# FROM SINGLE LINK MIMO TO MULTI-USER MIMO

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**ABSTRACT**

For any given single link strategy for transmission over a MIMO channel with channel knowledge at both transmitter and receiver, we propose a general approach to reduce the interference when the strategy is used in an interference limited system. This solution has several interesting interpretations. The structure of the solution is shown to include the global optimum, even though it may be difficult to determine all parameters. However, a suboptimal choice is presented that can be implemented in practical systems using only local information. A numerical example illustrates the performance when trying to minimize the summed mutual information.

## 1. INTRODUCTION

The use of multiple antennas at both transmitters and receivers can provide both increased data rate and improved diversity to combat fading. This has been proved both in theoretical studies and with practical algorithms, see for example [1] for an overview. However, most of the results focus on the performance of a single link. For multi-user scenarios, the capacity region has been investigated for cases such as the multiple access channel (single receiving base station) or broadcast channel (single transmitting base station) [2, 3] but is largely unknown for general networks with multiple transmitters and receivers. Some practical approaches have been suggested for the broadcast channel, [4–6] and for general networks using ideas of optimal downlink beamforming [7] or opportunistic beamforming [8], but this topic is still in its infancy.

Herein, we consider systems that exploit channel knowledge at the transmitter. For a single MIMO link, several approaches have been proposed to provide optimal performance in terms of different Quality of Service (QoS) definitions [9–11]. We search to extend these algorithms to take interference into account both at the transmitter and receiver. Noise prewhitening is a well-known technique to take care of interference at a receiver, modeling the interference as spatially colored noise. Here, we show that a similar prewhitening technique can be applied at the transmitter to reduce the amount of interference transmitted to known receivers in the surroundings. This solution has several interesting interpretations as will be shown below.

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## 2. PRELIMINARIES

Assume a MIMO system with one or more transmitters, each communicating with one or several receivers. Both the transmitters and receivers are equipped with array antennas with $N$ and $M$ antenna elements each, respectively. We study only users that share the same carrier frequency and for simplicity of notation, we will assume a narrowband system and purely spatial processing, even though the results easily can be extended to space-time processing for frequency selective situations. Also, assume a downlink situation where each receiver $r$ is allocated to a single transmitter $\tau(r)$. The discrete time equivalent complex valued baseband channel from transmitter $t$ to receiver $r$ is described by the $M \times N$ channel matrix $\mathbf{H}_{t,r}$. Thus, the signal at receiver $r$ is given by the $M \times 1$ vector

$$\mathbf{y}_r(n) = \sum_t \mathbf{H}_{t,r}\mathbf{x}_t(n) + \mathbf{n}_r(n) \tag{1}$$

where $\mathbf{x}_t(n)$ denotes the $N \times 1$ vector of signals emitted from the antennas of transmitter $t$. We will assume that the additive noise $\mathbf{n}_r(n)$ is spatially and temporally white, $\mathrm{E}[\mathbf{n}_r(n_1)\mathbf{n}_r^*(n_2)] = \sigma_r^2\mathbf{I}\delta_{n_1,n_2}$. Since each transmitter may communicate with several receivers,

$$\mathbf{x}_t(n) = \sum_{r:\tau(r)=t} \mathbf{z}_r(n) \ , \tag{2}$$

where $\mathbf{z}_r(n)$ is the vector of signals intended for receiver $r$.

## 3. ALGORITHM AND ANALYSIS

Assume that we have some algorithm that determines the processing at the transmitter and receiver of a single MIMO link based on knowledge of the channel matrix $\mathbf{H}$ and on the noise (including interference) covariance matrix $\mathbf{R}^{\mathrm{IN}}$ (if the algorithm is designed only for spatially white noise, it can be applied to ${\mathbf{R}^{\mathrm{IN}}}^{-1/2}\mathbf{H}$, so-called noise prewhitening). Assume furthermore that this algorithm minimizes some cost function, $C(\mathbf{\Theta};\mathbf{H},\mathbf{R}^{\mathrm{IN}})$, under constraints on the total transmitted power $\mathrm{Tr}[\mathbf{R}_z(\mathbf{\Theta})] \leq P_{\mathrm{tot}}$. Here $\mathbf{\Theta}$ denotes the parameters that determine the transmit and receive processing and $\mathbf{R}_z(\mathbf{\Theta}) = \mathrm{E}[\mathbf{z}(n)\mathbf{z}^*(n)]$ the resulting covariance matrix of the transmitted data. For linear precoders and detectors, for example, we have $\mathbf{\Theta}^T = [\mathrm{vec}^T\{\mathbf{U}\}, \mathrm{vec}^T\{\mathbf{V}\}]$ and $\mathbf{R}_z(\mathbf{\Theta}) = \mathbf{U}\mathbf{U}^*$, where the vector of data $\mathbf{d}(n)$ is transmitted using $\mathbf{z}(n) = \mathbf{U}\mathbf{d}(n)$ and received using $\hat{\mathbf{d}}(n) = \mathbf{V}\mathbf{y}(n)$. Several examples of such cost functions and the resulting transmitters and receivers can be found, for example, in [9].

In a multi-user setting, we first note that the interference plus noise covariance matrix at receiver $r$ is given by

$$\mathbf{R}_r^{\mathrm{IN}} = \sum_{i\neq r} \mathbf{H}_{\tau(i),r}\mathbf{R}_z(\mathbf{\Theta}_i)\mathbf{H}^*_{\tau(i),r} + \sigma_r^2\mathbf{I} \tag{3}$$

and can be estimated at the receiver, so it is easy to incorporate (spatial) interference reduction at each receiver. In order to extend the algorithm to actively reduce interference also at the transmitters, we propose the following general strategy,

$$\begin{aligned} &\min C(\mathbf{\Theta}_r;\mathbf{H}_{\tau(r),r},\mathbf{R}_r^{\mathrm{IN}}) \\ &\text{s.t. } \mathrm{Tr}[\mathbf{R}_z(\mathbf{\Theta}_r)\mathbf{T}_r] \leq \tilde{P}_r \end{aligned} \tag{4}$$

where

$$\mathbf{T}_r = \sum_{k\neq r} \mathbf{H}^*_{\tau(r),k}\mathbf{\Lambda}_k\mathbf{H}_{\tau(r),k} + \alpha_r\mathbf{I} \tag{5}$$

and the parameters $\mathbf{\Lambda}_k$ and $\alpha_r$ remain to be determined. Below, two different derivations of this general solution strategy are presented, providing somewhat different guidelines on the choice of these parameters.

It is easy to see that this strategy is closely related to prewhitening, but applied at the transmitter instead of the receiver. For algorithms that determine a linear prefilter $\mathbf{U}_r$, for example, the solution can be obtained using the following steps,

- $\tilde{\mathbf{H}}_r = \mathbf{H}_{\tau(r),r}\mathbf{T}^{-1/2}$
- $\{\tilde{\mathbf{U}}_r, \tilde{\mathbf{V}}_r\} = \arg\min\ C(\{\tilde{\mathbf{U}},\tilde{\mathbf{V}}\};\tilde{\mathbf{H}}_r,\mathbf{R}_r^{\mathrm{IN}})$, under the constraints $\mathrm{Tr}[\tilde{\mathbf{U}}\tilde{\mathbf{U}}^*] \leq \tilde{P}_r$.
- $\mathbf{U}_r = \mathbf{T}^{-1/2}\tilde{\mathbf{U}}$.

### 3.1. First Derivation – Global Optimality

It is not obvious how to even define a cost function for the problem of joint design of all transmitters and receivers in a network. Here, we choose to use the sum of the costs of the individual links and a constraint of the total power used together by all transmitters. Consequently, the joint design is formulated in the form of the optimization problem,

$$\begin{aligned} \min_{\mathbf{\Theta}_r,\mathbf{R}_r^{\mathrm{IN}}} &\sum_r C(\mathbf{\Theta}_r;\mathbf{H}_r,\mathbf{R}_r^{\mathrm{IN}}) \\ \text{s.t. } &\mathbf{R}_k^{\mathrm{IN}} \succeq \sum_{r\neq k}\mathbf{H}_{\tau(r),k}\mathbf{R}_z(\mathbf{\Theta}_r)\mathbf{H}^*_{\tau(r),k} + \sigma_k^2\mathbf{I} \\ &\sum_r \mathrm{Tr}[\mathbf{R}_z(\mathbf{\Theta}_r)] \leq P_{\mathrm{tot}}\ . \end{aligned} \tag{6}$$

Here, the notation $\mathbf{A} \succeq \mathbf{B}$ denotes that $\mathbf{A}-\mathbf{B}$ is positive semidefinite. Note that the constraint on $\mathbf{R}_k^{\mathrm{IN}}$ just as well could have been expressed with an equality sign without changing the solution since any reasonable cost function (such as MSE or BER) will increase with an increasing noise level. However, the formulation in (6) allows for some interesting conclusions below. Introducing Lagrange multipliers $\mathbf{\Lambda}_k$ (Hermitian matrices) and $\lambda$, respectively, for the first and second lines of constraints, we get the Lagrangian

$$\begin{aligned} &\sum_r C(\mathbf{\Theta}_r;\mathbf{H}_r,\mathbf{R}_r^{\mathrm{IN}}) - \sum_k \mathrm{Tr}[\mathbf{\Lambda}_k\mathbf{R}_k^{\mathrm{IN}}] \\ &\quad + \sum_k \mathrm{Tr}\big[\mathbf{\Lambda}_k\big(\sum_{r\neq k}\mathbf{H}_{\tau(r),k}\mathbf{R}_z(\mathbf{\Theta}_r)\mathbf{H}^*_{\tau(r),k} + \sigma_k^2\mathbf{I}\big)\big] \\ &\quad + \lambda\sum_r \mathrm{Tr}[\mathbf{R}_z(\mathbf{\Theta}_r)] - \lambda P_{\mathrm{tot}}. \end{aligned} \tag{7}$$

Rearranging the sums and using $\mathrm{Tr}[\mathbf{AB}] = \mathrm{Tr}[\mathbf{BA}]$, it is easy to see that finding the optimal $\mathbf{\Theta}_r$ for fixed $\mathbf{\Lambda}_k$, $\lambda$ and $\mathbf{R}_k^{\mathrm{IN}}$ decouples into separate problems for each $r$ of the form

$$\begin{aligned} &\min_{\mathbf{\Theta}_r} C(\mathbf{\Theta}_r;\mathbf{H}_r,\mathbf{R}_r^{\mathrm{IN}}) \\ &\quad + \mathrm{Tr}\big[\mathbf{R}_z(\mathbf{\Theta}_r)\big(\sum_{k\neq r}\mathbf{H}^*_{\tau(r),k}\mathbf{\Lambda}_k\mathbf{H}_{\tau(r),k} + \lambda\mathbf{I}\big)\big] + \text{const.} \end{aligned} \tag{8}$$

which is equivalent to the Lagrangian of (4) with $\mathbf{T}_r$ defined by (5), $\alpha_r = \lambda$ and a suitable choice of $\tilde{P}_r$. This shows the optimality of the solution structure given in (4) since any stationary point of (6) is also a stationary point of (4).

Note that finding the optimal set of $\mathbf{\Lambda}_k$ and $\lambda$ is still in general a very difficult optimization problem. However, we can draw some conclusions on the choice of $\mathbf{\Lambda}_k$. Assume that receiver $r$ projects the signal $\mathbf{y}_r$ linearly onto a subspace $\mathcal{A}_r$. Then we can set

$$\mathbf{R}_k^{\mathrm{IN}} = \sum_{r\neq k}\mathbf{H}_{\tau(r),k}\mathbf{R}_z(\mathbf{\Theta}_r)\mathbf{H}^*_{\tau(r),k} + \sigma_k^2\mathbf{I} + \mathbf{B}\mathbf{B}^*$$

in (6) without changing the optimality as long as $\mathbf{B}$ is orthogonal to $\mathcal{A}_r$. The complementarity conditions [12] state that

$$\mathrm{Tr}[\mathbf{\Lambda}_k(\mathbf{R}_k^{\mathrm{IN}} - \sum_{r\neq k}\mathbf{H}_{\tau(r),k}\mathbf{R}_z(\mathbf{\Theta}_r)\mathbf{H}^*_{\tau(r),k} + \sigma_k^2\mathbf{I})] = 0$$

at the optimum, which implies that $\mathbf{B}^*\mathbf{\Lambda}_k\mathbf{B} = 0$, so $\mathbf{\Lambda}_k$ should be low rank and belong to the subspace $\mathcal{A}_r$.

### 3.2. Second Derivation – Practical Ad Hoc Solution

Here, we assume for simplicity that each receiver contains an initial linear spatial processing step, $\hat{\mathbf{d}}_r(n) = \mathbf{V}_r \mathbf{y}_r(n)$. The signal $\mathbf{z}_r(n)$ intended for receiver $r$ will cause interference at all other receivers in the surroundings. Assume that transmitter $\tau(r)$ has knowledge about the channels $\mathbf{H}_{\tau(r),k}$ to these interfered receivers and the corresponding spatial receive filters $\mathbf{V}_k$. An ad-hoc approach to reduce the interference is to use the original single-link approach but add a constraint on the total interference caused by $\mathbf{z}_r(n)$ (ignoring the fact that further processing such as temporal equalization and multi-user detection may be used in the receivers to further reduce the interference in $\hat{\mathbf{d}}_k(n)$),

$$\begin{aligned}\min\ & C(\mathbf{\Theta}_r; \mathbf{H}_r, \mathbf{R}_r^{\text{IN}}) \\ \text{s.t.}\ & \operatorname{Tr}[\mathbf{R}_z(\mathbf{\Theta}_r)] \le P_r \\ & \sum_{k \neq r} \operatorname{Tr}[\mathbf{V}_k \mathbf{H}_{\tau(r),k} \mathbf{R}_z(\mathbf{\Theta}_r) \mathbf{H}^*_{\tau(r),k} \mathbf{V}_k^*] \le P_r^{\text{int}}\ .\end{aligned} \tag{9}$$

The Lagrangian is

$$\begin{aligned} & C(\mathbf{\Theta}_r; \mathbf{H}_r, \mathbf{R}_r^{\text{IN}}) - \lambda P_r + \lambda \operatorname{Tr}[\mathbf{R}_z(\mathbf{\Theta}_r)] - \lambda_{\text{int}} P_r^{\text{int}} \\ & \quad + \lambda_{\text{int}} \operatorname{Tr}[\mathbf{R}_z(\mathbf{\Theta}_r) \underbrace{\sum_{k \neq r} \mathbf{H}^*_{\tau(r),k} \mathbf{V}_k^* \mathbf{V}_k \mathbf{H}_{\tau(r),k}}_{\triangleq \mathbf{T}_r}] \\ & = C(\mathbf{\Theta}_r; \mathbf{H}_r, \mathbf{R}_r^{\text{IN}}) - \lambda_{\text{int}}(P_r^{\text{int}} + \frac{\lambda}{\lambda_{\text{int}}} P_r) \\ & \quad + \lambda_{\text{int}} \operatorname{Tr}[\mathbf{R}_z(\mathbf{\Theta}_r)(\mathbf{T}_r + \frac{\lambda}{\lambda_{\text{int}}} \mathbf{I})] \end{aligned} \tag{10}$$

which is equivalent to the Lagrangian of (4) with $\mathbf{\Lambda}_k = \mathbf{V}_k^* \mathbf{V}_k$ and $\alpha = \lambda / \lambda_{\text{int}}$.

Since the solution depends on the $\mathbf{V}_k$ for all other communication links, a practical implementation of this scheme requires an iterative solution where the transmit and receive processing at each link is determined based on the solution of the other links from the previous iteration. This can be implemented in a decentralized algorithm using a feedback link from each receiver with channel state information and information on the spatial receive filter. An alternative is to use a shared dedicated pilot channel where each receiver transmits a signal using its own receive filter $\mathbf{V}_k$. Then, $\mathbf{T}_r$ can be estimated directly as the covariance matrix of the data received on the pilot channel at the transmitters. Because of the path loss, the relative scaling of the terms in $\mathbf{T}_r$ will be different from (5) but the solution will anyway provide reduced interference. This solution is closely related to the so-called virtual uplink problem used in optimal downlink beamforming [13]. The connection between the virtual uplink problem and the Lagrange multipliers has also been observed in [3].

Finally, it is worth noting that the choice $\mathbf{\Lambda}_k = \mathbf{V}_k^* \mathbf{V}_k$ lies in exactly the same subspace as was depicted at the end of Section 3.1. The parameter $\alpha$ can be seen as a tuning parameter of the algorithm.

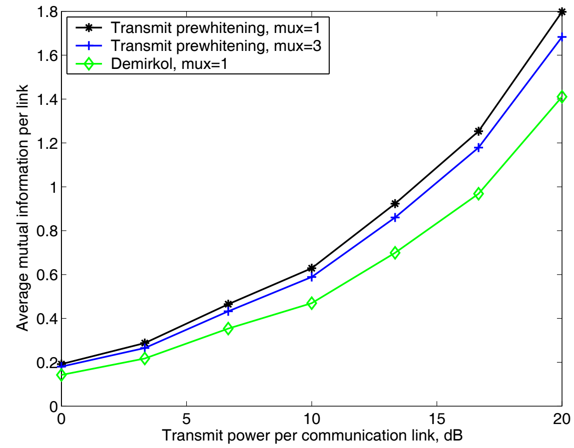


**Fig. 1**. Average mutual information per link, 4 transmitters serving 8 receivers, all with 4 element antenna arrays.

## 4. EXAMPLE

As an example of this general strategy, we have tried to find a solution that maximizes the sum of the mutual information on all the links. Note that this solution in general will not be on the boundary of the capacity region since that typically requires non-linear processing such as "writing on dirty paper" techniques [2, 3]. The iterative implementation outlined in Section 3.2 was used. In each step, $\mathbf{R}_z(\mathbf{\Theta}_r)$ was calculated using ordinary water-filling on the doubly prewhitened channel matrix $\tilde{\mathbf{H}}_r = \mathbf{R}_r^{\text{IN}^{-1/2}} \mathbf{H}_{\tau(r),r} \mathbf{T}^{-1/2}$ but with the "water level" calculated to give $\operatorname{Tr}[\mathbf{R}_z(\mathbf{\Theta}_r)] = P_r$ instead of $\operatorname{Tr}[\tilde{\mathbf{R}}_z(\mathbf{\Theta}_r)] = \tilde{P}_r$. The same power constraint $P_r$ was used for all communication links. Also, based on the ideas from [14], we tried to limit the maximum number parallel data streams to be multiplexed over the same physical link. For each receiver, $\mathbf{V}_k$ was determined as the MMSE solution but based on numerical experiments $\mathbf{\Lambda}_k = \mathbf{V}_k^*(\mathbf{V}_k \mathbf{V}_k^*)^{-1} \mathbf{V}_k$ was used instead of $\mathbf{\Lambda}_k = \mathbf{V}_k^* \mathbf{V}_k$ since it provided better performance.

The resulting algorithm was evaluated numerically on a simulated narrowband system with four fixed transmitting access points serving eight randomly placed receiving terminals. Both the access points and terminals were equipped with 4 element antenna arrays. The channels were described by random matrices with i.i.d. Gaussian elements, corresponding to an idealized rich scattering propagation. However, the gain of each channel matrix was determined based on a simple geometrical model with distance dependent path loss and additive lognormal fading. Each terminal was allocated to the access point with highest average path gain.

The parameter $\alpha$ was set to $\alpha = 0.001$ based on numerical experiments.

The results, presented in Figure 1, include a comparison with an algorithm by Demirkol and Ingram [15], which is similar but does not take interference reduction into account at the transmitters since prewhitening is only used at the receivers.

Ideally, the waterfilling should determine the optimal number of parallel spatial channels to multiplex over each physical link, but it turns out that the performance is further improved by explicitly limiting the maximum number streams to one in this specific scenario. Figure 1 shows the results when the maximum number of multiplexed streams is 1 or 3, respectively. Allowing for the maximum 4 multiplexed streams provides performance very similar to that of the Demirkol algorithm. Also, we tried to apply the algorithm from [7] using the resulting SINR of the above solution as the target SINR. The results (not included in the graph) provided a slight improvement of about 1–2%. Both these results indicate clearly that the proposed ad-hoc scheme, as expected, does not reach the globally optimum solution.

## 5. CONCLUSIONS

A generalized prewhitening at the transmitters has been suggested to improve the performance of MIMO link level algorithms in congested systems. Similar ideas are well-known in the field of downlink beamforming but have, to our knowledge, not previously been applied in a MIMO setting. Two different derivations are included, one showing that the proposed solution structure includes the globally optimal solution, another providing a practically implementable algorithm. Numerical experiments show a significantly increased performance but also the potential for further improvements. The results also confirm the conclusions in [14] that beamforming is better than spatially multiplexing several data streams over each link in interference limited scenarios.

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