ANTENNA SELECTION FOR SPACE TIME CODING OVER FREQUENCY-SELECTIVE FADING CHANNELS

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ABSTRACT

In this paper, we deal with antenna selection at the receiver side for space-time coded systems over frequency-selective fading channels. We reveal that introducing antenna selection based on the signal-to-noise-ratio (SNR) observed can still achieve the full diversity available, if the underlying space-time code (STC) is full-rank (i.e., if it achieves full diversity without antenna selection over the frequency-selective fading channel). We also argue that if the code is not full-rank, antenna selection results in a loss in the diversity of the system.

1. INTRODUCTION

The capacity of wireless communication systems increases considerably with the use of multiple antennas at the receiver and/or transmitter. Specifically, the capacity increases linearly with the number of transmit antennas as long as there are at least as many receive antennas available [1, 2]. A practical means of achieving this capacity limit is the use of space-time coding techniques [3, 4].

One of the disadvantages of multiple input multiple output (MIMO) systems is the cost of implementing multiple RF chains at the transmitter and the receiver. In particular, with the strict space requirements of the wireless devices, it is a significant challenge to place them on a single unit while proper isolation is achieved. A method to alleviate this problem is the application of "antenna selection" techniques. For example, with antenna selection at the receiver, one can monitor the received SNR at each receive antenna periodically, and select the one with the largest SNR to be used for each frame of data. Antenna selection at the transmitter requires exact knowledge of the CSI at the transmitter, which may not be practical due to channel estimation errors and bandwidth constraints. A more pragmatic scenario is to apply antenna selection at the receiver, which is the focus of this paper.

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In [5], the author presents a general overview of the capacity and performance of MIMO systems with antenna selection at the receiver. Antenna selection algorithms/analysis by considering the minimization of error probability of the STCs is provided in [6]. In [7] and [8], the authors consider antenna selection at the receiver based on maximizing the SNR over quasi-static flat fading channels. It is shown that with antenna selection, the diversity order is maintained compared to that of the full complexity system, provided that the underlying STC is full-rank. However, it is also shown that, if the STC is rank-deficient, the resulting diversity order deteriorates and becomes a function of the number of selected antennas.

If we select the symbol duration to be small in an attempt to achieve a larger transmission rate, the frequency flat fading channel model may no longer apply. If the symbol duration is reduced beyond the multipath spread of the channel, a more suitable wireless channel model is that of frequency-selective fading. While most of the results on space-time coding concentrate on flat fading channels, space-time code design principles are developed and the performance of specific coding techniques are explored for frequency selective fading channels as well [9-11].

In this paper, we investigate the diversity gain that STCs can offer over frequency-selective fading channels when antenna selection is employed. Specifically, we perform a pairwise error probability analysis by exploiting the results available for flat fading channels [7]. We show that if the space-time code used achieves full-rank over the frequency-selective fading channel, then antenna selection based on the largest SNR observed does not degrade the diversity gain compared to that of the full complexity system. Furthermore, we show that if the code does not achieve full diversity, then performing antenna selection results in a loss in the diversity order. Note that the results are very general, and apply for different space-time codes, and even for concatenated coding schemes.

The remainder of the paper is organized as follows: Sec-

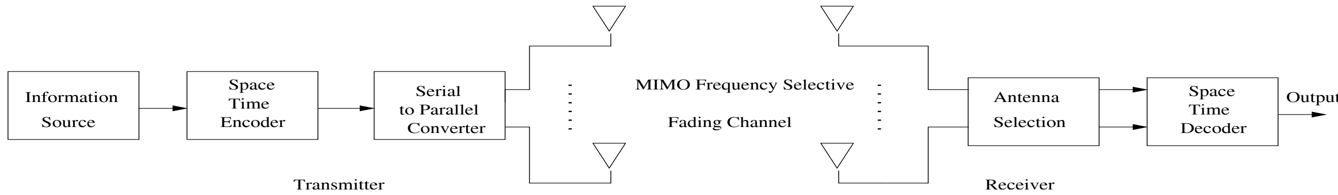

Fig. 1. Space-Time Coded Multiple Antenna System with Antenna Selection.

tion 2 describes the system model. Section 3 discusses space-time codes over flat and frequency-selective fading channels, and presents our main results with antenna selection. We present some examples and simulation results in Section 4. Finally, Section 5 concludes the paper.

2. SYSTEM MODEL

Figure 1 shows the space-time coded system with antenna selection at the receiver side. The information sequence is encoded by the space-time encoder and the output is divided by a serial-to-parallel converter into several data streams. The resulting data streams are then modulated and transmitted through different antennas simultaneously. The channel is modelled as a quasi-static frequency-selective MIMO Rayleigh fading channel where the sub-channels fade independently. At the receiver, a subset of the antennas is selected based on their received SNR, and space-time decoding is performed. We note that this is a relatively general system model which includes the flat fading channel as a special case.

The received signal at the receive antenna r at time k is

$$y_k^r = \sqrt{\frac{\rho}{LN_T}} \sum_{l=0}^{L-1} \sum_{t=1}^{N_T} h_t^r(l) c_{k-l}^t + n_k^r \tag{1}$$

where L is the total number of intersymbol interference (ISI) taps, $h_t^r(l)$ is the fading coefficient between transmit antenna t and receive antenna r from the lth ISI tap, c_k^t is the transmitted symbol from antenna t at time k, N_T is the number of transmit antennas, N_R is the number of receive antennas, and n_k^r is the noise sample at the receive antenna r at time k, ($k = 1, \cdots, K$). $h_{r,t}(l)$ and n_k^r are i.i.d. complex Gaussian samples having zero mean and variance $1/2$ per dimension. ρ is the expected SNR at each receive antenna. Clearly, the case with $L = 1$ corresponds to flat fading channel. The received signals at all antennas can be stacked in a matrix form as

$$Y = \sqrt{\frac{\rho}{LN_T}} HC + N \tag{2}$$

where C is the codeword matrix whose elements are c_k^t and is shown in the next section in detail, the $N_R \times K + L - 1$ noise matrix N contains the noise samples n_k^r, and the $N_R \times LN_T$ channel coefficients matrix is given by

$$H = \begin{pmatrix} h_1^1(0) & \dots & h_1^1(L-1) & \dots & h_{N_T}^1(0) & \dots & h_{N_T}^1(L-1) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ h_1^{N_R}(0) & \dots & h_1^{N_R}(L-1) & \dots & h_{N_T}^{N_R}(0) & \dots & h_{N_T}^{N_R}(L-1) \end{pmatrix}.$$

3. ANTENNA SELECTION FOR MIMO FREQUENCY-SELECTIVE FADING CHANNELS

Space-time code design principles for quasi-static Rayleigh fading channels have been developed in [3]. There are mainly two criteria: Rank criterion, and determinant criterion. Let C denote that codeword matrix, which is given by

$$C = \begin{pmatrix} c_1^1 & \dots & c_K^1 \\ \vdots & \ddots & \vdots \\ c_1^{N_T} & \dots & c_K^{N_T} \end{pmatrix}_{N_T \times K} \tag{3}$$

and E be the erroneously decoded codeword matrix given by

$$E = \begin{pmatrix} e_1^1 & \dots & e_K^1 \\ \vdots & \ddots & \vdots \\ e_1^{N_T} & \dots & e_K^{N_T} \end{pmatrix}_{N_T \times K} \tag{4}$$

The rank criterion states that over all the codeword pairs, the rank of the codeword difference matrix

$$B = \begin{pmatrix} (e_1^1 - c_1^1) & \dots & (e_K^1 - c_K^1) \\ \vdots & \ddots & \vdots \\ (e_1^{N_T} - c_1^{N_T}) & \dots & (e_K^{N_T} - c_K^{N_T}) \end{pmatrix}_{N_T \times K}$$

should be N_T (assuming that $K > N_T$). The determinant criterion states that over all the codes achieving the full diversity, the minimum determinant of BB^H should be maximized. For the case of flat fading channels, the maximum diversity available is $N_T N_R$.

When the channel is frequency-selective in multiple antenna systems, then the $LN_T \times (K+L-1)$ codeword matrix C in (2) can be shown as

$$C = \begin{pmatrix} c_1^1 & \dots & c_K^1 & 0 & \dots & 0 \\ 0 & c_1^1 & \dots & c_K^1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & c_1^1 & \dots & c_K^1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_1^{N_T} & \dots & c_K^{N_T} & 0 & \dots & 0 \\ 0 & c_1^{N_T} & \dots & c_K^{N_T} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & c_1^{N_T} & \dots & c_K^{N_T} \end{pmatrix}. \tag{5}$$

We observe from the above formulation that space-time codes over frequency-selective fading channels can be considered as codes over frequency flat fading where there are effectively $N_T L$ (virtual) transmit antennas with the code structure given as above [9]. Therefore, it can be shown [9, 10] that in order to achieve the maximum available diversity order of $N_R L N_T$, the codeword difference matrix $B(\mathbf{c}, \mathbf{e})$ for all possible codeword pairs should have full-rank of LN_T.

The properties of the space-time codes over frequency-selective fading channels are derived in [10], and some suitable codes are developed. Space-time trellis codes based on an algebraic framework that achieve full diversity are considered in [9]. The codes in [9] are $1/N_T$ nonsystematic convolutional codes (NSCC) for N_T transmit antennas. For these codes with maximal memory order ν, the diversity order over the frequency-selective fading channel having L taps is $\nu + L$, however, it cannot exceed LN_T (Corollary 5 in [9]).

As L increases, it becomes more complicated to find full-rank space-time codes. It may not even be possible to find such codes for a given memory order of the convolutional encoder (see [9]). This motivates the use of rank-deficient space-time codes over frequency-selective fading channels as well. However, we note that, since the maximum diversity available is very large for frequency-selective fading channels, this may not pose a serious problem in practice.

For flat fading channels, if N_S out of N_R receive antennas which observe the largest received powers are used for the received frames and full-rank STCs are employed, i.e., the rank of the codeword difference matrix B is N_T, then full diversity gain is still achievable as proved in [7] and [8]. In [8], by several approximations, it is shown that the diversity order will remain the same with antenna selection if the B matrix is full-rank and the coding gain decreases by up to $10\log_{10}(N_R/N_S)$ dB. The complete proof of the conservation of diversity gain is provided in [7] by computing an upper bound for pairwise error probability without resorting to an approximate analysis. When the underlying space-time code is rank-deficient, i.e., $q = rank(B) < N_T$, then the diversity gain for the full complexity system becomes qN_R, whereas, antenna selection for the rank-deficient space-time coded system decreases the diversity gain to qN_S.

The lengthy proof in [7] can be extended to the case of frequency-selective fading in a straightforward manner by the use of the equivalence of the frequency-selective fading system with the flat fading one using virtual antennas. Therefore, similar to the flat fading case, the antenna selection at the receiver side maintains the diversity gain if the rank of the codeword difference matrix B of the underlying space-time code is LN_T. For example, full-diversity STTCs found in [9], or the delay diversity scheme of [11], when used in conjunction with the SNR based antenna selection at the receiver, will have the same diversity order (of $N_T L N_R$) as that of the full complexity system.

As noted earlier, when the number of channel taps is large, it may be a challenge (or, even impossible) to find full diversity space-time codes (e.g., space-time codes in [9]) due to the increased complexity of the convolutional encoder. For instance, when the memory of the code is limited to only 2, and if $L = 3$, there are no full diversity codes from algebraic designs of [9]. For those cases, i.e., when the code is rank-deficient, we also need to consider antenna selection for the frequency-selective MIMO channel. In this case, the result of [7] readily applies as well, which mainly states that if the underlying code has a rank $q = rank(B) < LN_T$ then the diversity order with antenna selection is qN_S, as opposed to that of the full complexity system which is qN_R. Therefore, there is a significant reduction in the overall available diversity order when antenna selection is employed.

4. EXAMPLES

In this section, we give several examples to illustrate our main points. The STCs developed in [9] are used over frequency selective quasi-static fading channels. The input bits are passed through the nonsystematic convolutional encoder and each output bit is transmitted from a transmit antenna at the same time interval. At the receiver, we employ a Viterbi decoder that is matched to the trellis of the combined encoder and channel to perform maximum-likelihood decoding. Each transmitted frame from different transmit antennas consists of 100 BPSK symbols. The generator polynomials of the convolutional codes used in the simulations are shown in octal form on the figures. Whenever applicable, antenna selection is based on maximizing the received SNR over one received frame. For example, when $N_S = 1$, the selected antenna is the one for which $\sum_{t=1}^{N_T}\sum_{l=0}^{L-1}|h_t^r(l)|^2$ is the largest.

Figure 2 shows the frame error rate plots for the system with 2 transmit, 2 receive antennas and 2 equal power ISI taps. We use two different codes, namely, (5,7) nonsystematic convolutional code (NSCC) and (4,2) NSCC. When the number of channel taps is equal to $L = 2$, the (5,7) code achieves full-rank ($2 \times 2 \times 2 = 8$), whereas the other one achieves a diversity order of only $3 \times 2 = 6$, i.e., it is rank-deficient. We observe from the plots that the system with antenna selection preserves the diversity of the full-rank code, as expected. However, the diversity order of the rank-deficient code is reduced to 3 (from 6) as expected from the theoretical results of the previous section.

A similar observation can be made for the case of $L = 3$. Figure 3 shows the frame error rate plots for the system having 2 transmit, 2 receive antennas and 3 equal power ISI taps. We also consider the same system but with 1 receive antenna for comparison purposes. If no selection is applied, the (64,74) NSCC achieves full diversity of $N_R L N_T = 12$, with and without antenna selection. However, the use of (5,7) NSCC results in a diversity gain of 10. The STC code

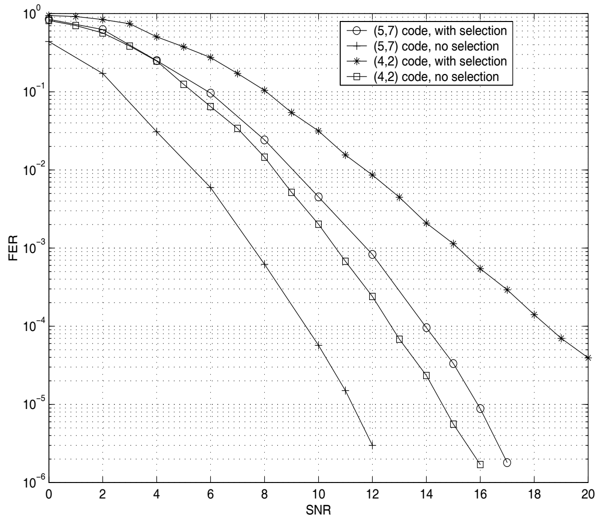

Fig. 2. 2 transmit, 2 receive antennas, 2 ISI taps, with and without antenna selection.

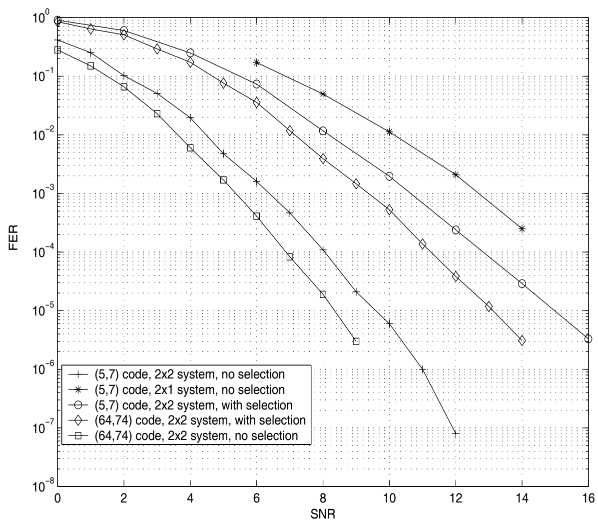

Fig. 3. 2 transmit, 1 or 2 receive antennas, 3 ISI taps, with and without antenna selection.

obtained with the (5,7) NSCC is a rank-deficient code. With antenna selection, the resulting diversity order becomes $qN_S = 5$. Although the diversity gains for 1 receive antenna and 2 receive antennas with selection are equal when a rank-deficient code is used, the case with antenna selection provides considerable gain (2 dB). Note that the simulation results for this case do not always show the diversity orders achieved precisely this is because longer simulations are needed at much smaller frame error rates to see them clearly (which is not feasible due to the increased computational complexity).

5. CONCLUSIONS

In this paper, we considered the application of antenna selection (based on the maximum received powers) at the receiver side for space-time coded MIMO system over frequency-selective fading channels, where only the receiver has knowledge of the CSI. Utilizing the results of [7] which consider frequency flat fading channels, we demonstrated that by applying antenna selection one can still achieve full available diversity which is determined by the number of antennas and number of ISI taps provided that the underlying code is full-rank for the frequency-selective channel. Otherwise, the diversity order is reduced and it becomes a function of the number of selected antennas.

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