SPECTRUM EFFICIENCY OF MIMO MULTIPLE-ACCESS WIRELESS SYSTEMS EXPLORING ONLY CHANNEL SPATIAL CORRELATIONS: AN ASYMPTOTIC APPROACH

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ABSTRACT

In this paper, we use the replica method originally developed in statistical physics to investigate the asymptotic sum-rate of a Gaussian antenna-array-based multiple-input multiple-output (MIMO) multiple-access wireless channel having spatial correlations at both the transmitters and the receiver. The asymptotic solution is not only rigorously valid for systems with large array sizes, but it also produces highly accurate ergodic results for systems with only a few antenna elements at each transmitter and receiver. Furthermore, with the asymptotic solution, we provide an efficient iterative water-filling algorithm to determine the optimum transmit signal covariance matrices when only the slow-varying channel spatial covariance information is available.

1. INTRODUCTION

Recent research results from the perspective of information theory have shown excellent spectral efficiency in wireless systems with multiple-input-multiple-output (MIMO) channels using antenna arrays [1]. Great progress has been made in recent years toward understanding the information-theoretical capacity of a point-to-point wireless system with a MIMO channel [1]-[4]. Such point-to-point MIMO systems can employ orthogonal multiple-access protocols, such as the TDMA or the FDMA, to serve each individual user independently in a multi-user network. Not until recently is the use of multiple antennas considered a possible choice for designing multi-access protocols to simultaneously serve multiple cochannel users with a high sum-rate (total rate of all users) [5].

With perfect channel state information, an efficient technique for finding the capacity-achieving transmit signal covariance matrices for multiple users, called iterative water-filling (IW), is developed by Yu, *et al.* [6]. Still little is known about the sum-rate of the MIMO multiple-access channel (MAC) with only the knowledge of the channel covariance information (CCI) [5], [7]-[10].

Following the pioneering work on CDMA systems in [11],[1] we present a framework to analyze the spectral efficiency of a MIMO-MA system with spatially-correlated channels and with only the knowledge of the CCI. Application of the *replica method*, a tool developed for macroscopic statistical physics, allows us to derive an asymptotic sum-rate when both the transmit and the receive channel spatial correlations exist. It is shown through numerical simulations that these results are not only rigorously valid for systems with large array sizes, but they also produce highly accurate ergodic results even for systems with only a few antenna elements at each transmitter and the receiver. With the asymptotic results, we develop an iterative algorithm to find the capacity-achieving transmit signal covariance matrix for each user. It is shown that when only the CCI is available, the joint optimization of signaling power and signaling directions for each user can be carried out with a traditional single-user water-filling algorithm.

Notations: For any general matrix $\mathbf{A}$, $\mathbf{A}^*$ denotes the conjugate transpose of $\mathbf{A}$, $\mathtt{Tr}(\mathbf{A})$ denotes the trace of $\mathbf{A}$, and $\lambda_{\mathbf{A},i}$ denotes the i^{th} eigenvalue of $\mathbf{A}$. In addition, $\mathbf{I}$ denotes the identity matrix, $\mathbf{0}$ denotes the zero matrix, $\|\cdot\|_2$ denotes the Euclidean norm, and $\mathtt{E}\{\cdot\}$ represents the expectation operator.

2. CHANNEL MODEL

We consider a MIMO-MA system, as shown in Fig. 1, with N receive antennas and K users respectively with $M_1, \cdots, M_K$ transmit antennas. The channel response of the k^{th} user from its transmit antenna m to the receive antenna n, denoted by $h_{n,m}^{(k)}$, can be assembled to form a channel matrix $\mathbf{H}_k$ of size $N \times M_k$. Let $\mathbf{s}_k \in C^{M_k\times 1}$ be the transmit signal of user k, let $\mathbf{x} \in C^{N\times 1}$ be the receive signal, and let $\mathbf{u} \in C^{N\times 1}$ be a noise vector with its covariance matrix being $\sigma^2\mathbf{I}$. The receive signal can then be represented as

$$\mathbf{x} = \sum_{k=1}^{K} \frac{1}{\sqrt{M_k}} \mathbf{H}_k\mathbf{s}_k + \mathbf{u} = \mathbf{H}\mathbf{s} + \mathbf{u}, \qquad (1)$$

[1]Subsequently, this work is generalized by Guo and Verdú [12].

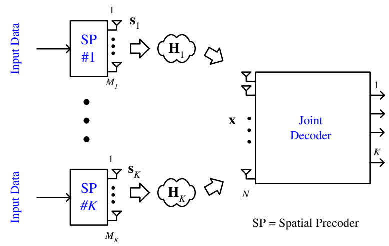

Fig. 1. Block diagrams of a MIMO MAC system.

where $\mathbf{H} = [\frac{1}{\sqrt{M_1}}\mathbf{H}_1 \; \frac{1}{\sqrt{M_2}}\mathbf{H}_2 \cdots \frac{1}{\sqrt{M_K}}\mathbf{H}_K]$ and $\mathbf{s} = [\mathbf{s}_1^T \; \mathbf{s}_2^T \cdots \mathbf{s}_K^T]^T$. The covariance matrices of the transmit signal are defined as

$$\mathrm{E}\{\mathbf{s}_k\mathbf{s}_{k'}^*\} = \begin{cases} \mathbf{\Omega}_k, & k = k' \\ \mathbf{0}, & k \neq k' \end{cases} \qquad (2)$$

with the total transmit power of the k^{th} user limited to P_k, i.e., $\mathrm{Tr}(\mathbf{\Omega}_k) \leq P_k$. Obviously, if the transmitters are allowed to cooperate, the MIMO-MA system reduces to a single-user MIMO system with a channel matrix of size $N \times M$, where $M = \sum_{k=1}^{K} M_k$. In a spatially correlated wireless channel, the MIMO channel matrix for the k^{th} user can be written as [3]

$$\mathbf{H}_k = \mathbf{R}_k^{\frac{1}{2}}\mathbf{W}_k\mathbf{T}_k^{\frac{1}{2}}, \qquad (3)$$

where $\mathbf{R}_k$ and $\mathbf{T}_k$ are, respectively, the $N \times N$ receive spatial correlation matrix and $M_k \times M_k$ transmit spatial correlation matrix of the k^{th} user, and $\mathbf{W}_k$ is an $N \times M_k$ i.i.d. (independent identically distributed) complex matrix.

The design of the transmit signaling schemes based on the long-term channel covariance matrices is more practical. Therefore, we consider the maximization of the ergodic sum-rate [5]:

$$\begin{aligned} &\max_{\{\mathbf{\Omega}_k\}} \quad \mathrm{E}_{\mathbf{W}}\left\{ \log\det\left(\mathbf{I} + \sum_{k=1}^{K}\mathbf{H}_k\mathbf{\Omega}_k\mathbf{H}_k^*\right) \middle| \mathbf{R},\, \mathbf{T} \right\}; \\ &\text{s.t.} \quad \begin{cases} \mathrm{Tr}(\mathbf{\Omega}_k) \leq P_k, & k = 1, \ldots, K \\ \mathbf{\Omega}_k \succeq \mathbf{0}, & k = 1, \ldots, K, \end{cases} \end{aligned}$$

where $\mathbf{W} \equiv \{\mathbf{W}_1, \cdots, \mathbf{W}_K\}$, $\mathbf{T} \equiv \{\mathbf{T}_1, \cdots, \mathbf{T}_K\}$ and $\mathbf{R} \equiv \{\mathbf{R}_1, \cdots, \mathbf{R}_K\}$. To simplify the sum-rate derivation, we focus only on large-system regimes, where both M_k and N tend to infinity; but $\frac{M_k}{N}$, known as the system load of each user, is fixed at a positive number ρ_k. We define $\rho = \sum_{k=1}^{K}\rho_k$ as the total system load and $\mu_k = \frac{M_k}{M}$ as the percentage system load of the k^{th} user. With this, we apply the *replica method* to derive a closed-form asymptotic solution for $\mathrm{E}_{\mathbf{W}}\{\log\det(\mathbf{I}+\sum_{k=1}^{K}\mathbf{H}_k\mathbf{\Omega}_k\mathbf{H}_k^*)|\mathbf{R}, \mathbf{T}\}$. Based on the asymptotic solution, we then develop an efficient algorithm to determine the optimum transmit signal covariance matrix for each user to maximize the sum-rate.

3. SUM-RATE

If the input signal statistics is fixed, the sum-rate of the MIMO-MA system is equal to the joint mutual information of the MIMO-MA system conditioned on the channel matrix $\mathbf{H}$

$$I_{\mathrm{MA}}(\mathbf{s};\mathbf{x}|\mathbf{H}) = \mathrm{E}_{\mathbf{s},\mathbf{x}}\left\{ \log\frac{p(\mathbf{x}|\mathbf{s},\mathbf{H})}{p(\mathbf{x}|\mathbf{H})} \middle| \mathbf{H} \right\}, \qquad (4)$$

where the expectation is taken over the joint conditional distribution $p(\mathbf{s},\mathbf{x}|\mathbf{H})$. Note that $p(\mathbf{x}|\mathbf{s},\mathbf{H})$ denotes the conditional probability density function (pdf) of $\mathbf{x}$ conditioned on $\mathbf{s}$ and $\mathbf{H}$. Similar notation will be used throughout this paper. In (4), $p(\mathbf{x}|\mathbf{H})$ is the marginal distribution of $p(\mathbf{x},\mathbf{s}|\mathbf{H}) = p(\mathbf{x}|\mathbf{s},\mathbf{H})p(\mathbf{s})$. From (1), the characteristics of the Gaussian MIMO-MA system can be described as

$$p(\mathbf{x}|\mathbf{s},\mathbf{H}) = \frac{\exp\left[-\frac{1}{2\sigma^2}\|\mathbf{x}-\mathbf{H}\mathbf{s}\|_2^2\right]}{(2\pi\sigma^2)^{-N}}. \qquad (5)$$

Noticing that (5) is a Gaussian probability density function, we have

$$\mathrm{E}_{\mathbf{s},\mathbf{x}}\{\log p(\mathbf{x}|\mathbf{s},\mathbf{H})|\mathbf{H}\} = -N\log(2\pi\sigma^2 e). \qquad (6)$$

Define

$$Z(\mathbf{x},\mathbf{H},\sigma) = \mathrm{E}_{\mathbf{s}}\left\{ \exp\left[-\frac{1}{\sigma^2}\|\mathbf{x}-\mathbf{H}\mathbf{s}\|_2^2\right] \middle| \mathbf{H} \right\}. \qquad (7)$$

Therefore, by using (4), (6), and (7) the sum-rate of the MIMO-MA system can be written as

$$\begin{aligned} &I_{\mathrm{MA}}(\mathbf{s};\mathbf{x}|\mathbf{H}) \\ &= -M \cdot \mathrm{E}_{\mathbf{x}}\left\{ \frac{1}{M}\log Z(\mathbf{x},\mathbf{H},\sigma) \middle| \mathbf{H} \right\} - N, \end{aligned} \qquad (8)$$

which is closely related to $Z(\mathbf{X},\mathbf{H},\sigma)$. Note that, if $p(\mathbf{s})$ is Gaussian, the sum-rate of the MIMO-MA system (8) is thus $\log\det(\mathbf{I} + \sum_{k=1}^{K}\mathbf{H}_k\mathbf{\Omega}_k\mathbf{H}_k^*)$.

In the statistic mechanics, the free energy is defined as

$$\frac{1}{M}\log Z(\mathbf{x},\mathbf{H},\tilde{\sigma}), \qquad (9)$$

which includes all the statistics of the observables in the system [11]. A standard trick used in statistical mechanics in order to compute the asymptotic free energy is the *replica method*. In the remaining part, we present some results without proof. Readers interested in details are referred to [14]. After the asymptotic free energy is derived, we then have the following proposition:

Proposition 1 *[14] The mutual information of a MIMO MAC with Gaussian distributed inputs is given by*

$$\begin{aligned}\mathcal{I}_{\mathrm{MA}} &= \frac{1}{N}\sum_{k=1}^{K}\log\det(\mathbf{I}+\varepsilon_k\mathbf{T}_k\mathbf{\Omega}_k)\\ &+\frac{1}{N}\log\det\left(\mathbf{I}+\sum_{k=1}^{K}\frac{1-a_k}{\sigma_0^2}\mathbf{R}_k\right)\\ &-\sum_{k=1}^{K}\rho_k\varepsilon_k(1-a_k), \qquad (10)\end{aligned}$$

where ε_k and a_k satisfy the saddle-point equations

$$\begin{aligned}\varepsilon_k &= \frac{1}{M_k}\mathrm{Tr}\left\{\left[\sigma_0^2\mathbf{I}+\sum_{l=1}^{K}(1-a_l)\mathbf{R}_l\right]^{-1}\mathbf{R}_k\right\},\\ 1-a_k &= \frac{1}{M_k}\mathrm{Tr}\left\{[\mathbf{I}+\varepsilon_k\mathbf{T}_k\mathbf{\Omega}_k]^{-1}\mathbf{T}_k\mathbf{\Omega}_k\right\}. \qquad (11)\end{aligned}$$

It should be noted that we have represented $\mathcal{I}_{\mathrm{MA}}$ in **Proposition 1** in a quantized format of as if the number of the input and the output arrays are finite.

4. SUM-RATE MAXIMIZATION

Through numerical simulations in Section 5, we observe that the asymptotic mutual information of the MIMO MAC is extremely close to its ergodic sum-rate, even for systems with as few as two or three antennas at each transmitter and receive. This observation offers the asymptotic solution of $\mathcal{I}_{\mathrm{MA}}$ a practical value on developing an efficient algorithm to identify the optimal signal covariance matrix for each user.

With no knowledge of the CSI, the optimal signal covariance matrix $\{\mathbf{\Omega}_k\}$ in a MIMO MAC to maximize $\mathcal{I}_{\mathrm{MA}}$ are identity matrices [13]. On the other hand, when only the slow-varying CCI is available, the optimal $\mathbf{\Omega}_k$ that maximize $\mathcal{I}_{\mathrm{MA}}$ is the water-filling solution to an equivalent system with $\varepsilon_k\mathbf{T}_k$ being its stand-along MIMO channel.

Proposition 2 *[14] When only the CCI is known, the signal transmission strategy for the k^{th} user that maximizes $\mathcal{I}_{\mathrm{MA}}$ is the water-filling solution to an equivalent system with $\varepsilon_k\mathbf{T}_k$ being its stand-alone MIMO channel, where $\{\varepsilon_k\}$ are a set of positive roots of joint equations of (11).*

Obviously, the interaction among users is through $\{\varepsilon_k\}$. The signal covariance matrix of each user is not affected by the channel structures of the other users. The transmit channel structure of the k^{th} user, $\mathbf{T}_k$, affects only the eigenvalue distribution of its own $\mathbf{\Omega}_k$. Since all the signal covariance matrices $\{\mathbf{\Omega}_k\}$ are involved in (11) in solving for ε_k, we might expect that the optimal covariance matrices to be found with an iterative algorithm.

Table 1. Comparison of the mutual information between the analytic result (10) and the corresponding simulation results for $N = 2$, $M_1 = M_2 = M_3 = M_4 = 2$ and for different SNRs.

SNR(dB)	2	10	18
Analytical (bps/Hz)	5.17	9.91	16.43
Empirical (bps/Hz)	5.20	9.97	16.50
Difference (bps/Hz)	0.64%	0.64%	0.43%

Algorithm 1 *CCI-based iterative water-filling (IW) algorithm for a joint-decoding MIMO MAC system:*

1. *Initialize* $\varepsilon_k = \frac{1}{\sigma_0^2}$, $k = 1, \cdots, K$.
2. $\mathbf{\Omega}_k = \arg\max_{\mathrm{Tr}\{\mathbf{\Omega}_k\}\le P_k}\log\det(1+\varepsilon_k\mathbf{T}_k\ \mathbf{\Omega}_k)$, $k = 1, \cdots, K$.
3. *Solve $\{\varepsilon_k\}$ according to the joint equations of (11) with the new $\{\mathbf{\Omega}_k\}$.*
4. *Go to step 2.*

Such an iterative min-max procedure is not guaranteed to converge even when a problem has a concave-convex structure. But the iterative procedure appears to work well in practice for this particular problem. In fact, the proposed algorithm provides a viable approach to maximize the sum-rate in a practical wireless MIMO MAC system. Not that, for the k^{th} user, the associated transmit channel spatial correlation matrix $\mathbf{T}_k$ can be estimated through the receiver of the k^{th} user's downlink communication channel. A base station only need to deal with Step 3 of the proposed algorithm and then send the k^{th} user his corresponding ε_k. After the k^{th} user receive ε_k, Step 2 of the proposed algorithm is the only required process for the k^{th} user. With the same procedures, Step 2 and Step 3 are repeated.

5. EXPERIMENTS

For all the experiments, we assume the spatial correlation is generated from an uniform linear array with half wavelength spacing in a wireless environment where there is one propagation path cluster with Gaussian power azimuthal distribution having mean angle of θ_k and angle spread of δ_k. In the remaining part of this section, θ_k and δ_k each with subscripts T and R, respectively, refer to the concerned values at the transmit side and at the receive side.

Experiment 1: Accuracy of (10)

Table 1 compares the mutual information between analytical results according to (10) and the corresponding simulation results obtained from 10,000 realizations of $\{\mathbf{H}_k\}$.

Without loss of generality, we assume $\mathbf{\Omega}_k = \mathbf{I}$ in this experiment. It is obvious that, regardless of the SNR, (10) produces highly accurate results even for systems with only a few antenna elements at each transmitter and receiver.

Experiment 2: Channel structure effects on the sum-rate

An intuitive question is whether the spatial channel structures, either at the transmit side or at the receive side, affect the sum-rate. To investigate the influence of the channel correlation matrices, we focus on a two-user scenario.

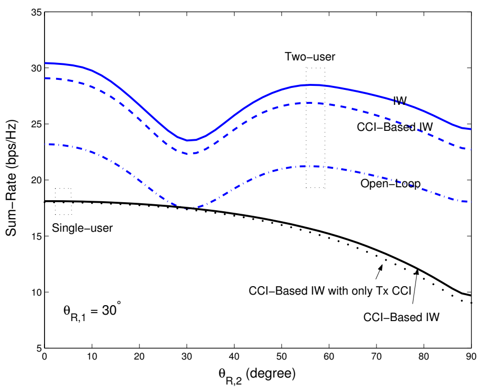

Fig. 2. Effects of the receiver-side spatial channel structure on the system sum-rate.

Fig. 2 depicts the sum-rates when $\theta_{\mathrm{R},1} = 30^\circ$ with $\theta_{\mathrm{R},2}$ being a control parameter. The curves from top to bottom are 1) the sum-rates of a joint-decoding MIMO MAC system employing the IW algorithm with perfect CSI, 2) the sum-rates of a joint-decoding MIMO MAC system employing the CCI-based IW algorithm with only the knowledge of the CCI, 3) the sum-rates of an open-loop MIMO MAC system with $\mathbf{\Omega}_1 = \mathbf{\Omega}_2 = \mathbf{I}$, and 4) the sum-rates of a separate-decoding MIMO MAC system employing the linear MMSE spatial equalizer at the receiver and use the same use the same signal covariance matrices as those in 2). Obviously, there are always dips around $\theta_{\mathrm{R},2} = 30^\circ$ for all the curves in Fig. 2 when the signals of the two users collide at the receive side. Fig. 2 also depicts the ergodic capacity of a single-user system, i.e., $K = 1$, by removing the first user from the system. Numerical results show that, with only the knowledge of $\mathbf{T}_2$, assuming $\mathbf{R}_2 = \mathbf{I}$ is enough to obtain a near-optimal ergodic capacity. This phenomenon is quite different from that in the MIMO MAC scenarios. We conclude these experiment results with the following property.

Property 1 *In a MIMO MAC system, signal covariance matrix optimization using the IW algorithm with full knowledge of the CSI, performances only a little better than the CCI-based IW algorithm. When only the CCI is available: 1) For each individual user, knowledge of the transmit-side CCI is enough to approach the maximum data-rate. 2) For a general MIMO MAC system, on the other hand, knowledge of the receive-side CCI is crucial for maximizing the sum-rate.*

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