BICM FOR MIMO SYSTEMS USING LOW-DENSITY GENERATOR MATRIX (LDGM) CODES

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ABSTRACT

We propose a Bit Interleaved Coded Modulation with Iterative Decoding (BICM-ID) scheme for MIMO systems using systematic linear codes with Low-Density Generator Matrix (LDGM). We compare the performance of the proposed scheme with the (constrained input) channel capacity and show its ability to perform close to the theoretical limit. The main advantage of the proposed structure relies on its low encoding and decoding complexity.

1. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) systems resulting from the use of several antennas at both transmission and reception have concentrated much attention in the last years [1]. Different signaling techniques have been specifically proposed to suit the characteristics of the MIMO channel. Most of them focus on allowing very simple symbol detection at reception [2]. However, it is necessary to incorporate an outer code in order to obtain a performance as close as possible to the channel capacity. One way is to jointly design the coding and the signaling schemes, in an analogous way as in Trellis Coded Modulation (TCM) in single-antenna systems. Space-time trellis codes [2] are an instance of such approach. The drawback of these schemes is that they are designed by hand for each particular case and their decoding complexity is exponential in the number of transmitting antennas.

In this paper we explore an alternative coding and signaling method for MIMO channels that is based on Zehavi's concept of Bit-Interleaved Coded Modulation (BICM) [3]. BICM consists of a channel code followed by a bit-interleaver and a symbol-mapper. Contrarily to TCM, BICM schemes cannot be designed by optimizing the coding/modulation system as a whole, since the codewords resulting from the bit-interleaving process are very long. On the other hand, the use of standard convolutional codes within a BICM scheme showed a reasonable performance over fading channels. Nevertheless, for the case of AWGN channels (where no diversity gain can be obtained from the interleaving process) the performance of Zehavi's scheme was worse than that of TCM.

This limitation of BICM can be overcome by using iterative decoding techniques similar to those employed for Turbo codes [4]. Since the bit-to-symbol mapping in BICM can be seen as a rate-1, binary-input, complex output inner code, an iterative decoding process can be applied in BICM schemes by separately computing the *a posteriori* probabilities of the coded symbols at both the demapper and the channel decoder. For AWGN channels, iterative decoding of BICM schemes with convolutional codes [5] results in a performance loss with respect to Turbo-TCM systems, although their lower decoding complexity and simpler structure at transmission[1] might make worthy this performance loss. For Rayleigh fading channels, the performance of BICM-ID is very close to that of Turbo-TCM systems.

The previous discussion indicates that the application of the concept of BICM with iterative decoding to MIMO systems is an attractive idea. This was first proposed using convolutional codes in [6] under the term Space-Time BICM (STBICM). The main impairment of BICM-ID for MIMO systems is the exponential complexity in the number of transmitting antennas associated with the demapping processs. This can be overcome by means of suboptimum, very powerful demapping methods such as that presented in [7]. There, the authors showed the ability of a BICM-ID MIMO system, using Turbo codes as channel codes, to perform very close to the capacity limit for a wide range of signaling techniques and number of transmit and receiving antennas.

In this paper we consider BICM-ID for MIMO systems using a concatenation of two linear block codes with Low-Density Generator Matrix (LDGM codes) [8]. The idea of using LDGM codes for bandwidth-efficient modulation was explored by Cheng and McEliece in [9] but only for high-rate codes which do not present error floors and therefore do not need the concatenation. In [10] we proposed the use of LDGM codes for BICM over AWGN channels in a general perspective, allowing lower code rates by eliminating their associated error floors by means of the concatenated scheme. Here we extend that BICM-ID scheme to MIMO systems. The main advantange of LDGM with respect to Turbo codes (as those used in [7]) is that they have much less encoding and decoding complexity. The results presented here show that BICM-ID with concatenated LDGM codes performs close to the theoretical limit.

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[1]Note that coding is performed just by means of a standard code, optimized for the binary case

2. LDGM CODES IN MIMO SYSTEMS

We consider the case of systematic LDGM codes, which are linear codes with a sparse generator matrix, $\mathbf{G}_{K\times N} = [\mathbf{I}_K \ \mathbf{P}_{K\times L}]$, where $N = K + L$ and $\mathbf{P}_{K\times L}$ is a sparse matrix. Given an input codeword $\mathbf{u} = (u_1, u_2, ..., u_K)$, the parity bits $\mathbf{c} = (c_1, c_2, ..., c_L)$ are obtained as $\mathbf{c} = \mathbf{uP}$. Notice that LDGM codes are particular cases of standard Low-Density Parity Check (LDPC) codes, since it is obvious that their parity check matrices are also sparse. As indicated before, the advantage over standard LDPC codes is that their encoding complexity is very low.

Decoding of LDGM codes is performed by representing them graphically as a Bayesian Network, and applying the Pearl's Belief Propagation (BP) algorithm [11] over the resulting graph. Specifically, each node in the graph represents a random variable which describes either a systematic or a parity bit. The connections among the nodes are specified by the generator matrix $\mathbf{G}$. Since a connection is only possible between a systematic bit node and a parity bit node, the graph is bipartite. We use the notation (L, X, K) LDGM code to indicate that the number of parity and input bits is, respectively, L and K, and the degree of the systematic bit nodes (the number of parity bits that depend on a given systematic bit node) is X.

2.1. Eliminating the error floor: concatenated LDGM codes

Except for the case of high code rates [9], LDGM codes present error floors when used as standard channel codes. However, these error floors can be substantially reduced (and practically eliminated) by using very simple concatenated schemes [8]. The basic idea is to use an additional, serially concatenated, outer code that will allow to correct the few errors remaining after the decoding of the inner code. Note that any kind of code could be used as outer code, but in this paper we assume that it is another systematic LDGM code. The rate of the outer code can be extremely high, since the number of errors to be corrected is very low. The output of the outer code $(u_1, ..., u_K, c^o_1, ..., c^o_{L_o})$ is fed into the inner code, resulting in the sequence to be mapped

$$(c_1, c_2, ..., c_{N_o}) = (u_1, ..., u_K, c^o_1, ..., c^o_{L_o}, c^i_1, ..., c^i_{L_i})$$

We will denote as $[(L_o, X_o, K_o), (L_i, X_i, K_i)]$ the whole code resulting from the concatenation of an outer (L_o, X_o, K_o) LDGM code and an inner (L_i, X_i, K_i) LDGM code.

In order to keep the overall rate fixed, we have to adjust the rate between the inner and the outer code. We consider that no puncturing is performed on the systematic bits, so we transmit K_o systematic bits and $L_o + L_i$ parity bits. The overall rate is given by $R_c = K_o/(K_o + L_o + L_i)$. We can build up the inner code by means of slightly modifications in the number of systematic and parity bits of a simple (non-concatenated) LDGM code which presents good properties of threshold and error floor. In other words, if we know that a given (L, X, K) simple LDGM code presents good performance, then we can use as inner code in the concatenated scheme a $(L - L_o, X, K + L_o)$ simple LDGM code, where L_0 is small, and an $(L_o, X_o, K_o = K)$ simple LDGM code as outer code.

2.2. Bit-to-symbol mapping

Given a block of $N_o = K_o + L_o + L_i$ bits at the output of the coding stage, we consider that the assignment of bits to symbols is performed pseudorandomly, by means of an interleaver. Let us denote by $(c'_1, c'_2, ..., c'_{N_o}) = \pi(c_1, c_2, ..., c_{N_o})$ the coded bit sequence after the interleaving process. The mapping to the transmitted symbol $\mathbf{x}(i) = [x_1(i), x_2(i), ..., x_M(i)]^T$ is then performed according to a given mapping rule

$$\mathbf{x}(i) = \text{map}(\mathbf{c}'(i)) \quad i = 1, 2, ..., N_o/(MM_c)$$

where

$$\mathbf{c}'(i) = (c'_{MM_c(i-1)+1}, c'_{MMc(i-1)+2}, ..., c'_{MMc(i-1)+MMc})$$

and M is the number of transmitting antennas and M_c denotes the number of bits carried by each symbol for the employed modulation. For instance, $M_c = 2, 3, 4$ bits for QPSK, 8PSK and 16QAM, respectively. In particular, for each subblock of MM_c bits we can perform the mapping by assigning the first M_c bits to the first transmitting antenna, the following M_c bits to the second, and so on. Then, for each transmitting antenna we can use a usual mapping, such as Gray, Set Partitioning or Semi-Set Partitioning.

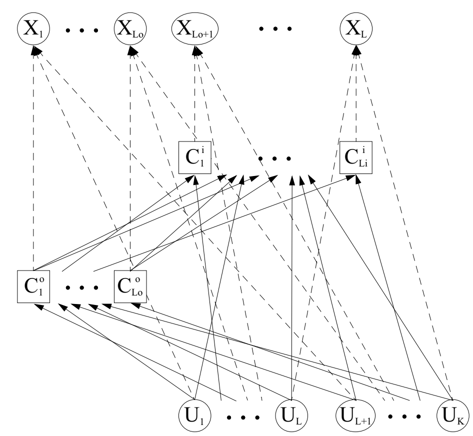

Fig. 1. Bayesian network representing the concatenated coding and modulation scheme. $\{U_i\}$ represents the information bits, $\{C^o_i\}$ the outer parity bits, $\{C^i_i\}$ the inner parity bits and $\{X_i\}$ the symbols transmitted through the channel.

The received signals can be written as

$$\mathbf{y}(i) = \mathbf{H}(i)\mathbf{x}(i) + \eta(i) \quad i = 1, 2, ..., N_o/(MM_c) \qquad (1)$$

where $\mathbf{H}(i)$ is the $N \times M$ matrix which models the channel. We assume that each complex coefficient of the channel matrix is circularly-invariant, Gaussian distributed (thus being its modulus Rayleigh distributed) with unit variance. The channel coefficients are independent in both the spatial and the temporal dimensions. The components of the noise vector $\eta(i)$ are both spatial and temporally uncorrelated, circularly-invariant, Gaussian distributed with variance $N_0/2$ per complex dimension.

As shown in Figure 1, the overall concatenated coding and modulation scheme can be represented using a Bayesian network. Each node represents a random variable and the connection between two nodes means that there is a statistical dependence between them. In our case, the dependences are deterministic, because they are either parity constraints or refer to the mapping between bits and symbols. Both cases are easily handled by defining the corresponding probability mass functions as indicator functions.

3. ITERATIVE DETECTION AND DECODING USING PEARL'S BELIEF PROPAGATION ALGORITHM

In this section we describe how to apply the Pearl's Belief Propagation (BP) algorithm [11] over the graph representing the BICM scheme with the concatenation of two LDGM codes. The goal of the decoding process is to compute the so-called *belief* for each information bit node, denoted by $BEL(u_i)$. If the Bayesian network had no cycles[2], $BEL(u_i)$ would equal $P[u_i|\mathbf{e}]$, the *a posteriori* probability of the bit node U_i being equal to u_i given the available *evidence* (i.e., the observations and the probabilistic constraints contained in the Bayesian network). Once $BEL(u_i)$ is computed, a decision is made according to the usual rule

$$\hat{u}_i = \begin{cases} 0 & \text{if} \quad BEL(u_i = 0) > BEL(u_i = 1) \\ 1 & \text{if} \quad BEL(u_i = 0) \leq BEL(u_i = 1). \end{cases} \tag{2}$$

In general, for a given node U, the belief of $U = u$ is computed by

$$BEL(u) \overset{\alpha}{=} \lambda(u)\pi(u), \tag{3}$$

where $\overset{\alpha}{=}$ stands for equality except for a multiplicative constant and $\lambda(u) = P[\mathbf{e}_U^-|u]$ and $\pi(u) = P[u|\mathbf{e}_U^+]$ are, respectively, the *likelihood* of $U = u$ given the evidence contained in the tree below the node U, and the *a posteriori* probability of $U = u$ given the evidence contained in the network above U. We will denote by Y_i, $i = 1, ..., m$, and W_j, $j = 1, ..., n$, the children and the parent nodes of the node U, respectively. Given the messages received from its children, $\lambda_{Y_i}(u)$, $i = 1, ..., m$, and from its parents, $\pi_U(w_i)$, $j = 1, ..., n$, $\lambda(u)$ and $\pi(u)$ can be computed as

$$\lambda(u) = \prod_{i=1}^{m} \lambda_{Y_i}(u) \tag{4}$$

$$\pi(u) = \sum_{\mathbf{w}} P[u|\mathbf{w}] \prod_{j=1}^{n} \pi_U(w_j), \tag{5}$$

where $\mathbf{w} = (w_1, ..., w_n)$ denote the values taken by the parents of U.

Node U delivers messages to both its children and parents. The message $\lambda_U(w_j) = P[\mathbf{e}_U \backslash e_U^{W_j}|w_j]$ delivered by the node U to its parent W_j is the likelihood of $W_j = w_j$ given all the evidence observed in the node U except that coming from W_j, and is computed as

$$\lambda_U(w_j) \overset{\alpha}{=} \sum_u \lambda(u) \sum_{\mathbf{w}\backslash w_j} P[u|\mathbf{w}] \prod_{k \neq j} \pi_U(w_k), \tag{6}$$

where $\mathbf{w}\backslash w_j$ denotes the vector $\mathbf{w}$ without its j-th component. Similarly, the message delivered by U to its child Y_i, $\pi_{Y_i}(u) = P[Y_i|u_i, \mathbf{e}_U \backslash e_U^{Y_i}]$, is the *a posteriori* probability of Y_i given $U_i = u_i$ and all the evidence observed in the node U except that coming from Y_i, and is computed as

$$\pi_{Y_i}(u) \overset{\alpha}{=} \frac{\lambda(u)}{\lambda_{Y_i}(u)} \pi(u). \tag{7}$$

The particularization of the previous equations to the proposed scheme can be found in [10]. In particular, the computation carried out in the symbol nodes is

$$BEL(X_i) \overset{\alpha}{=} \lambda(X_i)\pi(X_i), \tag{8}$$

where $\lambda(x_i) = P[Y_i|x_i] \propto \exp(\|\mathbf{y}(i) - \mathbf{H}(i)\mathbf{x}(i)\|^2/N_0)$ is the likelihood of $X_i = x_i$ given the noisy observation from the MIMO channel, and

$$\pi(X_i) = \sum_{\mathbf{w}:\text{map}(\mathbf{w})=x_i} \prod_{j=1}^{n_s} \pi_{X_i}(w_j), \tag{9}$$

where w_j is the value taken by either a bit node, an outer parity bit node or an inner parity bit node. The messages delivered by a symbol node to its parents are

$$\lambda_{X_i}(w_j) \overset{\alpha}{=} \sum_{x_i} \lambda(x_i) \sum_{\mathbf{w}\backslash w_j:\text{map}(\mathbf{w})=x_i} \prod_{k \neq j} \pi_{X_i}(w_k). \tag{10}$$

Different schedules for message updating can be utilized. However, we have corroborated that most of them achieve a similar performance. The results presented in next section consider successive updating of inner parity, outer parity and systematic bit nodes.

4. SIMULATION RESULTS

Figure 2 shows the performance of the proposed BICM-ID scheme for a two-transmit, two-receive antenna (2x2) MIMO system. We consider a Rayleigh fading channel (see Eq. (1)) and perfect CSI at reception. The code used is a $[(500, 4, 15000), (14500, 6, 15500)]$ concatenated LDGM code of rate 1/2, thus being the block length $N_o = 30000$ bits, where $K_o = 15000$ bits are systematic and $L_o + L_i = 500 + 14500 = 15000$ bits are parity. A pseudorandom interleaver with length equal to the block length is used between the output of the concatenated LDGM and the bit-to-symbol mapping. For each subblock of MM_c bits we assign the first M_c bits to the first transmitting antenna, the following M_c bits to the second, and so on. Then, for each transmitting antenna we can use Gray labeling, which provides a sufficiently high "push" in the first iteration[3] to lead the LDGM decoder to the convergence region. At reception the MIMO channel is assumed to be known. We perform one iteration over the symbol nodes (which form the demapping module in our scheme) followed by ten iterations over the rest of the graph, which represents the concatenated LDGM code. The order in which the nodes of the concatenated LDGM code are activated is: first, the inner parity bit nodes, then the outer parity, and finally the systematic bit nodes. We allow a maximum of ten iterations over the symbol nodes, stopping the algorithm when the same decoded sequence is produced along three successive iterations. We also plot in Figure 2 the necessary E_b/N_0 to attain a (constrained input) channel capacity [7] in bit/s/Hz equal to the information rate corresponding to the use of QPSK, 8PSK and 16QAM, i.e., 2,3 and 4 bit/s/Hz respectively. It is apparent from Figure 2 that the proposed BICM-ID with concatenated LDGM codes performs close to the capacity limit for the various modulation techniques considered. The gap with respect to the capacity limit is around 1.2, 1.6 dB and 1.9 dB for QPSK, 8PSK and 16QAM, respectively.

[2] In our case, as it occurs with Turbo and LDPC coding schemes in general, the network has cycles, but, as we will see in Section 4, the resulting performance is still reasonable.

[3] This push can be measured in terms of the mutual information between the *a priori* L-values about the coded bits and the *a posteriori* extrinsic L-values at the output of the demapping module, as proposed by ten Brink [12]

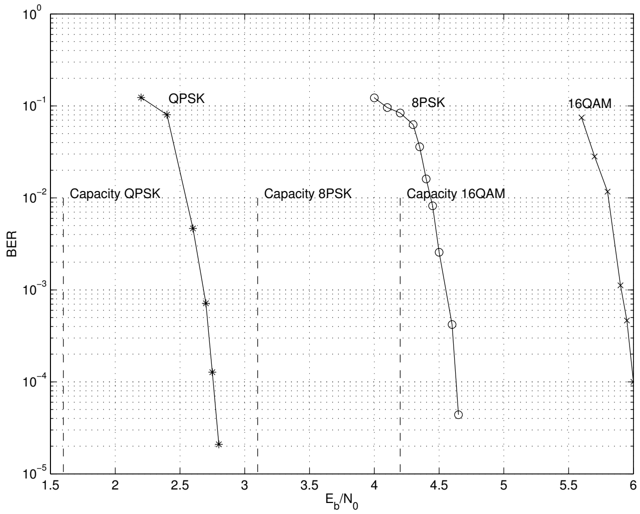

Fig. 2. Bit Error Rate of BICM using LDGM codes over a 2x2 MIMO channel

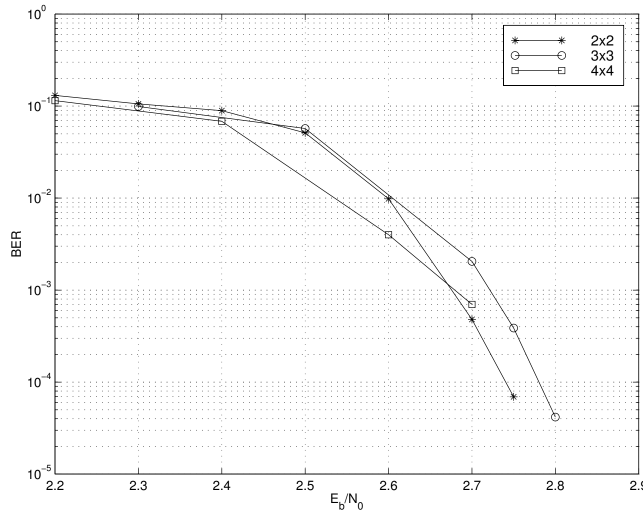

Fig. 3. Bit Error Rate of BICM using LDGM codes for QPSK and several (2x2,3x3 and 4x4) MIMO channels.

We also study the effect on performance of increasing the number of transmit and receiving antennas for a given modulation, namely, QPSK. Figure 3 plots the BER vs E_b/N_0 obtained using QPSK for several MIMO configurations: 2×2, 3×3 and 4×4. The rest of the parameters are the same as before. The difference in performance is around 0.05 dB and does not allow to conclude a performance loss when increasing the number of antennas. This results in a throughput gain, since we can increase the number of antennas, which increases the capacity of the MIMO channel allowing us to use a higher rate, without an increase in the required E_b/N_0 for sucessful decoding at reception.

Note that the complexity of the demapping process (10) is exponential in both the number of transmitting antennas, M, and the modulation order, M_c. As previously noted, a way to overcome this impairment is to use suboptimum demapping schemes.

5. CONCLUSIONS

This paper presents a novel scheme suitable for MIMO channels based on the use of Bit Interleaved Coded Modulation and codes with low-density generator matrix. Under the assumption of fast flat uncorrelated Rayleigh fading, the proposed approach performs close to the capacity limit of the MIMO channel. The main advantanges of the proposed approach are its very low encoding/decoding complexity and that it does not require optimization of the code parameters.

6. ACKNOWLEDGEMENT

The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U.S. Government.

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