# DESIGN TRADEOFFS IN OFDMA TRAFFIC CHANNELS

*Manyuan Shen, Guoqing Li and Hui Liu*

Department of Electrical Engineering, Box 352500
University of Washington, Seattle, WA 98195
{mshen, lgq, hliu}@ee.washington.edu

**ABSTRACT**

This paper studies uplink traffic channel design tradeoffs for broadband OFDMA systems. For fixed and portable users where the propagation channels are semi-static, we show that the uplink capacity can be maximized when users' traffic channels are configured with the maximum frequency selectivity. For mobile services on the other hand, we show that the optimality depends on the outage threshold associated with the "outage capacity." In low outage probability region, the outage capacity is maximized when the traffic channels are configured with the maximum frequency diversity. The opposite is true for the high outage probability region. Using the results presented, designers of OFDMA system can determine the optimum traffic channel configuration based on the types of services supported by the network.

## 1. INTRODUCTION

In the past decade, orthogonal frequency division multiplexing (OFDM) has emerged as one of the prime modem schemes for broadband wireless networks (e.g., DVB-T, Wi-Fi). For multiuser communications, one way of applying OFDM is to divide the total bandwidth into traffic channels, each comprised of a set of OFDM subcarriers, so that multiple access can be accommodated in an orthogonal frequency division multiple access (OFDMA) fashion [1], see for example, IEEE 802.16a.

Theoretically, each OFDM subcarrier can be assigned to a different user in multiple-access. In practice however, a single-subcarrier based traffic channel is hard to implement and often too small to provide the basic services. The traffic channel configuration (i.e., how a set of OFDM subcarriers is grouped into a traffic channel) is an important factor in OFDMA system design. Intuitively, a "tight" traffic channel comprised of consecutive subcarriers may be more desirable for fixed applications due to its rich "frequency-selectivity", which can transfer into higher capacity using dynamic resource allocation. The reverse might be true for mobile scenarios where "spread" traffic channels bearing rich "frequency diversity" are more essential in coping with rapid fading channels. For systems that support mixed services (fixed, portable, and mobile), conflicting requirements present an interesting design challenge.

The objective of this paper is to provide a quantitative analysis on the effect of traffic channel configuration on OFDMA system performance. The remainder of the presentation is organized as follows. In Section II, the system model is described and the design parameters are specified. Section III analyzes the total uplink data rate with channel allocation for fixed and portable applications. In Section IV, the capacity of mobile system is evaluated for fast varying channels. Section V demonstrates the design trade-offs for both fixed and mobile services through numerical results. The paper is then concluded in Section VI.

## 2. OFDMA DESIGN PARAMETERS

An OFDMA system is defined as one in which a subset of OFDM subcarriers forms an OFDMA *traffic channel*, each of which is assigned exclusively to one user at any time. Consider an OFDMA system with a total number of $N$ subcarriers and $K$ users. Since $N$ is typically a large number, we divide the $N$ subcarriers into $L$ traffic channels, each with $M$ subcarriers: $N = L \times M$.

Theoretically, the mapping from subcarriers into traffic channels can be arbitrary. In practice however, regular mapping is often utilized for easy implementation. In this paper, we concentrate on regular mapping of the traffic channel where the $M$ subcarriers in each traffic channel are further divided into $M/M_c$ clusters, with each cluster having $M_c$ consecutive subcarriers. Spacing between two clusters is $M_d = N/(M/M_c)$ – see Fig. 1 for illustration. Under these assumptions, a traffic channel configuration, $C$, can be uniquely defined as $\{(M, M_c)\}$. In this particular example, each traffic channel in Fig. 1 has 8 subcarriers, divided into two clusters each with 4 subcarrier; the distance between two clusters is 16. Hence, this configuration is denoted as $\{(8, 4)\}$. Generally speaking, an $\{(M, M)\}$ configuration has the tightest traffic channel, and thus the maximum frequency selectivity, whereas an $\{(M, 1)\}$ configuration has the loosest traffic channel with the highest frequency diver-

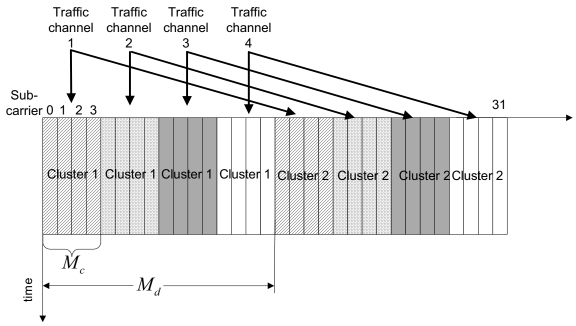


**Fig. 1**. Illustration of the traffic channel configuration: $\mathcal{C}$={(8,4)}.

sity.

For the OFDMA system under consideration, let $\Omega_l = \{l_1, ..., l_M\}$ be the configured subcarrier indices of traffic channel $l$. We define $\mathbf{h}_l = [h_{l_1}, h_{l_2}, ..., h_{l_M}]^T$ as the channel vector corresponding to the $l^{th}$ traffic channels. If needed a superscript $k$ will be added (i.e., $\mathbf{h}_l^k$) to denote the $k^{th}$ user's channel response on the $l^{th}$ traffic channel.

## 3. TRAFFIC CHANNEL CONFIGURATION FOR FIXED/PORTABLE APPLICATIONS

For fixed or portable services with static or semi-static channels, **h** is a random vector that can be treated as a constant in time. The fact that the channel response vectors are highly diverse among users indicates that "multiuser diversity" can be exploited through intelligent traffic channel loading [2].

In this section, we examine the OFDMA uplink capacity as a function of the traffic channel configurations with intelligent traffic channel loading. To this end, we derive the statistical characteristics of $\mathbf{h}_l$, based on which we calculate the uplink capacity.

Using ACM, the achievable data rate of the $k^{th}$ user in the $l^{th}$ traffic channel, $r_l^k$, can be expressed as a function of the average SNR, $v_l^k$, on that traffic channel, [1]: $r_l^k = M \cdot g(v_l^k)$. To perform channel allocation based on users channel response vectors , we normalize the channel, $\|\mathbf{h}\| = 1$, so that the propagation loss factor is disregarded. The average SNR experienced by the $k^{th}$ user on the $l^{th}$ traffic channel, $v_l^k$, is given as follows [2]: $v_l^k = \frac{p_l^k}{M\sigma^2}$, where the signal power over the $l^{th}$ traffic channel, $p_l^k$, is calculated as

$$p_l^k = \|\mathbf{h}_l^k\|^2. \tag{1}$$

Clearly, the uplink capacity is maximized when each traffic channel is allocated to the user with the maximum achievable data rate in that channel. The achievable rate over the $l^{th}$ traffic channel is given by

$$r_{l,\max} = \max\{r_l^1, r_l^2, ...r_l^K\},\ l = 1, \cdots, L. \tag{2}$$

Note that the rate-SNR function $g(\cdot)$ is non-decreasing, hence a traffic channel will always be allocated to the user with the highest SNR, or equivalently, the user with the highest power $p_l^k$. Then (2) can be rewritten as $r_{l,\max} = M \cdot g\left(\frac{p_{l,\max}}{M\sigma^2}\right)$, where $p_{l,\max} = \max\{p_l^1, p_l^2, ...p_l^K\}$. Using the total uplink data rate as a performance measure, the total uplink data rate is

$$r_{sys} = \sum_{l=1}^{L} r_{l,\max} = M \sum_{l=1}^{L} g\left(\frac{p_{l,\max}}{M\sigma^2}\right), \tag{3}$$

and the normalized throughput of the system (bits/s/Hz) is

$$\bar{r}_{sys} = r_{sys}/N. \tag{4}$$

Assuming that all uplink channels have the same statistics with normalized mean value (through power control), the correlation matrix $\mathbf{R}_{\mathbf{h}_l^k \mathbf{h}_l^k}$ is the same for all $k$ and all $l$. Therefore, all $p_l^k$ have the same probability density function (pdf) $f_p(x)$. It is easy to show that $p_l^k$ is a random variable with the characteristic function as follows

$$\psi_p(jw) = \prod_{m=1}^{M_1} \frac{1}{1 - jw\lambda_m}, \tag{5}$$

where $\lambda_1, ..., \lambda_{M_1}$ are the $M_1$ non-zero eigenvalues of correlation matrix $\mathbf{R}_{\mathbf{h}_l^k \mathbf{h}_l^k} = E\left\{\mathbf{h}_l^k \left(\mathbf{h}_l^k\right)^H\right\}$.

Since channels from different users are independent, it can be shown through straightforward manipulations that the pdf of $p_{l,\max}$ is,

$$f_{p_{l,\max}}(x) = K F_p(x)^{K-1} f_p(x), \tag{6}$$

where $F_p(x)$ is the CDF of random variable $p$. Clearly, all $p_{l,max}$ for $l = 1, ..., L$ have the same pdf, and we drop the subscript $l$ to simplify the notation. With the pdf of $p_{max}$ in hand, the average normalized capacity of the whole system is calculated from (3) and (4) as

$$E\left[\bar{r}_{sys}\right] = E\left[g\left(\frac{p_{max}}{M\sigma^2}\right)\right]. \tag{7}$$

[1] For simplicity, we assume that the average SNR of the subcarriers in a traffic channel is used to determine the ACM selection. Other more sophisticated schemes may be used as well without affecting the ensuing analysis. The actual rate-SNR function $g(\cdot)$ depends on the available ACM schemes and the BER targeted.

[2] Here we assume that no power control is used at the transmitter side and that the transmission power is unit on each subcarrier.

## 4. TRAFFIC CHANNEL CONFIGURATION FOR MOBILE APPLICATIONS

While the results from the previous section favor traffic channel configurations with higher frequency selectivity and lower frequency diversity. The situation is intuitively opposite for mobile applications. For users with fast fading channels, it is usually impractical for the base station to perform optimum channel allocation due to intensive overhead and difficulties in channel estimation. The rapid varying channel also makes the application of ACM less feasible.

Given that the channel vector is a random process, we resort to the the outage capacity [4] to analyze the system performance against different configurations. For convenience, we also assume that unit power is allocated to each subcarrier. Based on the formula for maximum mutual information of the parallel channels [3], the maximum mutual information of traffic $l$ is

$$I_l = \frac{1}{M}\sum_{k=1}^{M}\log(1+\frac{\|h_{l_k}\|^2}{N_0}).$$

The traffic channel configuration affects the correlation among $h_{l_k}$, and therefore, the capacity $I$ potentially. The outage probability for a given rate $r$, $P_{out}(r)$, is defined as the probability that $I$ falls below $r$: $P_{out}(r) = P(I < r)$, and the outage capacity, $r(\varepsilon)$, is the largest $r$ such that outage probability is less that a given probability $\epsilon$, i.e., $r(\varepsilon) = \sup_{\{r:\, P_{out}(r)<\varepsilon\}} r$. Denote $F_I$ as the CDF of the random variable $I$, then $P_{out}(r) = F_I(r)$, and $r(\varepsilon) = \sup_{\{r:\, F_I(r)<\varepsilon\}} r$.

When the SNR is small, we can approximate $I_l$ as$I_l \approx \frac{1}{M}\log(1+\frac{\sum_{k=1}^{M}\|h_k\|^2}{N_0})$.With the aid of (1) and (6), the CDF of $I_l$ can be calculated through CDF of $\|\mathbf{h}_l\|^2$:

$$P_{out}(r) = F_p(N_0(e^{Mr}-1)). \tag{8}$$

When the SNR is large, the following approximation can be invoked $I_l \approx \frac{1}{M}\sum_{k=1}^{M}\log(\frac{\|h_k\|^2}{N_0})$. In this case, the distribution of $I_l$ is difficult to obtain. In order to make the problem tractable, we approximate $log(x)$ as $ax+b$, and the value of $a$ and $b$ depend on the region of SNR. As a result, $I_l \approx \frac{1}{M}a\sum_{k=1}^{M}\frac{\|h_k\|^2}{N_0}+b$. Using the similar argument for small SNR regime, we obtain the outage probability for large SNR case:

$$P_{out}(r) = F_p(\frac{N_0M(r-b)}{a}). \tag{9}$$

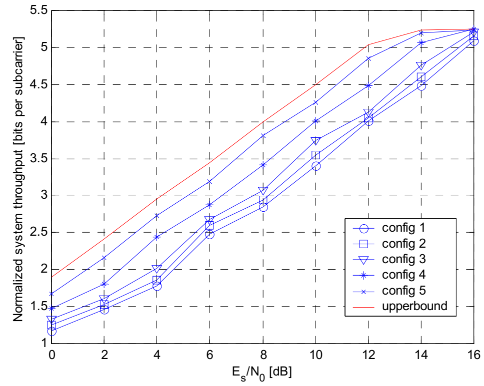


**Fig. 2**. Average normalized system capacity vs. SNR. K=64, TU channel and traffic channel coinfigurations are: $C_1$={(16, 1)}, $C_2$={(16, 2)}, $C_3$={(16, 4)}, $C_4$={(16, 8)}, $C_5$={(16, 16)}.

## 5. DESIGN TRADE-OFFS

In this section, we elaborate the effect of traffic channel configuration on system capacity using numeric examples.

### 5.1. Fixed and portable services

Fig. 2 compares the normalized system capacities of different traffic channel configurations with the upper bound, which is achieved by allowing transmitter side power allocation. In our simulation, we use the Typical Urban (TU, non-hilly) power delay profiles defined in COST207. The OFDMA system has a bandwidth of 8MHz, $N = 1024$ subcarriers, and $M = 16$ subcarriers in each traffic channel.

Comparing with the upper bound, the capacity corresponding to configuration $C_5$ ( $M_c = M$, i.e., consecutive subcarriers for most frequency selectivity) is only less than 1 dB below the upper bound. The small gap indicates an insignificant gain from transmitter side power allocation. In light of the trivial gain and complicated implementation, transmitter side power loading does not seem to have high practical values.

As expected, $C_5$ has the highest system capacity among all channel configurations. At $E_s/N_0 = 10$dB, the capacity of $C_5$ is about 25% higher than that of $C_1$ (maximum frequency diversity, least frequency selectivity). The 3-4 dB performance gap between $C_1$ and $C_5$ essentially quantifies the potential gain of OFDMA traffic channel design.

Based on the above observations, one can conclude that clusters should be grouped as tight as possible to enable higher frequency selectivity, leading to higher system capacity.

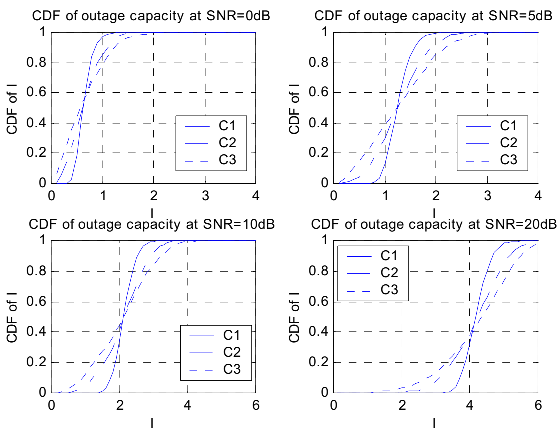


**Fig. 3**. CDF of $I$, i.e., $P_{out}(I < r)$. Three configurations are compared: $C_1$={(16, 1)}, $C_2$={(16, 8)}, $C_3$={(16, 16)}.

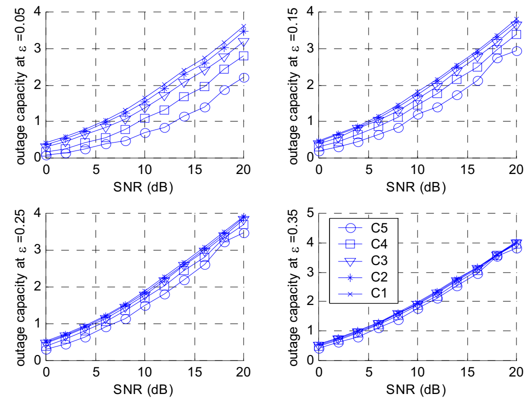


**Fig. 4**. Outage capacity vs. SNR. Traffic channel configurations are: $C_1$={(16, 1)}, $C_2$={(16, 2)}, $C_3$={(16, 4)}, $C_4$={(16, 8)}, $C_5$={(16, 16)}.

### 5.2. Mobile services

We now examine the effect of traffic channel configuration by evaluating (8) and (9). Fig. 3 gives the CDF of $I$ at different SNR levels. It is seen that for the outage probability range of practical interest (e.g., from 0 to 0.4), the outage capacity is the highest for configuration $C_1$ with the largest frequency diversity, and is the lowest for configuration $C_3$ with the least frequency diversity. However, if very high outage is permitted, the situation is the reverse. $C_3$ would yields the best performance and $C_1$would be the worst. This is explained by the fact that different configurations have the same expected capacity. For any two configurations, their CDF must have an intersection, otherwise, their expected capacity values cannot be the same.

Fig. 4 shows the outage capacity versus SNR at different outage requirements. The figure shows the situation when the outage capacity is in a reasonable region. It is seen that for the outage probability range of practical interest, the outage capacity is the highest for configuration $C_1$ with the largest frequency diversity, and is the lowest for configuration $C_5$ with the least frequency diversity. As the outage probability increases, the differences in system performance corresponding to different configurations diminish.

Based on the above observations, one can conclude that for applications with small output probability requirements, the clusters should be distributed to enable higher diversity; the opposite is true for applications that can tolerate high outage probabilities.

## 6. CONCLUSIONS

In this paper, we have studied the effect of traffic channel configuration on the OFDMA system performance. The uplink capacity with channel allocation and the outage capacity have been derived to evaluated the impact of different traffic channel designs. The following results are observed: (i) For static/semi-static channel with channel loading, the optimum traffic channel configuration is the one with the highest frequency selectivity (i.e., highest multi-user diversity) or least frequency diversity. (ii) For fast fading channel with a low outage probability requirement, the configuration with lowest frequency selectivity/highest frequency diversity gives the best performance. Although the conclusion for fixed/portable and mobile services are distinct, the results provide a guideline for OFDMA design. For networks that support mixed services, the best trade-offs will depend on many variables including the ratio of fixed/portable and mobile users, available adaptive coded modulation (ACM) schemes and spatial diversity techniques that can enhance the frequency diversity in a fast fading environment.

## 7. REFERENCES

[1] C. Y. Wong et. al. "Multiuser OFDM with Adaptive Subcarrier, Bit, and Power Allocation", *IEEE J. Select. Areas Commun.*, vol. 17, pp.1747-1758, Oct. 1999.

[2] D. Tse, "Optimal Power Allocation over Parallel Gaussian Broadcast Channels", *Proceedings of International Symposium on Information*, Ulm Germany,June 1997, p. 27.

[3] Thomas M. Cover and Joy A. Thomas, Elements of information theory, Wiley series in Telecommunications, 1991.

[4] I. Emre Telatar, "Capacity of Multi-antenna Gaussian Channels," Technical report, Bell lab, Rm. 2C-174.