prefix based asynchronous communication systems analytically and design per frequency bin antenna combining weights optimally considering the asynchronous interference [2]. However, this frequency domain interference suppression method is not effective for multicarrier systems such as OFDM due to ICI and IBI caused by the asynchronism. The work in [3] showed that a space-time domain filter gives better results when ICI due to Doppler spread exists in MIMO OFDM systems.

In this paper, the asynchronous multiuser OFDM signal model is established, and a space-time MMSE equalizer combined with interference cancellation is proposed. In Section 2, the model of asynchronous OFDM systems is explained. In Section 3, several MMSE-based equalization methods for asynchronous interference are developed. Section 4 presents the BER performance of the proposed algorithms obtained via computer simulations.

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2. ASYNCHRONOUS MULTIUSER OFDM SIGNAL MODEL

Consider the asynchronous multiuser OFDM transmission system where U active users transmit independent OFDM symbols in the same bandwidth without synchronization. Multiple receive antennas are employed to mitigate the multiple access interference, and M is the number of receive antennas. Without loss of generality, we assume that the channels from each user have the same length L, and the length of zero padding or cyclic prefix, L_g, equals $L-1$ for all users so that there is no IBI. Here, we will use the term *guard interval* to refer to both zero padding and cyclic prefix. The different distances from each user to the receiver cause the signals to arrive asynchronously. Therefore, we denote the interfering user u's delay relative to the beginning of the desired user's guard interval as $L_d^{(u)}$. In practice, OFDM systems do not use all the subcarriers. The null side carriers as frequency guard bands prevent the interference from adjacent frequency bands. Thus, we assume that only K' subcarriers out of K are used, and $\mathbf{X}^{(u)}(n_b)$ is the n_b^{th} OFDM block transmitted from user u with size $K' \times 1$. Let $\mathbf{y}^{(u)}[n_b] = \left[\mathbf{y}_1^{(u)}[n_b]^T \quad \cdots \quad \mathbf{y}_M^{(u)}[n_b]^T\right]^T$, where $\mathbf{y}_m^{(u)}[n_b]$ is the received $K \times 1$ time-domain signal vector at antenna m and ^{T} is the transpose operator. $\mathbf{y}^{(u)}[n_b]$ is obtained by synchronizing the observation window with the n_b^{th} block of user u and by removing the guard interval. Without loss of generality, user 1 is assumed to be the desired user so that our main focus is on $\mathbf{y}^{(1)}[n_b]$. For the simplicity of notation, from now on, the superscript of $\mathbf{y}^{(1)}[n_b]$ will be omitted.

The received OFDM signal, which is synchronized with respect to user 1's timing, is given as:

$$\mathbf{y}[n_b] = \mathbf{H}[n_b]\mathbf{F}\mathbf{X}(n_b) + \mathbf{v}[n_b], \tag{1}$$

where $\mathbf{X}(n_b)$ are transmitted data symbols in the frequency domain, $\mathbf{F}$ is the OFDM modulation matrix considering interferers' timing offsets, $\mathbf{H}[n_b]$ is a channel matrix, and $\mathbf{v}[n_b]$ is an $MK \times 1$

$$\mathbf{H}_m^{(1)}[n_b] = \begin{bmatrix} h_m^{(1)}[L-1;0;n_b] & \cdots & h_m^{(1)}[0;0;n_b] & & \\ & \ddots & & \ddots & \\ & & h_m^{(1)}[L-1;K-1;n_b] & \cdots & h_m^{(1)}[0;K-1;n_b] \end{bmatrix} \quad (2)$$

vector of additive Gaussian noise with power σ_v^2. Specifically,

$$\mathbf{X}(n_b) = [\ \mathbf{X}^{(1)}(n_b)^T \quad \mathbf{X}^{(2)}(n_b-1)^T \quad \mathbf{X}^{(2)}(n_b)^T \quad \cdots \quad \mathbf{X}^{(U)}(n_b-1)^T \quad \mathbf{X}^{(U)}(n_b)^T\ ]^T, \quad (3)$$

$\mathbf{F} = \text{diag}(\ \mathbf{F}^{(1)} \quad \mathbf{F}^{(2)} \quad \cdots \quad \mathbf{F}^{(U)}\ )$, and

$$\mathbf{H}[n_b] = \begin{bmatrix} \mathbf{H}_1^{(1)}[n_b] & \cdots & \mathbf{H}_1^{(U)}[n_b] \\ \mathbf{H}_2^{(1)}[n_b] & \cdots & \mathbf{H}_2^{(U)}[n_b] \\ \vdots & \vdots & \vdots \\ \mathbf{H}_M^{(1)}[n_b] & \cdots & \mathbf{H}_M^{(U)}[n_b] \end{bmatrix}. \quad (4)$$

$h_m^{(u)}[l;n;n_b]$ is the channel impulse response between user u and antenna m over the n_b^{th} block period of user u, where n represents the sampling time within the block and l is the index of multipaths. Then, $\mathbf{H}_m^{(u)}[n_b]$ is the $K \times (K+L_g)$ channel matrix related to $h_m^{(u)}[\cdot\,;\cdot\,;\cdot]$ during the n_b^{th} block time of the desired user. For example, $\mathbf{H}_m^{(1)}[n_b]$ is defined at (2). Similarly, $\mathbf{H}_m^{(u)}[n_b]$ for $u = 2, \cdots, U$, is constructed depending on the offset $L_d^{(u)}$. The modulation of OFDM is done by inverse discrete Fourier transform (IDFT), so we define $\mathbf{F}_K$ as the size K IDFT matrix with entries $e^{j\frac{2\pi kn}{K}}/\sqrt{K}$. Then, $\mathbf{F}_{K\times K'}$ is obtained by eliminating the first a and last b columns from $\mathbf{F}_K$, and $\mathbf{f}_i$ represents the i^{th} row of matrix $\mathbf{F}_{K\times K'}$. Considering cyclic prefix, the modulation matrix for the desired user is $\mathbf{F}^{(1)} = \left[\mathbf{f}_{K-L_g}^T \quad \cdots \quad \mathbf{f}_{K-1}^T \quad \mathbf{F}_{K\times K'}^T\right]^T$. Since the interfering users have timing offsets, signal samples related to the previous OFDM baud exist inside the observation window. Thus, $\mathbf{F}^{(u)}$ is given by

$$\mathbf{F}^{(u)} = \begin{bmatrix} \mathbf{F}_{n_b-1}^{(u)} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_{n_b}^{(u)} \end{bmatrix} \quad \text{for } u = 2, \cdots, U. \quad (5)$$

Here, $\mathbf{F}_{n_b-1}^{(u)}$ and $\mathbf{F}_{n_b}^{(u)}$ represent modulation of $\mathbf{X}^{(u)}(n_b-1)$ and $\mathbf{X}^{(u)}(n_b)$, respectively, and they are determined according to the timing offset. For example, if $0 < L_d^{(u)} \leqslant K$, we have $\mathbf{F}_{n_b-1}^{(u)} = \left[\mathbf{f}_{K-L_d^{(u)}}^T \quad \cdots \quad \mathbf{f}_{K-1}^T\right]^T$ and $\mathbf{F}_{n_b}^{(u)} = [\ \mathbf{f}_{K-L_g}^T \quad \cdots \quad \mathbf{f}_{K-1}^T \quad \mathbf{f}_0^T \ \cdots \ \mathbf{f}_{K-L_d^{(u)}-1}^T]^T$. For zero padded-OFDM (ZP-OFDM) [4], the rows corresponding to the location of cyclic prefix are filled with zero vectors.

The demodulated OFDM signal $\mathbf{Y}(n_b)$, is expressed as:

$$\begin{aligned} \mathbf{Y}(n_b) = \mathbf{Q}\mathbf{y}[n_b] = \\ & \mathbf{Q}\mathbf{H}^{(1)}[n_b]\mathbf{F}^{(1)}\mathbf{X}^{(1)}(n_b) \\ & + \sum_{u=2}^{U} \mathbf{Q}\mathbf{H}^{(u,1)}[n_b]\mathbf{F}_{n_b-1}^{(u)}\mathbf{X}^{(u)}(n_b-1) \\ & + \sum_{u=2}^{U} \mathbf{Q}\mathbf{H}^{(u,2)}[n_b]\mathbf{F}_{n_b}^{(u)}\mathbf{X}^{(u)}(n_b) + \mathbf{Q}\mathbf{v}[n_b], \end{aligned} \quad (6)$$

where $\mathbf{Q} = \mathbf{I}_M \otimes \mathbf{F}_{K\times K'}^H$ ($\mathbf{I}_M$ is an $M \times M$ identity matrix, $\otimes$ is the Kronecker product, and ^{H} is the conjugate transpose operator) and

$$\begin{aligned} \mathbf{H}^{(u)}[n_b] &= \left[\mathbf{H}_1^{(u)}[n_b]^T \quad \cdots \quad \mathbf{H}_M^{(u)}[n_b]^T\right]^T \\ &= \left[\mathbf{H}^{(u,1)}[n_b] \quad \mathbf{H}^{(u,2)}[n_b]\right]. \end{aligned} \quad (7)$$

(6) shows that the demodulated OFDM signal contains IBI induced by asynchronous interferers. In the next section, we presents various equalization schemes to suppress the interference from the asynchronous users.

3. ASYNCHRONOUS MULTIUSER OFDM RECEIVERS

3.1. Per-carrier (PC) MMSE diversity combining

At first, we examine per frequency bin MMSE diversity combining [2]. The received signal at frequency bin k is written as:

$$\mathbf{Y}(k, n_b) = \mathbf{\Gamma}_k \mathbf{Y}(n_b), \quad (8)$$

where matrix $\mathbf{\Gamma}_k$ consists of the rows of identity matrix $\mathbf{I}_{MK}$, which have the value '1' at the location corresponding to frequency bin k at each antenna. Then, the frequency-domain combining weights for the desired user are given by

$$\mathbf{W}_{PC,k}^{(1)} = \mathbf{R}_{\mathbf{Y}(k,n_b)\mathbf{Y}(k,n_b)}^{-1} \mathbf{\Gamma}_k \mathbf{Q}\mathbf{H}^{(1)}\mathbf{F}^{(1)}\mathbf{R}_{\mathbf{X}^{(1)}\mathbf{X}^{(1)}}, \quad (9)$$

where $\mathbf{R}_{(\cdot)(\cdot)}$ denotes the correlation matrix, and the estimates of the transmitted symbols are computed as

$$\hat{\mathbf{X}}^{(1)}(k, n_b) = dec\{\mathbf{W}_{PC,k}^{(1)}{}^H \mathbf{Y}(k, n_b)\}, \quad (10)$$

where $dec(\cdot)$ represents the decision process.

3.2. MMSE estimation in space-time domain

Frequency-domain estimation is attractive in terms of the computational complexity, but its performance is severely degraded by ICI and IBI. To see the effect of ICI, we investigate the elements of $\mathbf{QHF}$. It is usually assumed that a channel impulse response is invariant over one OFDM block duration, that is, $h[l;n;n_b] = h[l;n_b]$. For block invariant cyclic prefix-OFDM (CP-OFDM) channels, the element of $\mathbf{F}_{K\times K'}^H \mathbf{H}_m^{(1)}[n_b]\mathbf{F}^{(1)}$ corresponding to input frequency bin k' and output k is given by:

$$\left[\mathbf{F}_{K\times K'}^H \mathbf{H}_m^{(1)}[n_b]\mathbf{F}^{(1)}\right]_{k,k'} = H_m^{(1)}(k', n_b)\delta(k-k'), \quad (11)$$

where DFT$\{h_m^{(1)}[l;n_b]\} = H_m^{(1)}(k, n_b)$ and $\delta(\cdot)$ is a delta function. Therefore, for the desired user, the output signal at subcarrier k is not influenced by data symbols from other subcarriers. Next, let us consider the second term of (6), which represents the IBI due to asynchronous interferers. When $0 < L_d^{(u)} \leqslant K$, we have

$$\left[\mathbf{F}_{K\times K'}^H \mathbf{H}_m^{(u,1)}[n_b]\mathbf{F}_{n_b-1}^{(u)}\right]_{k,k'} = \frac{1}{K}\sum_{n=0}^{L_d^{(u)}-1} e^{-j\frac{2\pi n}{K}(k-k')} e^{-j\frac{2\pi k'}{K}(L_d^{(u)}-L_g)} \sum_{l=\max(0,L-L_d^{(u)}+n)}^{L-1} h_m^{(u)}[l;n_b-1]e^{-j\frac{2\pi k' l}{K}}. \quad (12)$$

The above equation shows that the matrix $\mathbf{F}_{K\times K'}^H \mathbf{H}_m^{(u,1)}[n_b]\mathbf{F}_{n_b-1}^{(u)}$ is not diagonal, that is, ICI exists. Thus, as is well known, this implies that the received signals corrupted by asynchronous interferers cannot be decoupled using DFT as conventional CP-OFDM systems. Thus, we propose space-time domain MMSE estimation to optimally suppress the ICI at the expense of increased complexity.

From now on, the OFDM block index, n_b, will be omitted for notational simplicity. The combining weights are given as

$$\mathbf{W}_{ST} = \mathbf{R}_{\mathbf{yy}}^{-1}\mathbf{HFR}_{\mathbf{XX}}, \quad (13)$$

where $\mathbf{R}_{\mathbf{yy}} = \mathbf{HFR}_{\mathbf{XX}}\mathbf{F}^H\mathbf{H}^H + \mathbf{R}_{\mathbf{vv}}$, and the estimates of the transmitted symbols are obtained by applying them to the time-domain received signals, that is, $\hat{\mathbf{X}} = dec\{\mathbf{W}_{ST}^H\mathbf{y}\}$.

3.3. Space-time MMSE estimation combined with interference cancellation

Finally, we propose a more sophisticated equalization method applying the space-time (ST) MMSE filter twice. At first, the asynchronous interference is estimated by synchronizing the observation window relative to the interferer's timing and employing the ST MMSE method. After that, all interfering signals are reconstructed and subtracted from $\mathbf{y}$ in (1). At that point, the previous OFDM block estimates are used to reconstruct the IBI. To reduce the processing delay, we can use the result in (13). In this case, however, the reliability of estimation of the interfering signals decreases as the interferer's timing offset becomes larger because of the increasing amount of IBI and ICI. The signals after interference cancellation can be written as

$$\tilde{\mathbf{y}} = \mathbf{H}^{(1)}\mathbf{F}^{(1)}\mathbf{X}^{(1)} + \sum_{u=2}^{U}\mathbf{H}^{(u)}\mathbf{F}^{(u)}\mathbf{e}^{(u)} + \mathbf{v}, \quad (14)$$

where $\mathbf{e}^{(u)} = \mathbf{X}^{(u)} - \hat{\mathbf{X}}^{(u)}$. Suppose that the interfering signals are accurately estimated and subtracted from the received signals, that is, $\mathbf{e} = 0$. Then, the ST MMSE filter after interference cancellation can be computed as follows:

$$\mathbf{W}_{ST-IC} = \mathbf{R}_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}}^{-1}\mathbf{H}^{(1)}\mathbf{F}^{(1)}\mathbf{R}_{\mathbf{X}^{(1)}\mathbf{X}^{(1)}} \quad (15)$$

The desired user's symbol estimates are obtained by filtering $\tilde{\mathbf{y}}$. Since the matrix $\mathbf{F}_{K\times K'}^H\mathbf{H}_m^{(1)}\mathbf{F}^{(1)}$ is diagonal for static or block invariant channels in CP-OFDM systems, it can be converted to the per-carrier MMSE diversity combining problem. Figure 1 shows the block diagram of the receiver which combines the ST MMSE with interference cancellation when two active users exist.

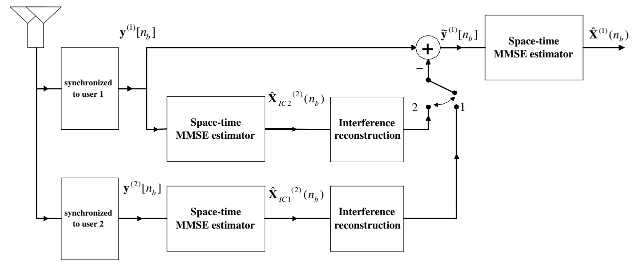


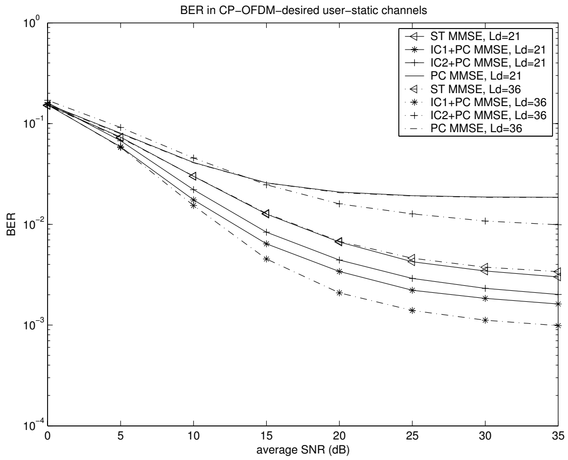
Fig. 1. Space-time MMSE receiver combined with the interference canceller

4. SIMULATION RESULTS

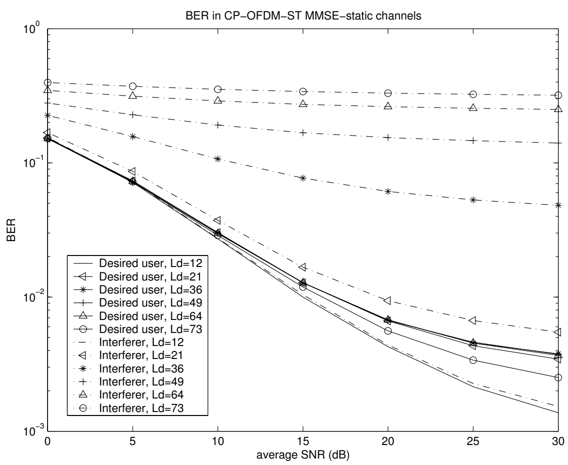
In this section, the performance of different equalization methods in Section 3 is evaluated in terms of bit error rate (BER) by computer simulations. Simulations were performed over static Rayleigh fading channels (time invariant over 10 OFDM blocks) with the Typical Urban delay profile Model of $L = 22$ [5]. The OFDM system has $M = 2$ receive antennas for two active users, and the guard interval is of length $L_g = 21$. The size of the DFT/IDFT is $K = 64$, but the first eight and last seven carriers are not used. The OFDM sub-symbols are taken from a QPSK constellation, and the interferer's signal arrives at the receiver with same strength as the desired user's. Perfect channel information and the interferer's timing offset are assumed available to all algorithms. The methods named 'IC1' and 'IC2' imply that the switch in Figure 1 is placed at the position '1' and '2', respectively. In other words, IC1 is to estimate "interfering" user's symbols from the received signal synchronized to "interfering" user, and IC2 from the received signal synchronized to "desired" user.

Fig. 2 shows a BER comparison of the different algorithms for $L_d^{(2)} = 21$ and $L_d^{(2)} = 36$ in CP-OFDM. As seen in Fig. 2 (a), the PC MMSE method yields poor BER performance. Meanwhile, note that ST MMSE curves have much lower BER compared to PC MMSE curves. Regarding the two-step procedures proposed in Sec. 3.3, PC MMSE combined with IC1 (IC1+PC MMSE) shows better performance than ST MMSE. Here, ST MMSE in the second step is replaced by PC MMSE owing to the circular convolution property of CP-OFDM sytems. When $L_d^{(2)} = 36$, ST MMSE outperforms 'IC2+PC MMSE'. This can be explained by the result in Fig. 2 (b), which demonstrates that interferer's symbol estimates are not so accurate as desired user's for $L_d^{(2)} = 36$. In addition, we observe that the performance of the ST MMSE weights for the interferer is highly affected by the interferer's offset.

Fig. 3 shows a BER comparison in ZP-OFDM. Since zero symbols are padded instead of cyclic prefix, the asynchronous interferer in ZP-OFDM induces less amount of ICI and IBI than in CP-OFDM. Fig. 3 (a) shows that 'IC1+ST MMSE' yields a dramatic performance improvement. From Fig. 3 (b), we infer that IC1 yields accurate estimates of interferer's symbols, but IC2 is not usable. To reduce the complexity, PC MMSE is applied after interference cancellation and the results are shown in Fig. 3 (a). Mostly, 'IC1+PC MMSE' has better performance than ST MMSE, but it has an error floor in high SNR. Because ZP-OFDM does not have the circular convolution property as CP-based systems, the performance of per-carrier operation is limited by ICI in high SNR.

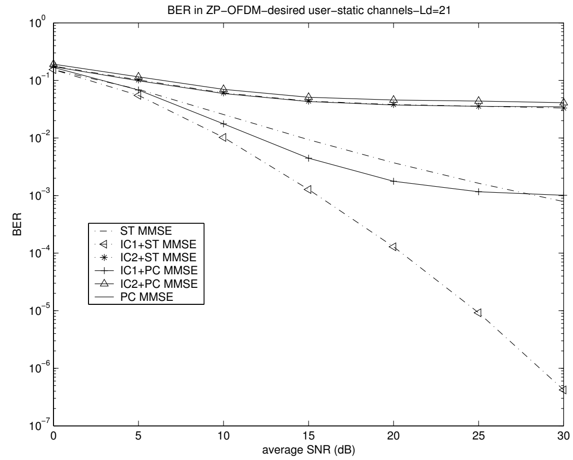


(a)

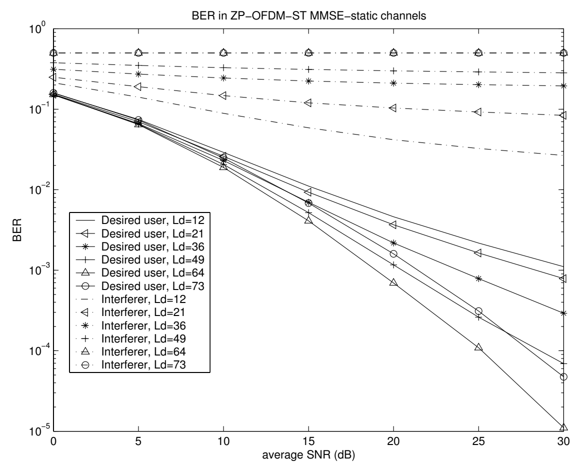


(b)

Fig. 2. BER in CP-OFDM (a) BER of the desired user when $L_d^{(2)} = 21$ and $L_d^{(2)} = 36$ (b) The BER performance of the space-time MMSE equalizer for various timing offsets of the interfering user.



(a)



(b)

Fig. 3. BER in ZP-OFDM (a) BER of the desired user when $L_d^{(2)} = 21$ (b) The BER performance of the space-time MMSE equalizer for various timing offsets of the interfering user.

5. CONCLUSION

This paper presents the generalized asynchronous OFDM signal model for both ZP- and CP-based systems, and the optimal space-time domain MMSE weights are derived based on this model. Furthermore, space-time MMSE estimation combined with interference cancellation is proposed. Since the space-time MMSE method can produce reliable estimates of the interference, insertion of the interference cancellation scheme improves the receiver performance. The performance of the ZP-OFDM system improves dramatically. However, the interference estimation algorithm needs to be carefully selected according to the interferer's timing offset.

6. REFERENCES

[1] M. B. Breinholt, M. D. Zoltowski, and T. A. Thomas, "Space-time equalization and interference cancellation for MIMO OFDM," in *Thirty-Sixth Annual Asilomar Conf. on Signals, Systems, and Computers*, Nov. 2002, pp. 1688–1693.

[2] T. A. Thomas and F. W. Vook, "Frequency-domain asynchronous interference suppression in cyclic-prefix communications," in *41st Annual Allerton Conf. on Communication, Control, and Computing*, Oct. 2003.

[3] A. Stamoulis, S. N. Diggavi, and N. Al-Dhahir, "Intercarrier interference in MIMO OFDM," *IEEE Trans. on Signal Processing*, vol. 50, no. 10, pp. 2451–2464, Oct. 2002.

[4] B. Muquet, Z. Wang, G. B. Giannakis, M. de Courville, and P. Duhamel, "Cyclic prefixing or zero padding for wireless multicarrier transmissions?," *IEEE Trans. on Communications*, vol. 50, no. 12, pp. 2136–2148, Dec. 2002.

[5] *3G TR 25.943: 3rd Generation Partnership Project; Technical Specification Group (TSG) RAN WG4; Deployment aspects*.