PERFORMANCE OF TRAINING-BASED OFDM SYSTEMS IN THE PRESENCE OF TIME VARYING FREQUENCY-SELECTIVE CHANNELS

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ABSTRACT

We consider a single user OFDM system that experiences time-varying and frequency selective channels. We study a training based scenario, where the channel is estimated based on pilots that proceed the transmission of the information bearing blocks, and then used to equalize the subsequent data blocks. In such a scheme, due to the time-varying channel, the BER will increase as the index of the data block increases. Considering commonly used models for time-varying channels, we derive an analytical expression for the bit error rate as a function of the block index. The proposed analysis is important in optimizing the system performance, as it can be exploited at the transmitter to determine how often pilots need to be transmitted, or what kind of modulation should be used on each carrier in order to maintain a certain error level.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is an increasingly popular multicarrier modulation technique, mainly because of its ability to combat multipath effects in wireless communication systems. It has been implemented in several wireline and wireless high-speed data communications standards (ADSL [1], IEEE 802.11 [3], HiperLAN [2]) and has been adopted by the European digital audio and video broadcasting standards (DAB and DVB).

The performance of the communications systems in general [11], OFDM systems [4], [7], and generalized OFDM systems [8] over the fading channels has been widely studied. However, very few works consider the general case of time-varying frequency selective channels. The time varying nature of the channels becomes very important in OFDM systems that use training data for channel estimation, e.g., the IEEE 802.11a standard. In such systems, the transmission alternates between training blocks and segments of data blocks, or frames. The channel coefficients are estimated based on the training data and then used for the equalization of the entire frame. In the mobile environment, where the multipath effects vary rapidly, the performance of such systems deteriorates as the number of OFDM symbols in the frame increases.

In this paper we derive an analytical expression for the bit error rate (BER) of the OFDM systems for frequency-selective and time varying Rayleigh fading channels. In particular, we analyze the BER performance as a function of block index in the OFDM frame. Such analysis has the potential to improve the performance of the OFDM system. For example, the expressions take as arguments the channel corresponding to the training data, ratio of the signal and noise variances and Doppler frequency. Based on these parameters the receiver can compute/predict the BER for the subsequent blocks, and based on it determine how often pilots need to be transmitted in order to maintain a low error level, or even what rate should be allocated on each carrier. The latter information could be communicated to the transmitter via a feedback channel, to be taken into account in subsequent transmissions.

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2. PROBLEM FORMULATION

Let us consider an OFDM system with N carriers. Let $s_i(n),\ n = 0, ..., N-1$ denote the information bearing symbols of the i-th OFDM block. We will assume that $s_i(n)$ are zero-mean i.i.d., temporally white, and spatially uncorrelated.

After OFDM modulation, transmission and demodulation, the received symbol over the m-th carrier equals:

$$y_i(m) = s_i(m)H_{m,i} + w_i(m) \qquad m = 0, \cdots, N-1 \tag{1}$$

where $H_{m,i}$ is the complex gain of the m-th carrier, and $w_i(m)$ is AWGN that is here assumed to be zero-mean with variance N_0.

Let H_m be the channel estimate obtained based on the training data, that will be used for the equalization of the entire frame. For simplicity, let us assume that H_m is identical to the true channel experienced by the pilots, i.e., $H_m = H_{m,0}$. Due to the time-varying channel, it holds:

$$H_{m,i} = H_m + \Delta_{m,i} \tag{2}$$

Using a zero-forcing equalizer, the recovered block equals:

$$\begin{aligned} \hat{s}_i(m) &= \frac{1}{H_m}(H_{m,i}s_i(m) + w_i(m)) \\ &= s_i(m) + \frac{\Delta(m,i)}{H_m}s_i(m) + \frac{1}{H_m}w_i(m) \end{aligned} \tag{3}$$

Assuming that signal and noise are independent, it follows from (3) that, at decision level, the signal to noise ratio at for the m-th carrier and i-th symbol equals:

$$SNR_{m,i} = \frac{E_s|H_m|^2}{N_0 + E_s|\Delta_{m,i}|^2} \qquad m = 0, \cdots, N-1 \tag{4}$$

where E_s is the average energy per symbol.

In the next section we establish the channel model and derive the analytical expression for the average BER of the described system.

3. SNR ANALYSIS

Let $h_{n,i}$ be the inverse DFT of $H_{m,i},\ m = 0, ..., N-1$, also referred to as channel impulse response. We make the following assumptions regarding the channel model:

(A1) We model the channel impulse response coefficients $h_{n,i}$ as a complex Gaussian random variables with zero mean.

(A2) We assume that the scattering is wide sense stationary and uncorrelated, that is:

$$E\{h_{n,i}h^*_{n+m,i+j}\} = r_{n,j}\delta(m) \qquad n = 0, ..., L-1 \tag{5}$$

where $\delta()$ is the Kronecker delta function.

(A3) We assume the Rayleigh fading conditions:

$$r_{n,j} = \sigma_n^2 J_0(2\pi f_d j) \tag{6}$$

where $J_0()$ denotes the 0-th order Bessel function of the first kind and f_d denotes the normalized Doppler frequency.

The channel response at each frequency bin is given as:

$$H_{m,i} = \mathbf{f}_m^H \mathbf{h}_i \tag{7}$$

with $\mathbf{h}_i \triangleq [h_{0,i}, \cdots, h_{L-1,i}]^T$ and $\mathbf{f}_i \triangleq [1, \cdots, e^{j\frac{2\pi}{N}(L-1)}]^T$.

Thus, according to our assumptions (A1)-(A3), $H_{m,i}$ is also a complex Gaussian random variable with zero mean and variance:

$$\begin{aligned} \sigma_G^2 &\triangleq E\{H_{m,i}H^*_{m,i}\} = \mathbf{f}_m^H E\{\mathbf{h}_i\mathbf{h}_i^H\}\mathbf{f}_m \\ &= \mathbf{f}_m^H diag\{\sigma_0^2, \cdots, \sigma_{L-1}^2\}\mathbf{f}_m = \sum_{n=0}^{L-1}\sigma_n^2 \end{aligned} \tag{8}$$

We can also see that the random variables H_m and $H_{m,i}$ are jointly complex Gaussian with the correlation factor:

$$\rho_G \triangleq \frac{1}{\sigma_G^2}E\{H_m H^*_{m,i}\} = J_0(2\pi f_d i) \tag{9}$$

Consequently, the random variables H_m and $\Delta_{m,i}$ are also jointly complex Gaussian processes with variances $\sigma_1^2 = \sigma_G^2$ and $\sigma_2^2 = 2\sigma_G^2(1-\rho_G)$, respectively, and correlation factor $\rho_{\tilde G} = \frac{\rho_G - 1}{\sqrt{2(1-\rho_G)}}$ (this can be obtained by expressing the random variables H_m and $\Delta_{m,i}$ as a linear combination of H_m and $H_{m,i}$).

In our analysis, we are interested in a joint distribution of $|H_m|$ and $|\Delta_{m,i}|$ (see (4)). From our discussion above, it follows that these parameters are Rayleigh distributed with joint probability density function known as bivariate Rayleigh distribution [9], i.e.,:

$$\begin{aligned} &f_{|H_m|,|\Delta_{m,i}|}(x,y) = \\ &= \frac{4xy}{(1-\rho_{\tilde R})\sigma_1^2\sigma_2^2} I_0\left(\frac{2\sqrt{\rho_{\tilde R}}xy}{(1-\rho_{\tilde R})\sigma_1\sigma_2}\right) e^{-\frac{1}{1-\rho_{\tilde R}}\left(\frac{x^2}{\sigma_1^2}+\frac{y^2}{\sigma_2^2}\right)} \end{aligned} \tag{10}$$

where $I_0()$ denotes the 0-th order modified Bessel function of the first kind and $\rho_{\tilde R} = \frac{Cov(x^2,y^2)}{\sqrt{Var(x^2)Var(y^2)}}$ (note that $\rho_{\tilde R}$ is defined as the power correlation factor).

Taking into account that H_m and $\Delta_{m,i}$ are jointly complex Gaussian, and after some manipulations it follows that:

$$\rho_{\tilde R} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x_r^2+x_i^2)(y_r^2+y_i^2)\frac{1}{\pi^2\sigma_1^4\sigma_2^4(1-\rho_{\tilde G}^2)} e^{-\frac{1}{1-\rho_{\tilde G}^2}\left(\frac{x_r^2+x_i^2}{\sigma_1^2}-2\rho_{\tilde G}\frac{x_r y_r + x_i y_i}{\sigma_1\sigma_2}+\frac{y_r^2+y_i^2}{\sigma_2^2}\right)} dx_i dx_r dy_r dy_i - 1$$

The bivariate Rayleigh distribution function appears relatively often in communications systems analysis and its infinite series representation has been analyzed in [10], [12]. Mathematical software packages such as Maple 8 allow us to numerically compute some of the complicated integrals involved in the above expression.

The last set of expressions provides an analytical tool for computing the effective SNR at the receiver of the OFDM system. By recalling (4) and using the expression given by (10) we are able to compute the average SNR as a function of the OFDM block i:

$$\overline{SNR}_{m,i} = \int_{x=0}^{\infty}\int_{y=0}^{\infty}\frac{E_s x^2}{N_0 + E_s y^2} f_{|H_m|,|\Delta_{m,i}|}(x,y)dxdy \tag{11}$$

This function for $\sigma_G^2 = 1$, $E_s = 10$ and $N_0 = 1$ and several values for f_d is shown in Figure 1. As can be seen, the average SNR decreases with i, especially in a highly mobile environment (i.e., large f_d). As an example, for a carrier frequency at 5.2 GHz and symbol duration $T_s = 4$ microseconds, $f_d = 5 \times 10^{-4}$ corresponds to the user speed of approximately 16 mph.

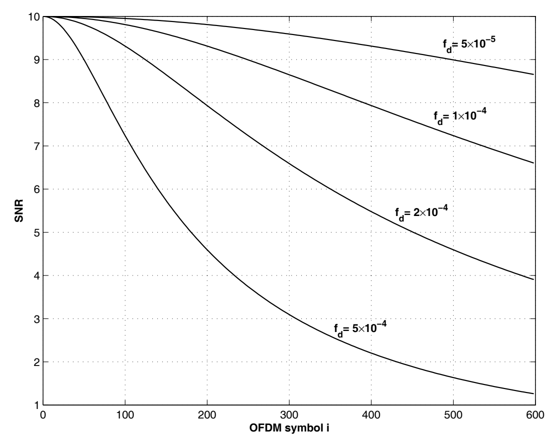

Figure 1: Average SNR.

4. BER ANALYSIS

As a consequence of decreasing SNR, the BER of the system also decreases. In this section we analyze the BER performance in the case of M-QAM constellation schemes. This turns out to be very difficult problem. The reason for this is not only the relatively complicated probability density functions related to the fading amplitudes, but also the fact that the exact analysis has to take into consideration the joint probability function of the phases of H_m and $H_{m,i}$ (this joint probability function can be found in [5]). We will follow the simplified approach presented in [13] and analyze only the effects of the amplitude estimation errors.

Using similar analysis as in the previous section we conclude:

$$\begin{aligned} &f_{|H_m|,|H_{m,i}|}(x,y) = \\ &= \frac{4xy}{(1-\rho_R)\sigma_G^4} I_0\left(\frac{2\sqrt{\rho_R}xy}{(1-\rho_R)\sigma_G^2}\right) e^{-\frac{1}{1-\rho_R}\left(\frac{x^2}{\sigma_G^2}+\frac{y^2}{\sigma_G^2}\right)} \end{aligned} \tag{12}$$

with

$$\rho_R = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x_r^2+x_i^2)(y_r^2+y_i^2)\frac{1}{\pi^2\sigma_G^8(1-\rho_G^2)}$$

$$e^{-\frac{1}{1-\rho_G^2}(\frac{x_r^2+x_i^2}{\sigma_G^2}-2\rho_G\frac{x_r y_r+x_i y_i}{\sigma_G^2}+\frac{y_r^2+y_i^2}{\sigma_G^2})} dx_i dx_r dy_r dy_i - 1$$

Due to a limited space, we will here consider only the case of a 16-QAM constellation scheme. The same analysis can be performed for all other constellations. The BER expression for 16-QAM with Gray encoding and taking into consideration the amplitude error only has been derived in [13]. Using this result the average BER for different number D of OFDM symbols in a frame can be expressed as:

$$BER_{16} = \int_{x=0}^{\infty}\int_{y=0}^{\infty} \frac{1}{ND} \sum_{m=0}^{N-1}\sum_{i=1}^{D}\sum_{p=1}^{6} c_p^{(16)} Q(a_p^{(16)} y + b_p^{(16)} x) f_{|H_m|,|H_{m,i}|}(x,y)dxdy \qquad (13)$$

a, b and c coefficients for the constellation of interest can be found in [13] (note that the coefficients need to be expressed in appropriate terms of E_s and N_o; in particular, a and b coefficients given in [13] are actually multiples of $\sqrt{\frac{E_s}{5N_o}}$ for 16-QAM scheme.

Since the channel estimates at the beginning of the frame are available at the receiver, we are able to take a step further in the BER analysis as will be shown in the next section.

5. IMPROVING THE THROUGHPUT IN THE OFDM SYSTEMS

Let us consider a specific OFDM system, e.g., the IEEE 802.11a standard. The analysis performed in the previous sections indicates that the equalization as currently performed in the 802.11a based OFDM systems (fixed equalizer throughout the frame) introduces errors in the equalizer in the case of time varying channels, that might significantly increase the BER at the receiver. The only mechanism of the 802.11a standard for coping with the bad transmission environment is to reduce the data transmission rate, which, on the other hand, reduces the system throughput. Our analysis points out to another possible approach to combat the bad transmission environment: to operate in *a channel aware mode* that allows the reduction of the OFDM DATA segment size (number of OFDM symbols per frame) when the channel is bad, so that the channels can be estimated more frequently.

Since the channel estimates at the beginning of the frame are available at the receiver, it is possible for the receiver to anticipate the system performance for the given parameters. In particular, for the given equalizer coefficients (based on the channel estimates that are assumed in our model to equal the exact channel coefficients at the beginning of the frame), f_d, constellation scheme, and number of OFDM symbol in a frame, the receiver can calculate the conditional BER and compare the current performance with different constellation schemes and different number of OFDM symbols in a frame. It can then send the information to the transmitter which set of parameters (constellation and DATA segment size) best utilizes the bandwidth.

Let us assume that the frequency selective channel is correctly measured at the beginning of the OFDM frame. The corresponding conditional BER equations, similar to those presented in the previous section, but this time based on the conditional probability density function equal:

$$BER_{16} = \int_{y=0}^{\infty} \frac{1}{ND} \sum_{m=0}^{N-1}\sum_{i=1}^{D}\sum_{p=1}^{6} c_p^{(16)} Q(a_p^{(16)} y + b_p^{(16)} |H_m|) f_{|H_{m,i}|||H_m|}(y|x=|H_m|)dy \qquad (14)$$

with

$$f_{|H_{m,i}|||H_m|}(y|x=|H_m|) = \frac{f_{|\Delta_{m,i}|,|H_m|}(x,y)}{f_{|H_m|}(x)}$$
$$= \frac{2y}{(1-\rho_R)\sigma_G^2} I_0(\frac{2\sqrt{\rho_R} y|H_m|}{(1-\rho_R)\sigma_G^2}) e^{-\frac{\sigma_G^2 y^2 + (1-\rho_R)\sigma_G^2 |H_m|^2}{\sigma_G^4(1-\rho_R)}} \qquad (15)$$

The above expression could be rewritten as:

$$BER_{16} = \frac{1}{ND} \sum_{m=0}^{N-1}\sum_{i=1}^{D}\sum_{p=1}^{6} A c_p^{(16)} \cdot$$
$$\cdot \int_{y=0}^{\infty} y Q(a_p^{(16)} y + b_p^{(16)} |H_m|) e^{\frac{y^2}{(1-\rho_R)\sigma_G^2}]} I_0(By)dy \qquad (16)$$

with

$$A = \frac{2}{(1-\rho_R)\sigma_G^2} e^{-\frac{\rho_R |H_m|^2}{(1-\rho_R)\sigma_G^2}} \qquad (17)$$

$$B = \frac{2\sqrt{\rho_R}|H_m|}{(1-\rho_R)\sigma_G^2} \qquad (18)$$

Using the alternative expression for the Marcum Q function:

$$Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\frac{x^2}{2sin^2\theta}} d\theta \quad , \qquad x \geq 0 \qquad (19)$$

to avoid having the argument y appear in the lower limit of the integral, the bit error rates can numerically be calculated for the given set of parameters.

The average BER results for one fixed carrier with $|H_m| = 1$, as a function of the OFDM symbol i for $\sigma_G^2 = 1$, $f_d = 2 \times 10^{-4}$ and several values for $\gamma \triangleq 10log\frac{E_s}{N_0}$ are shown in Figure 2.

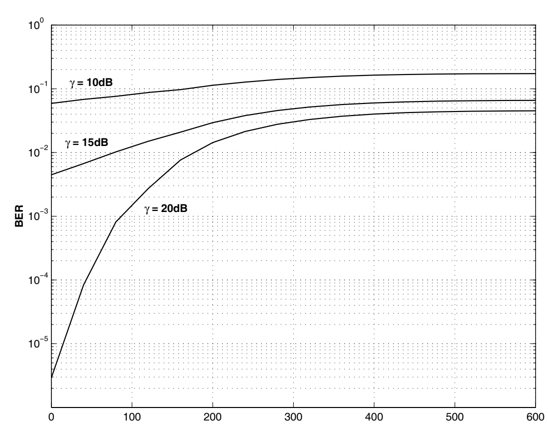

Figure 2: BER for 16-QAM constellation and fixed carrier.

Let us use the channel shown in Figure 3 (its magnitude) as the channel measured at the beginning of the OFDM frame using the training symbols. The corresponding conditional BER results obtained using (14), assuming $\sigma_G^2 = 1$, $f_d = 2 \times 10^{-4}$ and $N = 16$ for different D are shown in Figure 4.

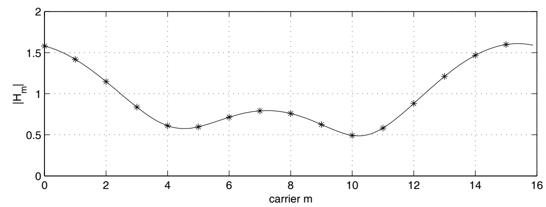

Figure 3: Channel frequency response (example).

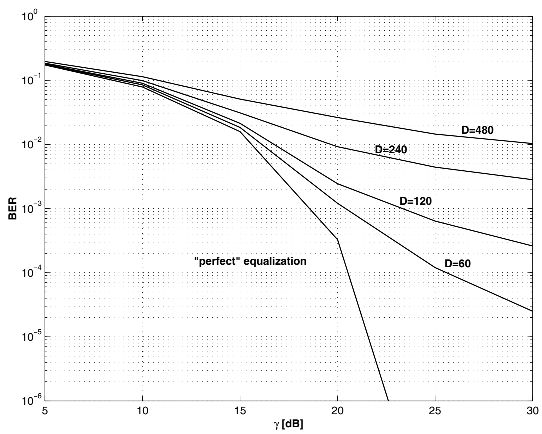

Figure 4: BER for 16-QAM constellation and given equalizer.

Let us now consider the potential consequences of the obtained results. Let us assume that 2160 bytes of data (user data plus all the required overheads form the upper layers such as TCP/IP header and MAC header) are to be sent. This information could be sent using one OFDM frame with 120 data symbols with 16-QAM and encoding rate R=3/4 (36 Mbps connection). The performance of such transmitter is shown in Figure 4 ($D = 120$ BER curve). It takes 532 microseconds to transmit the data (120 data symbols + 13 symbols for the PLCP header).

Let us now assume that the BER becomes too high. One possible course of actions (currently adopted) is to try to transmit the data using the lower constellation scheme, namely 4-QAM with encoding rate R=3/4 (18 Mbps connection). In this case it takes 1012 microseconds for the transmission (240 data symbols + 13 symbols for the PLCP header). This produces throughput reduction of approximately 47.43 %.

However, we can improve the BER performance by simply using 2 OFDM frames for the transmission with 16-QAM and R=3/4. The performance of this transmitter is also shown in Figure 4 ($D = 60$ BER curve). This time it takes 584 microseconds for the transmission (2×60 data symbols + 2×13 symbols for the PLCP header). This produces throughput reduction of approximately 8.9 %.

Thus, the improved BER at the receiver for the given SNR can be achieved by reducing the number of symbols per frame and only slightly reducing the throughput, instead of dropping the data rate and significantly reducing the throughput.

6. CONCLUSIONS

In this paper we analyze the performance of the OFDM systems in the presence of frequency-selective and time varying Rayleigh channels. We consider the scenario where the channel coefficients are estimated at the beginning of the OFDM frame and used for the equalization for the duration of the entire frame. We derive an analytical expression for the BER in such environment taking into account not only the fading effects but also the effects of the errors in the equalizer. We also analyze the possible improvements of the throughput of the OFDM systems that can be accomplished using the developed BER analysis. We show that in the presence of time varying channels a better throughput efficiency could be achieved by reducing the OFDM DATA segment size instead of switching to the lower constellation scheme.

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