PRACTICAL MULTIUSER DIVERSITY WITH FAIR CHANNEL ASSIGNMENT

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ABSTRACT

Inspired by the information theoretic results concerning multiuser diversity, we address practical issues in implementing multiuser diversity in a realistic multiple access wireless setting. The users are allowed to have different average channel powers and the important issue of fairness is investigated in this paper. Using a fair channel assigning strategy that assigns the channel to only one user with the greatest instantaneous SNR-to-average-SNR ratio, our emphasis is on the effect of channel feedback delay in downlink transmissions. A novel optimization criterion based on an outage probability constraint is proposed. Applying the outdated channel feedback model, we illustrate the impact of channel feedback delay on the achievable multiuser diversity gain with the number of users. A robust constant power, variable rate M-QAM scheme that is less sensitive to feedback delay is proposed using the optimal set of switching thresholds, which is derived to maximize the average throughput subject to an outage probability constraint.

1. INTRODUCTION

The conventional diversity techniques over point-to-point links, such as spatial diversity and frequency diversity, offer improvements in spectral efficiency. Recent studies indicate that, there is another form of diversity, called multiuser diversity, inherent in multiuser wireless systems [1]. For the uplink multiuser scenario, the information theoretic results in [2] show that the optimal policy to maximize the total system capacity defines a multiple access strategy similar to TDMA: at any moment, only the user with the best channel is scheduled to transmit. We term this scheme as the *greedy* scheme. The same access scheme is shown to be valid for the downlink from the base station (BS) to mobile users [3]. The information theoretic analysis for multiuser diversity provides a benchmark for practical systems. However, it does not reflect the performance achieved by actual systems with practical constraints. Adaptive transmission techniques are means to achieve system capacity [4]. Inspired by the capacity results in [2] and [3], we develop practical adaptive modulation techniques for multiple access wireless transmission, emphasizing practical issues to realize multiuser diversity in downlink transmissions.

The greedy scheme assumes the ideal situation where users' fading statistics are the same. However, in practical systems, the users' statistics are not identical. The greedy scheme amounts to assigning the channel always to the statistically stronger user and would be highly unfair. In this paper, we consider a fair channel assigning strategy where the channel is assigned to the user with the greatest instantaneous SNR-to-average-SNR ratio. Therefore the probability of being assigned to use the channel is the same for each user, even when each user has a different average SNR. It is noted that a similar fair assigning algorithm which employs normalization over the average throughput is proposed in [1]. In our proposed scheme, a discrete finite set of M-QAM constellations is adopted and a constant transmit power is assumed. Avoiding power adaptation eases the hardware complexity, and the peak-to-average-power (PAR) problem. Since at any instant only one user is allowed to use the channel, the adaptation policy is similar to the single-user scenario.

For single user systems, optimal and practical adaptive modulation techniques have been well established [4], [5]. Assuming a perfect channel quality feedback, a constant-power adaptive scheme to maximize the system throughput with an average BER requirement is investigated in [6]. Good performance of adaptive modulation requires accurate channel estimation at the receiver and a reliable feedback path between the receiver and the transmitter. However, the channel feedback information will become outdated if the channel is changing rapidly. The effect of channel feedback delay on the average BER over Nakagami fading channels is briefly addressed in [7]. Under an instantaneous BER constraint, the influence of outdated channel estimates on the adaptive schemes is investigated in [8].

Using the greedy channel assigning strategy in a multiuser system, a robust rate adaptation scheme to achieve multiuser diversity under the outdated channel feedback model is proposed in [9] and an average BER constraint is used to optimize the switching thresholds. However, some applications sensitive to instantaneous BER may not perform well under the average BER constraint, where the constraint of outage probability for instantaneous BER might be more suitable. While the users are assumed to have the same average channel power in [9], we allow for different average powers and address the important issue of fairness in this paper. With the objective of maximizing the average sum throughput under an outage probability constraint for instantaneous BER with respect to the switching thresholds, a robust constant power, variable rate M-QAM scheme that is less sensitive to feedback delay is proposed. The impact of channel feedback delay on the achievable multiuser diversity gain is illustrated as a function of the number of users.

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2. SYSTEM MODEL

We address downlink transmission in a multiuser system with a channel shared by N users. Each mobile receiver tracks its own channel SNR and feeds back the channel quality to the BS. The BS adapts the transmission rate for each user's signal under criteria based on the feedback channel quality. We assume that uncoded transmission with a constant transmit power is used and we normalize it to 1. Dropping the time index for simplicity, the baseband channel model is given by

$$z_n = h_n b + w_n, \tag{1}$$

where z_n is the received signal of the nth user, b is the transmitted signal broadcasted from the BS, h_n is the frequency flat fading channel gain between the BS and the nth user's receiver, and w_n represents the additive white Gaussian noise (AWGN) with zero mean and variance N_0. We define the channel SNR for user n as $\gamma_n = |h_n|^2/N_0$. Assuming each user undergoes Rayleigh fading independently, and the average channel SNR for each user is $E(\gamma_n) = \bar{\gamma}_n$, the probability density function (pdf) of SNR γ_n is given by the exponential distribution, which is $p_{\gamma_n}(\gamma_n) = \frac{1}{\bar{\gamma}_n} e^{-\gamma_n/\bar{\gamma}_n}, \gamma_n \geq 0$.

It is proved in [2] that the optimal channel assignment strategy in the sense of maximizing the throughput of a multiuser system is assigning the channel to only one user who has the best channel gain. Using this greedy scheme, a user may not be assigned to the channel if the user never has the best channel quality. Since each user may be located at a different distance from the BS, different users may have different average SNRs. To mitigate the unfairness existing in the greedy scheme, we consider a fair channel assigning strategy where the channel is assigned to the user with the greatest instantaneous SNR-to-average-SNR ratio $\beta_n := \frac{\gamma_n}{\bar{\gamma}_n}$. In this fair strategy, the channel is assigned to the user that has the largest normalized SNR in every time slot, and thus the probability of being assigned to use the channel is the same for each user. The user with the highest normalized SNR will be transmitted to, and therefore a certain degree of fairness is achieved among the users.

Multiuser diversity can be achieved by means of adaptive transmission techniques. Since the BS transmits data to only one user at a specific time, (1) is simplified to a single user model with b denoting the signal of the selected user. Thus adaptive modulation techniques for single-user systems can be extended to this multiuser setting, to increase the individual throughput as well as the system throughput. For practical reasons, a discrete finite set of M-QAM constellations is adopted, and we utilize a discrete set of candidate constellation sizes $\mathcal{M} = \{M_0, M_1, ..., M_{J-1}\}$ for Gray-coded square M-QAM rate-adaptive schemes, where M_0 denotes no-transmission. Given a set of switching thresholds $\mathbf{t} = [t_0, ..., t_J]$, the constellation size M_j is selected and used for transmission when $t_j \leq \hat{\gamma}_n < t_{j+1}$, where $\hat{\gamma}_n$ is the outdated SNR fed back from the nth user and will be discussed in detail in Section 3. Generally, it is assumed $t_0 = 0, t_J = \infty$.

3. OUTDATED CHANNEL FEEDBACK MODEL

In our model, the BS communicates with the user with the best instantaneous normalized SNR at any given time, and adapts the constellation size as a function of the channel feedback measure from the selected user. However, uncertainty of the feedback information arises either due to the delay of feedback path or the estimation error at the mobile receiver. Assuming perfect channel estimation at the receiver, we will focus on the uncertainty caused specifically by the outdated channel feedback. In the sequel, we will focus on the nth user. The estimate of SNR, $\hat{\gamma}_n$, is obtained from a feedback path with time delay τ_n. In a time-varying channel, the assigned user n and its corresponding constellation size is selected based on SNR at time t while the BS transmits data to it at time $t + \tau_n$. Therefore the estimate $\hat{\gamma}_n$ denotes the nth user's channel SNR $\gamma_n(t)$ at time t while the actual channel SNR γ_n during transmission is the nth user's channel SNR $\gamma_n(t + \tau_n)$ at time $t + \tau_n$.

Investigating the impact of feedback delay requires the second-order statistics of the channel variation. Suppose the nth user is selected, the estimated channel SNR $\hat{\gamma}_n$ is the only known information for the nth user at the BS and we are interested in the conditional pdf of the nth user's SNR, γ_n, given its estimate $\hat{\gamma}_n$, $p_{\gamma_n|\hat{\gamma}_n}(\gamma_n|\hat{\gamma}_n)$. For the nth user, the pdf of the current fading amplitude is Ricean when conditioned on the outdated channel feedback estimate. The conditional pdf of γ_n knowing $\hat{\gamma}_n$, $p_{\gamma_n|\hat{\gamma}_n}(\gamma_n|\hat{\gamma}_n)$, can be derived as [9][10]

$$\begin{aligned} p_{\gamma_n|\hat{\gamma}_n}(\gamma_n|\hat{\gamma}_n) \quad &= \quad \frac{1}{\bar{\gamma}_n(1-\rho_n^2)} \exp\left(-\frac{\rho_n^2\hat{\gamma}_n + \gamma_n}{\bar{\gamma}_n(1-\rho_n^2)}\right) \\ &\mathrm{I}_0\left(\frac{2\rho_n\sqrt{\hat{\gamma}_n\gamma_n}}{\bar{\gamma}_n(1-\rho_n^2)}\right), \quad \hat{\gamma}_n, \gamma_n \geq 0, \end{aligned} \tag{2}$$

where $\mathrm{I}_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind, and $\rho_n := J_0(2\pi f_{D_n}\tau_n)$ in which $J_0(\cdot)$ is the zeroth-order Bessel function, and f_{D_n} is the Doppler spread of the nth user.

4. AVERAGE THROUGHPUT

Based on the introduced model, in this section, we derive closed-form expressions for the average throughput for the nth user. At any given time slot, the BS compares all $\hat{\beta}_k := \frac{\hat{\gamma}_k}{\bar{\gamma}_k}$ from the users and selects the user with the largest such ratio. Under the outdated channel feedback model discussed in Section 3, for Rayleigh fading, $\hat{\beta}_k$ is exponentially distributed with unit mean. For a fixed $\hat{\gamma}_n = y$, the probability that the nth user has the largest normalized SNR can be expressed as

$$Pr\left(\bar{\gamma}_n \max_{k \neq n} \beta_k \leq y\right) = \left(1 - e^{-y/\bar{\gamma}_n}\right)^{N-1}. \tag{3}$$

Defining $p^f_{\gamma_n}(y) = Pr\left(\bar{\gamma}_n \max_{k\neq n} \beta_k \leq y\right) p_{\gamma_n}(y)$, the average throughput for the nth user can be shown to depend on the set of thresholds $\mathbf{t}$ as

$$\begin{aligned} \bar{R}_n(\mathbf{t}) &= \sum_{j=0}^{J-1} \int_{t_j}^{t_{j+1}} R_j p^f_{\gamma_n}(y) dy \\ &= \sum_{j=0}^{J-1} \frac{R_j}{N} \left[\left(1 - e^{-\frac{t_{j+1}}{\bar{\gamma}_n}}\right)^N - \left(1 - e^{-\frac{t_j}{\bar{\gamma}_n}}\right)^N\right], \end{aligned} \tag{4}$$

where $R_j := \log_2 M_j$. It is formulated as the weighted sum of the throughput of the individual M-QAM constellation weighted by its probability, and is independent of ρ_n. The throughput for the nth user is also independent of other users' average SNR.

Note that $p^f_{\gamma_n}(y)$ can be simplified as

$$p^f_{\gamma_n}(y) = \frac{1}{\bar{\gamma}_n} \sum_{i=0}^{N-1} (-1)^i \binom{N-1}{i} e^{-(1+i)y/\bar{\gamma}_n}, \tag{5}$$

where $\binom{N-1}{k} = \frac{(N-1)!}{(N-k-1)!k!}$.

Since in selecting the users relative SNRs are compared, the average time of access for any user is $\frac{1}{N}$, independent of the average SNR of the user, which mitigates the unfairness in the greedy scheme. We notice that (5) is independent of other users' average SNR and every user will benefit from multiuser diversity [1], thus increasing the system sum throughput. However, compared to the greedy scheme, the system sum throughput would be less using the fair channel assignment.

When the number of users N becomes large, the binomial coefficient term $\binom{N-1}{k}$ in (5) will become difficult to compute due to the factorial. Therefore a simple computable form for (5) is required for a system in the limit of large number of users. Through asymptotic analysis [9], it is shown that with a large value of N, (5) will converge to:

$$\frac{1}{N\bar{\gamma}_n}\exp\left(\frac{\bar{\gamma}_n \ln N - y}{\bar{\gamma}_n}\right)\exp\left[-\exp\left(\frac{\bar{\gamma}_n \ln N - y}{\bar{\gamma}_n}\right)\right]. \quad (6)$$

Using the asymptotic expression (6), a simple accurate closed-form approximations for the average throughput with a large number of users can be obtained:

$$\bar{R}_n(\mathbf{t}) \approx \sum_{j=0}^{J-1}\int_{t_j}^{t_{j+1}} \frac{R_j}{N}\left[\exp\left(-\exp\left(-\frac{t_{j+1}-\bar{\gamma}_n \ln N}{\bar{\gamma}_n}\right)\right)\right.$$
$$\left. - \exp\left(-\exp\left(-\frac{t_j-\bar{\gamma}_n \ln N}{\bar{\gamma}_n}\right)\right)\right]. \quad (7)$$

5. BER FOR M-QAM OVER AWGN CHANNELS

The BER expressions over AWGN channels are required to optimize the throughput in (4) with respect to the thresholds $\mathbf{t}$. Therefore in this section, we introduce the BER expressions of M-QAM over AWGN channels, which are necessary in the system design.

The exact BER of a Gray-coded square M-QAM scheme over AWGN channels as a function of received SNR γ and a constellation size M can be expressed as [6]

$$BER(M,\gamma) = \sum_{i=1}^{\pi(M)} G_{M,i}\ Q(\sqrt{g_{M,i}\gamma}), \quad (8)$$

where $Q(x)$ is the Gaussian Q-function [10], $\pi(M)$ is the number of terms for summation and $G_{M,i}, g_{M,i}$ is a pair of mode dependent constants, which can be found in [6]. The exact BER expressions are not easily inverted with respect to power and rate. Therefore tight BER approximations of M-QAM over AWGN channels are introduced for design of adaptive systems [4] [5] . A tight approximation of BER expression for Gray-coded square M-QAM over AWGN channels is given as [4]

$$BER(M,\gamma) \approx 0.2e^{-\frac{3\gamma}{2(M-1)}}, \quad (9)$$

which is tight to within $1dB$ for $M \geq 4$ and $0 \leq \bar{\gamma} \leq 30dB$.

6. THRESHOLD OPTIMIZATION SUBJECT TO OUTAGE PROBABILITY CONSTRAINT

Threshold optimization subject to the average BER constraint is considered in [9]. However, some applications sensitive to instantaneous BER may not perform well under the average BER constraint, where the outage probability constraint for instantaneous BER might be more suitable. Our goal here is to optimize the vector of thresholds $\mathbf{t}$ that maximizes the average throughput $\bar{R}(\mathbf{t})$ under the outage probability constraint $P[BER > BER_0] \leq \epsilon$ using Lagrange multipliers, where BER_0 is the desired target BER.

With a feedback path with time delay τ_n, the outage probability $P_{out}(\mathbf{t}) = P[BER > BER_0]$ is defined as

$$P_{out}(\mathbf{t}) = \sum_{j=1}^{J-1} P[BER > BER_0, t_j \leq \hat{\gamma}_n < t_{j+1}]$$
$$= \sum_{j=1}^{J-1}\int_{t_j}^{t_{j+1}}\int_0^{\chi_j} p_{\gamma_n|\hat{\gamma}_n}(\gamma_n|y)d\gamma_n p^f_{\gamma_n}(y)dy, \quad (10)$$

where χ_j satisfies $BER(M_j,\chi_j) = BER_0, j = 1,...,J-1$. For a given constellation size M_j when $\gamma_n < \chi_j$, the instantaneous BER becomes greater than BER_0.

Since we assume that the boundary thresholds are fixed to $t_0 = 0$ and $t_J = \infty$, we are concerned with a $J-1$ dimensional optimization problem. Using a Lagrange multiplier λ, the $J-1$ dimensional optimization is converted into a one dimensional optimization problem as follows. Simplifying (10), the cost function can be expressed as

$$\Phi(\mathbf{t}) = \bar{R}(\mathbf{t}) + \lambda(P_{out}(\mathbf{t}) - \epsilon)$$
$$= \sum_{j=0}^{J-1}\int_{t_j}^{t_{j+1}} R_j p^f_{\gamma_n}(y)dy$$
$$+\lambda\left(\sum_{j=1}^{J-1}\int_{t_j}^{t_{j+1}}(1-Q(\alpha,\beta_j))\,p^f_{\gamma_n}(y)dy - \epsilon\right), \quad (11)$$

where $Q(\cdot,\cdot)$ is the Marcum Q-function [10],

$$\alpha := \sqrt{\frac{2\rho_n^2 y}{\bar{\gamma}_n(1-\rho_n^2)}}, \quad (12)$$

and

$$\beta_j := \sqrt{\frac{2\chi_j}{\bar{\gamma}_n(1-\rho_n^2)}}. \quad (13)$$

The optimal vector of thresholds $\mathbf{t}_{opt}$ should satisfy $\frac{\partial\Phi(\mathbf{t})}{\partial\mathbf{t}} = 0$, and $P_{out}(\mathbf{t}) - \epsilon = 0$. After some manipulations, the relationship for all thresholds $t_j, j = 1,...,J-1$ are shown to satisfy

$$\frac{R_j - R_{j-1}}{\lambda} = Q(\alpha_j,\beta_j) - Q(\alpha_j,\beta_{j-1}), \quad (14)$$

where β_j is given by (13) and

$$\alpha_j := \sqrt{\frac{2\rho_n^2 t_j}{\bar{\gamma}_n(1-\rho_n^2)}}, \qquad j = 1,...,J-1. \quad (15)$$

Relating all thresholds $t_j, j = 2,...,J-1$ to t_1 using (14), we get

$$\frac{Q(\alpha_j,\beta_j) - Q(\alpha_j,\beta_{j-1})}{R_j - R_{j-1}} = \frac{Q(\alpha_1,\beta_1) - 1}{R_1}. \quad (16)$$

Therefore the optimum vector $\mathbf{t}_{opt}$ is completely determined by t_1, which is selected to satisfy the constraint

$$P_{out}(\mathbf{t}(t_1)) = \epsilon. \quad (17)$$

Note that (16) and (17) are both one dimensional root finding problems and numerical methods can be used to find the solution for this nested root finding problem. It is also noted that the computation of optimal switching thresholds is all done off-line.

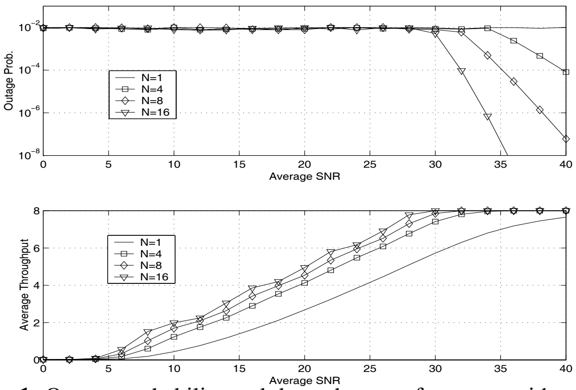

Fig. 1. Outage probability and throughput performance with optimal thresholds ($BER_0 = 10^{-4}$, $\epsilon = 0.01$, $f_d\tau = 0.01$)

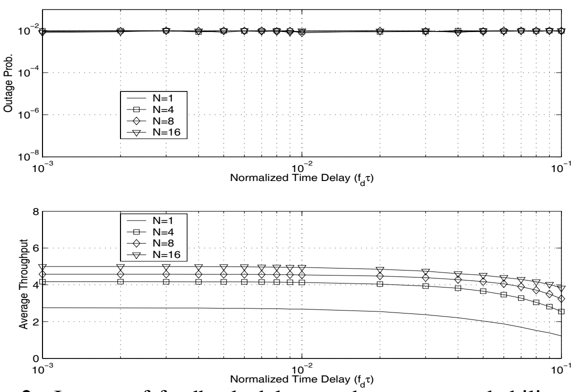

Fig. 2. Impact of feedback delay on the outage probability and throughput with optimal thresholds ($BER_0 = 10^{-4}$, $\epsilon = 0.01$, $\bar{\gamma}_n = 20dB$)

7. NUMERICAL RESULTS

In this section we present numerical results for a multiuser system with $N = 1, 4, 8, 16$ users with the optimal thresholds subject to the outage probability constraint for instantaneous BER while maximizing the average throughput in Rayleigh flat channels. The target outage probability is set to $\epsilon = 0.01$ and the target BER is set to $BER_0 = 10^{-4}$. For practical reasons, the constellation size is assumed to adjust over the limited set of $\mathcal{M} = \{0, 4, 16, 64, 256\}$. In order to give a fair comparison, for N users, there are N independent channels provided in the system, where the channels are assigned to the users independently using the fair channel-assigning strategy.

Fig. 1 depicts the outage probability and average throughput for the nth user with optimal thresholds subject to the outage probability constraint for instantaneous BER with a normalized feedback delay $f_d\tau = 0.01$. The average SNR $\bar{\gamma}_n$ for the nth user changes from $0dB$ to $40dB$. A system with more users provides higher throughput for the nth user as expected due to the multiuser diversity effect. The required target outage probability is maintained until the outage probability curve of the highest order constellation (i.e. 256-QAM) is reached. The impact of feedback delay on the outage probability and throughput is presented in Fig. 2. The scheme keeps maintaining the outage probability constant as the feedback delay increases. From Fig. 2, it is observed that a normalized delay up to $f_d\tau = 0.01$ can be tolerated without a noticeable degradation in average throughput while the outage probability constraint is satisfied. With more users, the system is shown to be less sensitive to feedback delay. Hence, with a given carrier frequency and vehicle speed, the tolerable time delay can be calculated using our framework. The scheme with optimal thresholds subject to the outage probability constraint is immune to feedback delay and the average throughput under the outage probability constraint increases with the number of users in the system due to multiuser diversity.

8. CONCLUSIONS

Using a fair channel assigning strategy that assigns the channel to only one user based on the normalized SNR fed back, we have developed and analyzed a practical rendition of a multiuser diversity system for the downlink multiple access wireless transmission. The impact of feedback delay on multiuser diversity is investigated and a robust adaptive scheme that is immune to the feedback delay is proposed under an outage probability constraint while maximizing the average throughput. The tolerable feedback delay that causes no noticeable system degradation can be calculated as a function of the number of users using our framework.

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