OFDM VS. SINGLE-CARRIER: A MULTI-ANTENNA COMPARISON

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Abstract— **There is an ongoing discussion in the broadband wireless world about the respective benefits of Orthogonal-Frequency-Division-Multiplexing (OFDM) and Single-Carrier with Frequency-Domain equalization (SC-FD). SC-FD allows for more relaxed front-end requirements, of which the power amplifier efficiency is very important for battery-driven terminals. OFDM, on the other hand, can yield improved performance at low complexity. Both schemes have extensions to multiple antennas to enhance the spectral efficiency and/or the link reliability. Moreover, both schemes have non-linear versions using Decision-Feedback Equalization (DFE) to further improve performance of the linear equalizers. In this paper, we compare these high-performance OFDM and SC-FD schemes using multiple-antennas and DFE, while also accounting for the power amplifier efficiency. To make a realistic comparison, we also consider most important digital imperfections such as channel and noise estimation, transmit and receive filtering, clipping and quantization as well as link layer impact. Our analysis shows that for frequency-selective channels the relative performance impact of the power amplifier is negligible compared to the frequency diversity impact. The higher frequency diversity exploitation of SC-FD allows it to transmit more efficiently than OFDM in most cases. Therefore, SC-FD is a suitable candidate for broadband wireless communication, especially for battery-powered up-link transmission.**

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a popular, standardized technique for broadband wireless systems: it is used for Wireless LAN [1], [2], Fixed Broadband Wireless Access [3], Digital Video & Audio Broadcasting [4], [5],... Several authors, such as [6], have shown that OFDM can reach high spectral efficiency at low equalization complexity. In recent years, Single-Carrier with Frequency-Domain equalization (SC-FD) has received a lot of attention as an alternative technique for broadband wireless communications [7]. Studies [8], [9] show that SC-FD can allow for a more power efficient transmitter, which is very important for battery operated mobile terminals. Recently, this comparison has gained more attention, since both schemes have been included in the IEEE 802.16a standard for Fixed Broadband Wireless Access [3].

Multiple antennas allow to increase the spectral efficiency and/or to improve the link reliability. Space-Division-Multiple-Access (SDMA) implements multi-user access spectrally efficiently [10]. Moreover, [10] shows that Decision-Feedback Equalization (DFE) improves the performance of OFDM-SDMA, namely by applying the so-called per-carrier Successive-Interference-Cancellation (OFDM-pcSIC). In [11], we derived a multiple-antenna SC-FD-DFE based on the SISO SC-FD-DFE as introduced by [12], [13]. We compare the performance between the SC-FD-DFE and OFDM-pcSIC.

Two very important points of comparison are the performance in a multi-path environment and the impact of the Power Amplifier (PA). OFDM and SC-FD are designed for transmission over a frequency-selective channel and their cyclic extension is useful only over such a channel. On the other hand, the back-off of the power amplifier determines the power efficiency of the transmitter, an important aspect for a wireless up-link as the terminal is battery-powered.

Previous comparisons have not investigated the impact of the power amplifier on the Bit Error Rate (BER) performance over a multi-path channel. For example, [7], [12], [13] only compare the ideal multi-path performance and do not consider the impact of the power amplifier. [9] only considers the spectral regrowth due to the power amplifier, but not the impact on the performance, while [8] considers the effect on the BER performance only for AWGN channels.

In this paper, we compare SC-FD and OFDM in a realistic multi-user scenario with Decision-Feedback-Equalization. In other words, we compare SC-FD-DFE and OFDM-pcSIC, taking into account the impact of the power amplifier and most important digital imperfections, such as channel and noise estimation, clipping and quantization, transmit and receiver filtering. Moreover, we account for coding and retransmission enabling us to compare the useful throughput.

The paper is structured as follows. In section II, we briefly introduce OFDM and SC-FD as well as the multi-antenna DFE concept for both schemes. Section III evaluates the performance of both SC-FD-DFE and OFDM-pcSIC under realistic multi-antenna conditions. Finally, the conclusions obtained in this paper are presented in section IV.

II. OFDM VS SINGLE-CARRIER WITH FREQUENCY-DOMAIN EQUALIZATION

In this section, we compare the basic properties of OFDM and SC-FD and indicate the main differences and similarities. We introduce the decision-feedback concept for both schemes.

A. OFDM

OFDM offers an elegant way for low-complexity equalization in frequency-selective channels by dividing the total bandwidth into smaller flat-fading sub-bands.

However, uncoded OFDM loses all frequency diversity inherent in the channel: a dip in the channel erases the information data on the sub-carriers affected by the dip and this information cannot be recovered from the other carriers. This mechanism results in a poor BER performance. Adding sufficiently strong coding spreads the information over multiple sub-carriers. This recovers frequency diversity and improves the BER performance.

B. Single-Carrier with Frequency-Domain equalization

SC-FD [7] offers similar low-complexity multi-path mitigation. However, it transmits the data in the time domain. This has a very important consequence: the information of each SC-FD symbol is spread out over the complete frequency band.

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This means that dips in the channel do not wipe out complete symbols, because the information of each symbol can be recovered from the other carriers. Therefore, SC-FD exploits the frequency diversity in the channel and thus has a better uncoded performance than OFDM [7].

SC-FD comes in different flavours, as the channel can be made cyclic by either inserting a Cyclic Prefix (CP) or a Training Sequence (TS), as indicated in figure 1. The overhead is very similar for both approaches. However, SC-TS is able to exploit the full multi-path diversity, while this is not guaranteed for SC-CP [14]. Moreover, as the Training Sequence is a known sequence, it can be used for synchronization, tracking and training [15] and it eases the DFE start-up [13].

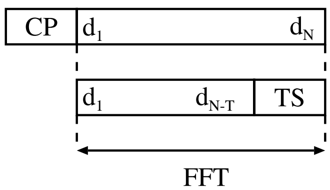

Fig. 1. Single Carrier with Cyclic Prefix (CP) vs. Training Sequence (TS).

C. Decision-Feedback Equalization

We aim to exploit Decision-Feedback Equalization (DFE) to eliminate interference caused by multi-path or by other users.

OFDM can eliminated multi-user-interference (MUI) e.g. by per-carrier successive interference cancellation or pcSIC as described in [10]. SC-FD can eliminate MUI by the Single-Carrier with Frequency-Domain processin and Decision-Feedback-Equalization (SC-FD-DFE) as we derived in [11]. Moreover, SC-FD-DFE can also eliminate multi-path interference (MPI) and thus also obtains an improvement over linear equalization for SISO communication [13], [12].

III. Multi-antenna SC-FD-DFE versus OFDM-pcSIC

OFDM and SC-FD react differently to the deviations from these ideal conditions, so they need to be considered in order to make a fair comparison between both schemes.

A. Power amplifier

For non-constant envelope signals a linear power amplifier is needed. We assume a class A power amplifier with back-off because of its linearity. The back-off determines the power consumption of the power amplifier and also its linear dynamic range. Since the linear dynamic range directly relates to the distortion, the back-off also determines the Bit Error Rate. The linearity of the power amplifier is quantified by the 1-dB-compression point P_{1dB}, defined as the input power at which the non-linearity lowers the output power by 1 dB compared to the ideal amplifier.

The baseband representation of a class A power amplifier with linear amplification G_a and a cubic non-linearity is [16]

$$\mathbf{y} = \mathbf{x} \cdot G_a \cdot (1 - \alpha \frac{3}{4} |\mathbf{x}|^2) \tag{1}$$

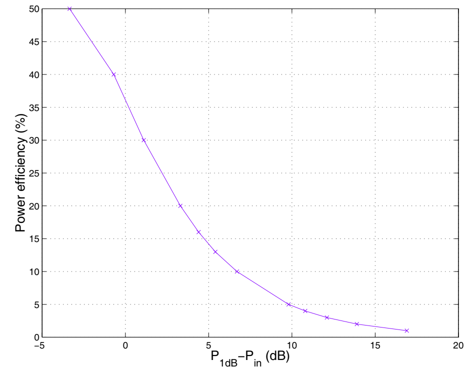

Fig. 2. Power efficiency of a class-A power amplifier as a function of the back-off $P_{1dB} - P_{in}$.

with $\mathbf{x}$ the input baseband signal and $\mathbf{y}$ the output and

$$\alpha = \frac{4}{3(1 - 10^{-1/20}) P_{1dB}^2} \tag{2}$$

In our setup, we set the average input power $P_{in} = 6dBm$; the linear gain of the power amplifier is $23dB$, such that we operate at $29dBm$ average output power, a specified maximum average output power for the 5GHz band [1]. The higher the P_{1dB} compression point, the further the signal is separated from the distortion area of the power amplifier transfer characteristic and thus the smaller the BER performance degradation. However, the larger the back-off between P_{in} and P_{1dB} the smaller the power amplifier efficiency, as can be seen in figure 2. A class A power amplifier has a theoretical maximum efficiency of 50%. This efficiency drops rapidly with increasing back-off.

OFDM has a large dynamic range compared to SC-FD [9]. Therefore, the P_{1dB}-P_{in} back-off needs to be larger for OFDM to accommodate the signal in the linear range of the power amplifier transfer function. Theory and simulations indeed show that in an AWGN channel the performance advantage of SC-FD over OFDM increases as the PA back-off decreases.

Coding helps to overcome the lack of frequency diversity, bringing the OFDM and SC-FD curves closer together. However, coding does not change the impact of the power amplifier back-off. The difference in performance between OFDM and SC-FD remains dominated by the difference in frequency diversity, which is determined by the code rate. For high code rates (and uncoded transmission) SC-FD outperforms OFDM ; for lower code rates, the performance becomes comparable.

B. Digital imperfections

To make a realistic comparison, we include channel and noise estimation, transmit and receive filtering and clipping and quantization in our simulation model as shown in figure 3.

Channel and noise estimation Since the channel has a limited number of taps (L) in the time domain, we know that all the power in the taps $\geq L + 1$ can be attributed to noise. This has 2 consequences: first, it allows to have an improved channel estimate by removing part of the noise. Second, the power of the

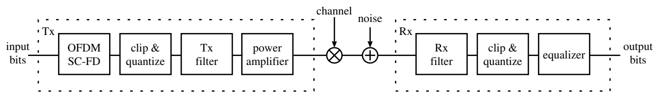

Fig. 3. The setup for the simulation.

noise which is removed is used to obtain a noise estimate. The analysis of these estimates can be found in [17].

In case of multiple transmit and/or receive antennas we repeat the above process for each antenna pair separately. [10] shows more performant multi-antenna channel estimation schemes exist. However, we just want to make a realistic and fair comparison between the OFDM and SC-FD schemes. The above frequency domain estimation can also be directly applied to SC-FD, ensuring a fair comparison. In practice, a time domain sequence will be used for SC-FD channel estimation, which is designed to have a flat frequency response, but this is beyond the scope of this paper.

Transmit and receive filtering The OFDM and SC-FD symbols need to be filtered to limit the out-of-band radiation. Therefore, we apply a Square-Root-Raised-Cosine (SRRC) filter at transmitter and receiver with a roll-off $\alpha = 0.25$, a delay of 5 taps and an oversampling by 4.

Clipping and quantization OFDM is clipped and quantized to limit the signal's dynamic range or the so-called the Peak-to-Average-Power-Ratio (PAPR). [16] shows clipping at 4σ and a quantization of 8 bits to be a good solution for a realistic OFDM system. SC-FD needs to be clipped and quantized as well, but because of its limited dynamic range the impact is a smaller.

C. *Performance*

We present the simulation results of the OFDM vs. SC-FD comparison for a WLAN case study: we use the system parameters for OFDM as described in the IEEE standard [1] and choose the equivalent SC-FD such that the comparison is fair. For both schemes the results for coded (R=3/4) BPSK transmission are presented. We assume uncorrelated 4-taps Rayleigh channels for all antenna pairs.

C.1 Single-user single-antenna

The simulation results shows that the frequency diversity dominates over the power amplifier back-off impact on performance. More specifically, SC-FD outperforms OFDM by 4.2 and 5.2 for linear MMSE equalization at goodput G=0.9 respectively 0.95 as shown in table 1. The goodput G is defined as the normalized effective throughput at MAC-level, taking into account transmission and protocol overhead and retransmissions.

C.2 Multi-User SDMA

In this scenario, we consider U users (each with 1 antenna) transmitting to a base station with A receive antennas, equal to the number of users. As the number of receive antennas and the number of users increases, so does the receive diversity and the MUI for both schemes. The receive diversity helps OFDM to overcome the lack of frequency diversity. Therefore, as the number of antennas goes up, the relative diversity difference between OFDM and SC-FD becomes smaller and their performances converge. Table 1 shows that the SC-FD advantage gradually decreases towards 0 as the number of users increases. The number in the following tables are given for $P_{1dB} = 4.4dBm$; the impact of P_{1dB} on the performance comparison turns out to be negligible. The SC-FD advantage increases if a larger goodput is targeted, because of the higher frequency diversity exploitation.

Number of Antennas	1	2	3	4
Goodput=0.90	4.2	2.8	1.5	0.5
Goodput=0.95	5.2	3.8	2.3	0.8

Table 1. SC-FD advantage (dB) over OFDM for MMSE SDMA as a function of the number of antennas (A)

Decision-Feedback Equalization allows to eliminate interference. In case of OFDM, pcSIC allows to eliminate pre- and post-cursor ISI, whereas in case of SC-FD, SC-FD-DFE only eliminates post-cursor ISI [13], [18]. This means OFDM possibly can eliminate the MUI completely, while SC-FD can only partly. Therefore, OFDM can outperform SC-FD for a large number of users, unlike the MMSE convergence for both schemes. Simulations show that for our setup, for 2-by-2 the advantage decreases rapidly to about 1.5 dB and OFDM performs better than SC-FD if $N_a \geq 3$ (table 2).

Number of Antennas	1	2	3	4
Goodput=0.90	5.4	1.6	-0.5	-1
Goodput=0.95	6	1.5	-1	-1.4

Table 2. SC-FD advantage (dB) over OFDM for DFE SDMA as a function of the number of antennas (A).

C.3 Multi-Antenna Receive Diversity

In this scenario, we consider 1 active user with 1 antenna while the base station has A receive antennas.

As the number of receive antennas increases, the receive diversity rapidly increases the performance of both schemes, since there is no extra inter-user-interference to counter the diversity benefit. Since the receive diversity also helps to compensate the lack of frequency diversity, both OFDM and SC-FD converge to the same performance. Simulations show that the SC-FD advantage over OFDM, rapidly decreases with increasing number of receive antennas:

Number of Antennas	1	2	3	4
Goodput=0.90	4.2	1.2	0.5	0
Goodput=0.95	5.2	1.8	0.6	0.2

Table 3. SC-FD advantage (dB) over OFDM for MMSE receive diversity as a function of the number of antennas (A).

In the single-user case, SC-FD performs the DFE through the SC-FD-DFE as described in [12], [13], while for single-user OFDM the pcSIC algorithm reduces to linear equalization. Therefore, the single-user SC-FD advantage over OFDM increases for the DFE compared to MMSE (table 4).

Number of Antennas	1	2	3	4
Goodput=0.90	5.4	1.6	0.7	0
Goodput=0.95	6	2.5	1	0.4

Table 4. SC-FD advantage (dB) over OFDM for DFE receive diversity as a function of the number of antennas (A).

Again larger goodputs increase the SC-FD advantage.

D. Interpretation of the results

The comparison between OFDM and SC-FD has yielded the following results.

- The relative impact of the power amplifier back-off on the multi-path performance is negligible compared to the frequency diversity impact.
- Comparing OFDM to SC-FD in a single-user single-antenna context, we observe that linear SC-FD outperforms OFDM by 4-5 dB because of its higher frequency diversity exploitation. Non-linear DFE provides SC-FD with an extra 1 dB advantage.
- For a multi-user, multi-antenna scenario (SDMA), the linear MMSE SC-FD advantage of 4-5 dB gradually decreases as the number of users increases, until the performance SC-FD and OFDM converge for about 4 antennas. In a multi-user context, DFE is relatively beneficial for OFDM than for SC-FD. Therefore, OFDM starts outperforming SC-FD for 3 or more users.
- In a single-user, multi-antenna context (receive diversity), the linear MMSE SC-FD advantage rapidly decreases as the number of antennas increases, until OFDM and SC-FD converge. In a single-user context, DFE is relatively more beneficial for SC-FD, thus it increases the advantage SC-FD has over OFDM.
- Targeting a larger goodput always increases the SC-FD advantage over OFDM.

E. Remarks

Code rate The difference in performance between OFDM and SC-FD depends also on the code rate R and the constellation size. In this paper, we have taken $R = 3/4$ as specified in both the IEEE802.11a and HIPERLAN-II standards for BPSK transmission. The code $R = 3/4$ makes a trade-off between code performance and code overhead. For code rates larger than $R = 3/4$, the frequency diversity advantage of SC-FD increases, while for smaller code rates it decreases. While this changes the absolute numbers of the respective degradations, the general conclusions as formulated in section III-D remain valid.

Modulation The BPSK transmission is the modulation scheme with the largest range. Moreover, for BPSK the power amplifier back-off creates the largest impact difference between SC-FD and OFDM. Since we have seen that this impact is small in a multi-path environment, BPSK simulations provide the strongest support for this assertion. Therefore, we believe the ($R = 3/4$, BPSK) setting is a relevant case study.

IV. Conclusions

In this paper, we compared the multiple-antenna performance of OFDM and SC-FD as modulation schemes for broadband wireless communication for linear and non-linear equalization. We have taken into account most important non-idealities to obtain a realistic comparison: the power amplifier of the transmitter, most digital imperfections as well as link layer efficiency.

Because of its higher frequency diversity exploitation, SISO SC-FD outperforms OFDM by 4-5 dB in our case study. Adding multiple antennas decreases the SC-FD advantage over OFDM until the linear equalizer performances converge for a large number of antennas. DFE increases the SC-FD advantage in single-user scenarios, while it decreases the advantage in a multi-user context, such that OFDM can outperform SC-FD for a large number of antennas.

In general, SC-FD is a suitable scheme for broadband wireless communication. especially for up-link transmission.

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