TRANSMIT OPTIMIZATION FOR MIMO CHANNELS WITH MIXED DELAY-CONSTRAINED AND NO-DELAY-CONSTRAINED SERVICES

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ABSTRACT

This paper studies the transmit optimization problem for MIMO channels with mixed delay-constrained (DC) and no-delay-constrained (NDC) services. Suppose that perfect channel state information is available to both transmitter and receiver sides, we propose a transmission scheme based on sub-channel grouping (SCG) technique together with different power control policies designed for the grouped channels generated by SCG to support either DC or NDC services. Optimal and sub-optimal power allocation strategies are proposed. Computer simulations are given to evaluate the capacity achieved by the proposed scheme as well as to evaluate the throughput performance when practical bit-loading algorithms are applied.

1. INTRODUCTION

Wireless transmission through MIMO channels has received considerable attention during the past years due to the multiplicative capacity of MIMO channels over SISO channels. When perfect channel state information (CSI) is available to both transmitter and receiver sides, a MIMO channel can be decoupled into multiple time varying SISO sub-channels through joint transmit and receive eigen-beamforming. For no-delay-constrained (NDC) case, the capacity in ergodic sense (or *ergodic capacity*) is of interest for which the codeword is required to be long enough to capture the ergodic property of the fading process. The ergodic capacity of a MIMO channel is maximized when the transmission powers to each sub-channel are allocated via water-filling policy [5]. For delay-constrained (DC) case, the codeword spans a frame of limited number of blocks, and thus there is a stringent delay constraint. One interesting capacity definition for this case is *delay limited capacity*, which defines the maximum rate of codes that can be reliably transmitted and received over all frames under a long term average power constraint (LTPC). If the MIMO channel is regular, the optimal delay-limited capacity is non-zero, and can be achieved through margin-adaptive water-filling policy [1].

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Capacity maximization has been studied either for single NDC service [5], or for single DC service [1], [4]. In practice, the property of high capacity provided by MIMO channels leads MIMO technology well suitable for multimedia transmissions, which may simultaneously consist of both DC service, such as voice transmission, and NDC service, such as delay insensitive packet data transmission. In fact, with the layer structure generated by joint transmit and receive beamforming, MIMO channels can be used to flexibly support those mixed transmissions.

In this paper, we propose a transmission scheme for MIMO channels to support mixed DC and NDC services. This scheme is based on the sub-channel grouping (SCG) technique proposed in [4], which first sorts the channel gains of all sub-channels in each frame in a decreasing order, and then groups the sub-channels with the same ordering number from all frames as one grouped channel. Through SCG, each grouped channel experiences a fading process with different statistics, ranging from less fluctuation for stronger channels to severe fading for weaker channels. We then employ overall channel inversion (OCI) based power control to transform less fluctuated channels into AWGN channels, so that these channels can be used for DC data transmission; and apply different constant powers to the channels with severe fading and transmit NDC data through those channels. Optimal and suboptimal power allocation strategies are derived for the proposed scheme.

This paper is organized as follows. Section II outlines the MIMO channel model and its optimal delay limited capacity. Section III presents the proposed MIMO scheme to support mixed DC and NDC services, and formulates the optimal power allocation problem. Suboptimal solutions are presented in Section IV. Computer simulation results are given in Section V. Conclusions are drawn in Section VI.

2. SYSTEM MODEL

2.1. Channel model

Consider a $M_r \times M_t$ MIMO block fading channel $\mathbf{H}$, and suppose the received signal $\mathbf{y}(n)$ at the frame of interest is given by

$$\mathbf{y}(n) = \mathbf{H}\mathbf{x}(n) + \mathbf{u}(n) \tag{1}$$

where $\mathbf{x}(n)$, $\mathbf{y}(n)$ and $\mathbf{u}(n)$ are the $M_t \times 1$ input vector, $M_r \times 1$ output vector and $M_r \times 1$ noise vector, respectively, at the nth time instant. The noise vector $\mathbf{u}(n)$ is assumed to be a circularly symmetric complex Gaussian (CSCG) vector, i.e., each element of the vector is iid, complex Gaussian, and both real and imaginary parts of each element are zero mean and with identical variance. We also assume that the variance of each element of $\mathbf{u}(n)$ is N_0, and that the input signal $\mathbf{x}(n)$ is statistically independent of $\mathbf{u}(n)$.

Let $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}^{1/2}\mathbf{V}^H$, where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices, $\mathbf{\Sigma} = \text{diag}(\lambda_1, \cdots, \lambda_M)$, with $M = \min(M_t, M_r)$ and $\lambda_i \geq 0, \forall\, i$. Suppose $\mathbf{H}$ is available to both transmitter and receiver, and set $\mathbf{x}(n) = \mathbf{V}\mathbf{s}(n)$, and $\mathbf{z}(n) = \mathbf{U}^H\mathbf{y}(n)$, we have

$$\mathbf{z}(n) = \mathbf{\Sigma}^{1/2}\mathbf{s}(n) + \tilde{\mathbf{u}}(n) \tag{2}$$

where $\tilde{\mathbf{u}}(n) = \mathbf{U}^H\mathbf{u}(n)$. Thus the MIMO channel is decoupled into M parallel eigenmode sub-channels using joint transmit and receive eigen-beamforming.

2.2. Delay-limited capacity

Delay-limited capacity [1], which is less popular than ergodic capacity, defines the maximum rate of codes that can be reliably transmitted and received over all frames. Suppose the channel is regular, i.e., the fading distribution is continuous and $E[1/\bar{\lambda}] < \infty$, where $\bar{\lambda}$ is the geometric mean of the λ_i's, $\bar{\lambda} = (\prod_i \lambda_i)^{1/M}$. For a target information rate R, the optimal power control policy to achieve zero-outage probability for the channel realization $(\lambda_1, \cdots, \lambda_M)$, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M \geq 0$, is given by [1]

$$\gamma_i(\lambda_1, \cdots, \lambda_M) = N_0 \left(\frac{2^R}{\prod_{i=1}^{g} \lambda_i}\right)^{1/g} - \frac{N_0}{\lambda_i} \tag{3}$$

where g is the largest integer such that $\gamma_i(\lambda_1, \cdots, \lambda_M) > 0$ for $i = 1, \cdots, g$. The above optimal power control policy is termed generalized water-filling (GWF). The minimum average transmission power $\mathcal{P}(R)$ to achieve the information rate R with zero-outage probability over all channel realizations is determined by: $\mathcal{P}(R) = \sum_{i=1}^{M} E\left[\gamma_i(\lambda_1, \cdots, \lambda_M)\right]$.

For a given rate R, and $M = 2$, the power allocation policy is as follows: if $\lambda_1/\lambda_2 \geq 2^R$, then $g = 1$, $\gamma_1 = (2^R - 1)N_0/\lambda_1$ and $\gamma_2 = 0$; if $\lambda_1/\lambda_2 < 2^R$, then $g = 2$, and $\gamma_i = N_0(2^{R/2}/\sqrt{\lambda_1\lambda_2} - 1/\lambda_i)$ for $i = 1, 2$. Thus the resultant SNRs for each grouped channel are not identical, and specifically-designed codes are required to achieve the optimal delay limited capacity [1], [3], [6].

3. PROPOSED TRANSMISSION SCHEME

In this section, we propose a transmission scheme for MIMO channels to support mixed DC and NDC services. This scheme first utilizes SCG to generate multiple grouped channels, each with different fading statistics; then applies OCI based power control for the channels designed for DC transmissions, and constant powers for those channels dedicated for NDC transmissions.

3.1. SCG

SCG is proposed in [4] to approach the optimal delay limited capacity together with the use of multi-target overall channel inversion (MT-OCI) based power control. Denote $\tilde{\lambda}_1(k), \cdots, \tilde{\lambda}_M(k)$ as the channel gains of the sub-channels within the kth frame. Sorting the gains in a decreasing order yields $\lambda_1(k), \cdots, \lambda_M(k)$ where $\lambda_1(k) \geq \lambda_2(k) \geq \cdots \geq \lambda_M(k) \geq 0$. Then the channels gains with ith ordering number from all frames, $\lambda_i(1), \lambda_i(2), \cdots$, are treated as the ith grouped channel. By doing so, we are able to generate a maximum of M grouped channels.

3.2. Power Control

With SCG, each grouped channel has different fading statistics. If the grouped channel is invertible with finite power, then OCI based power control can be applied to it, and DC services can be transmitted through this channel. However, OCI cannot be applied to the un-invertible channels, such as those with Rayleigh fading. To support mixed DC and NDC services, the proposed scheme utilizes less fluctuated grouped channels for DC transmissions, and severely faded channels for NDC transmissions.

Suppose the first K strongest grouped channels are dedicated for DC services, and the others are for NDC services. We apply OCI based power control to the first K strongest grouped channels. Let $\gamma_i(k) = \frac{\tilde{\lambda}_i}{\lambda_i(k)}\bar{\gamma}_i$, for $i = 1, \cdots, K$, where $1/\tilde{\lambda}_i = E[1/\lambda_i(k)]$, the ith grouped channel is then observed as an AWGN channel with effective SNR $\frac{1}{N_0}\tilde{\lambda}_i\bar{\gamma}_i$. Under this power control policy, the delay limited capacity for these K grouped channel is

$$C_{DC}(\bar{\gamma}_1, \cdots, \bar{\gamma}_K) = \sum_{i=1}^{K} \log_2\left(1 + \frac{1}{N_0}\tilde{\lambda}_i\bar{\gamma}_i\right) \tag{4}$$

On the other hand, we apply constant power $\bar{\gamma}_i$ to the ith grouped channel, where $i = K+1, \cdots, M$ and consider the ergodic capacity for those channels:

$$C_{NDC} = \sum_{i=K+1}^{M} \mathcal{E}\left[\log_2\left(1 + \frac{\lambda_i\bar{\gamma}_i}{N_0}\right)\right] \tag{5}$$

Thus we are interested in the maximization of the mixed capacity, i.e., the sum of the delay limited capacity in (4) and the ergodic capacity in (5),

$$\begin{aligned} C &= \sum_{i=1}^{K} \log_2 \left(1 + \frac{1}{N_0}\tilde{\lambda}_i \bar{\gamma}_i\right) \\ &\quad + \sum_{i=K+1}^{M} \mathcal{E}\left[\log_2\left(1 + \frac{\lambda_i \bar{\gamma}_i}{N_0}\right)\right] \end{aligned} \tag{6}$$

with LTPC: $\sum_{i=1}^{M} \bar{\gamma}_i \le E_s$. The above problem is a convex optimization problem, thus there exist computationally efficient algorithms to obtain the optimal transmission powers.

4. SUBOPTIMAL SOLUTIONS

In this section, two suboptimal solutions for power allocation are proposed by modifying the objective function in (6).

4.1. Method 1

Noticing that from (5)

$$C_{NDC} \le \sum_{i=K+1}^{M} \log_2 \left(1 + \frac{\bar{\lambda}_i \bar{\gamma}_i}{N_0}\right) \tag{7}$$

where $\bar{\lambda}_i = \mathcal{E}[\lambda_i]$, for $i = K+1, \cdots, M$, we turn to to solve the following optimization problem:

$$\bar{C}_1 = \max_{\bar{\gamma}_i : \sum_{i=1}^{M} \bar{\gamma}_i \le E_s} \sum_{i=1}^{M} \log_2 \left(1 + \frac{1}{N_0}\tilde{\lambda}_i \bar{\gamma}_i\right) \tag{8}$$

here we emphasize that $1/\tilde{\lambda}_i = \mathcal{E}[1/\lambda_i]$ for $i = 1, \cdots, K$, and $\tilde{\lambda}_i = \mathcal{E}[\lambda_i]$ for $i = K+1, \cdots, M$. The optimal $\bar{\gamma}_i$'s to achieve $\bar{C}_1$ are given by $\bar{\gamma}_i = \left(\mu - \frac{N_0}{\tilde{\lambda}_i}\right)^+$, $i = 1, \cdots, M$, where μ is the water level with which the equality power constraint is satisfied. This procedure for $\bar{\gamma}_i$ calculation is referred to as statistical water-filling (SWF), as it depends on the the statistical values, $\tilde{\lambda}_i$'s, of the grouped channels, rather than the instantaneous gains.

OCI is a two-layer power controller. The first layer sets a SNR target, $\tilde{\lambda}_i \bar{\gamma}_i / N_0$, to the ith grouped channel, where $\bar{\gamma}_i$'s are calculated based on SWF. The second layer then inverts the grouped channel into an AWGN channel, with SNR target being set by the first layer.

The grouped channels with OCI are referred to as DC channels, and those with constant power control are called NDC channels. We remark here that while the NDC channel cannot be used to carry DC services, the DC channels can be used to transmit either DC data or NDC data.

The proposed scheme is flexible to support different number of DC channels. For example, if $K = 0$, no DC channels are assigned. If $K = M$, all grouped channels can be

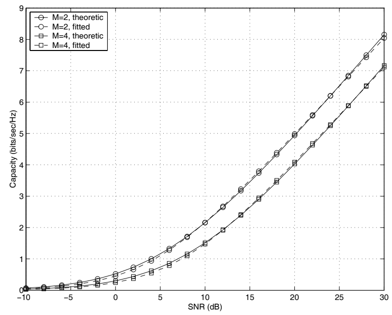

Fig. 1. Ergodic capacity for the Mth grouped channel.

used as DC channels, thus maximum delay limited capacity can be achieved for the proposed scheme, when jointly used with MT-OCI based power control [4].

4.2. Method 2

Method 2 is specifically designed for Rayleigh MIMO channels with square configuration and $K = M - 1$. Suppose the MIMO channel is zero-mean, unit variance CSCG (UV-CSCG). The PDF of the channel gain for the Mth grouped channel is [4]

$$f_M(\lambda) = M \exp(-M\lambda), \ \lambda \ge 0 \tag{9}$$

thus when a constant power $\bar{\gamma}_M$ is applied, the theoretic ergodic capacity for this channel is given by

$$C_M(\bar{\gamma}_M) = \log_2(e) \exp(MN_0/\bar{\gamma}_M)\mathrm{Ei}(MN_0/\bar{\gamma}_M) \tag{10}$$

where e is the natural log base, $\mathrm{Ei}(x) = \int_x^\infty e^{-t}/t dt$. This capacity can be approximated by

$$C_M(\bar{\gamma}_M) = \alpha \log_2 \left(1 + \frac{\tilde{\lambda}_M \bar{\gamma}_M}{N_0}\right) \tag{11}$$

where $\tilde{\lambda}_M = 0.8/M$ for $M \ge 2$ and $\alpha = 0.93$. From the the comparisons shown in Fig.1 for $M = 2, 4$, we see the approximation of (11) to (10) is quite close.

Then the optimization problem becomes

$$\begin{aligned} \bar{C}_2 &= \max_{\bar{\gamma}_i : \sum_{i=1}^{M} \bar{\gamma}_i \le E_s} \left[\sum_{i=1}^{M-1} \log_2 \left(1 + \frac{1}{N_0}\tilde{\lambda}_i \bar{\gamma}_i\right)\right. \\ &\qquad \left. + \alpha \log_2 \left(1 + \frac{\tilde{\lambda}_M \bar{\gamma}_M}{N_0}\right)\right] \end{aligned} \tag{12}$$

The optimal $\bar{\gamma}_i$'s are given by: $\bar{\gamma}_i = \left(\mu - \frac{N_0}{\tilde{\lambda}_i}\right)^+$, $i = 1, \cdots, M-1$; and $\bar{\gamma}_M = \left(\alpha\mu - \frac{N_0}{\tilde{\lambda}_M}\right)^+$, where μ is determined by the equality power constraint. For both methods, once the transmit powers are derived, the sum capacity can be calculated from (6).

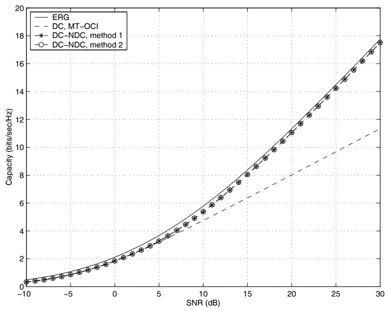

Fig. 2. Capacity comparisons for 2×2 MIMO channels.

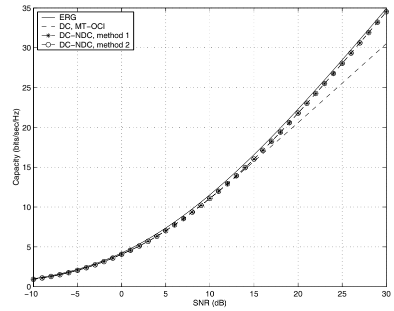

Fig. 3. Capacity comparisons for 4×4 MIMO channels.

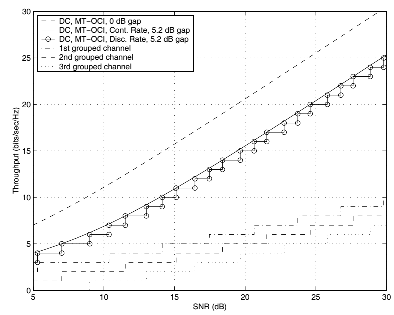

Fig. 4. Bit loading for 4×4 MIMO channels: $\Gamma = 5.2$ dB.

5. PERFORMANCE EVALUATIONS

We consider MIMO channels with square configurations. For comparison, we also simulate the optimal ergodic capacity, which serves as the upper bound for the related capacity results. Figs. 2 and 3 show the capacity comparisons for 2×2 and 4×4 UV-CSCG MIMO channels, respectively. In these figures, "ERG" denotes the optimal ergodic capacity. "DC, MT-OCI" represents the delay limited capacity achieved by MT-OCI based power control when all grouped channels are designed as DC channels $(K = M)$. "DC-NDC" denotes the mixed capacity achieved when the weakest channel is designed as NDC channel, while the others are DC channels $(K = M - 1)$.

It is seen that "DC, MT-OCI" has certain capacity loss from the optimal ergodic capacity for high SNRs. This is because for UV-CSCG square matrix channels, the weakest channel is un-invertible with finite power, thus it can never be used to carry DC service. However, this sub-channel can be used to carry NDC service, and it is evident that the "DC-NDC" suboptimal methods almost achieve the optimal ergodic capacity in all SNRs, and that both Method 1 and Method 2 achieve similar mixed capacity.

Finally, we evaluate the performance of practical bit-loading algorithms applied to our proposed scheme. We choose $K = M$ (all grouped channels are allocated as DC channels). The Levin-Campello algorithm [2], which is the optimal discrete rate allocation algorithm, is evaluated. A constant gap Γ of 5.2 dB is applied to all grouped channels, which corresponds to approximately 10^{-3} uncoded BER for MQAM modulations. The achieved throughput and loaded bits for each grouped channel are shown in Fig.4, where the information granularity is 1 bit. It is evident that the achieved throughput is quite close to that achieved with continuous rate modulations.

6. CONCLUSIONS

A transmission scheme based on sub-channel grouping and transmit power control is proposed for MIMO channels to support mixed DC and NDC services. Optimal and sub-optimal power allocation strategies are proposed. Computer simulations have shown that the achieved capacity for the proposed system is close to the optimal ergodic capacity and has considerable gain over the delay limited capacity achieved when all grouped channels are designed as DC channels, for UV-CSCG square matrix MIMO channels.

7. REFERENCES

[1] E. Biglieri, G. Caire and G. Taricco, "Limiting performance of block-fading channels with multiple antennas," *IEEE Trans. on IT,* vol.47, No.4, pp.1273-1289, May 2000.

[2] J.M. Cioffi, "Digital Communications," Course Reader, EE379, Stanford University, 2003.

[3] R. Knopp and G. Caire, "Power control and beamforming for systems with multiple transmit and receive antennas," *IEEE Trans. on Wireless Comm.,* vol.1, No.4, pp.638-648, October 2002.

[4] Y.-C. Liang, R. Zhang and J.M. Cioffi, "Sub-channel grouping and statistical water-filling for vector block-fading channels," submitted for publication, October 2003.

[5] I.E. Telatar, "Capacity of multi-antenna Gaussian channels," *Bell Labs Technical Memorandum*, 1995.

[6] R.D. Wesel and J.M. Cioffi, "Fundamentals of coding for broadcast OFDM," *Proc. of Asilomar'1996*, pp.2-6, 1996.