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# EFFICIENT NEAR-OPTIMUM SOFTBIT SOURCE DECODING FOR SOURCES WITH INTER- AND INTRA-FRAME REDUNDANCY

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**ABSTRACT**

The source codec parameters determined by modern speech, audio, or video encoders are, especially at high compression rates, highly sensitive to transmission errors and they usually exhibit natural residual redundancy due to delay and complexity limitations in the encoding process. Error concealment techniques like *softbit source decoding* (SBSD) utilize the residual redundancy as *a priori* knowledge for parameter estimation. Therewith, the annoying artifacts of transmission errors can considerably be concealed. But, the optimal utilization of all terms of residual redundancy requires large computational complexity.

In this paper, we derive a new efficient, nearly-optimal version to SBSD. Significant complexity savings become possible by exploiting the intra-frame and inter-frame redundancy separately using two nested *forward-backward* algorithms. Compared to the optimal joint evaluation, the performance loss is negligibly small.

## 1. TRANSMISSION MODEL

At time instant $\tau$ a source encoder extracts a frame $\underline{u}_\tau$ of $M$ codec parameters $u_{\kappa,\tau}$ from a short time segment of the original speech, audio, or video signal (see Fig. 1). Within this frame $\underline{u}_\tau = (u_{1,\tau}, \dots u_{M,\tau})$, each parameter is individually labeled by the position index $\kappa = 1, \dots M$. Each value $u_{\kappa,\tau}$ is quantized to $\bar{u}_{\kappa,\tau} \in \mathbb{U}_\kappa$ and mapped into a unique bit pattern $\mathbf{x}_{\kappa,\tau}$. Corresponding to a complete quantizer codebook $\mathbb{U}_\kappa$ the entire alphabet of valid bit patterns is given by $\mathbb{X}_\kappa$. The complete frame of bit patterns specified at time $\tau$ is denoted by $\underline{\mathbf{x}}_\tau = (\mathbf{x}_{1,\tau}, \dots \mathbf{x}_{M,\tau})$.

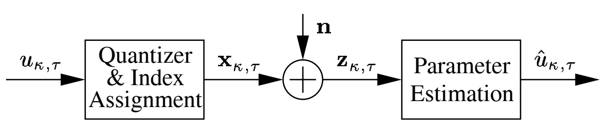


Figure 1: Transmission model

After BPSK (binary phase shift keying) transmission of the single bits of $\underline{\mathbf{x}}_\tau$ over a disturbed channel with additive noise $\mathbf{n}$ the sequence $\underline{\mathbf{z}}_\tau$ of real valued symbols is received. The term $\mathbf{z}_{\kappa,\tau}$ at time $\tau$ and position $\kappa$ denotes the received word for the bit pattern $\mathbf{x}_{\kappa,\tau}$. The channel may also comprise channel coding etc.

The aim of parameter estimation is to appropriately reconstruct the individual $u_{\kappa,\tau}$. For this purpose, the MMSE-optimal estimates $\hat{u}_{\kappa,\tau}$ (minimum mean squared error) are

$$\hat{u}_{\kappa,\tau} = \sum_{\bar{u}_{\kappa,\tau} \in \mathbb{U}_\kappa} \bar{u}_{\kappa,\tau} \cdot P(\mathbf{x}_{\kappa,\tau} \,|\, \mathbf{z}_{\text{SBSD}}) \quad . \tag{1}$$

The estimate $\hat{u}_{\kappa,\tau}$ is a weighted sum over all quantizer reproduction levels $\bar{u}_{\kappa,\tau}$ of the codebook $\mathbb{U}_\kappa$. The weights are given by the *a posteriori* probabilities $P(\mathbf{x}_{\kappa,\tau} \,|\, \mathbf{z}_{\text{SBSD}})$ for the bit patterns $\mathbf{x}_{\kappa,\tau}$ corresponding to $\bar{u}_{\kappa,\tau}$. At first, the set $\mathbf{z}_{\text{SBSD}}$ of observations shall comprise all past ($t = 1, \dots \tau - 1$), present ($t = \tau$), and possibly some future ($t = \tau + 1, \dots \Lambda$) received sequences $\underline{\mathbf{z}}_t$ up to a maximum look-ahead of $\Lambda$. In Section 2, where the different versions to *softbit source decoding* are reviewed, the set $\mathbf{z}_{\text{SBSD}}$ will be reduced to all those received words $\mathbf{z}_{k,t}$ ($k = 1, \dots M$) which effectively contribute to the reliability information about $\mathbf{x}_{\kappa,\tau}$.

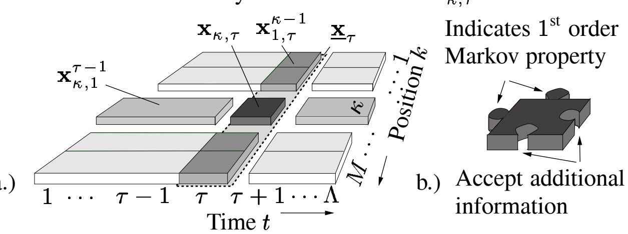


Figure 2: a.) Illustration of *time-position plane* of bit patterns $\mathbf{x}_{\kappa,\tau}$
b.) “Jigsaw puzzle” representing mutual dependencies

Fig. 2 a.) illustrates the *time-position plane* of bit patterns $\mathbf{x}_{k,t}$. The specific bit pattern $\mathbf{x}_{\kappa,\tau}$ under consideration (center element) is part of the frame $\underline{\mathbf{x}}_\tau$ at time $\tau$. Further on, we use a compact notation for series of bit patterns, e.g., $\mathbf{x}_{\kappa,1}^{\tau-1}$ denotes the *time*-related series $\mathbf{x}_{\kappa,1}, \dots \mathbf{x}_{\kappa,\tau-1}$ at a fixed position $\kappa$. Accordingly, we use $\mathbf{x}_{1,\tau}^{\kappa-1}$ for the *position*-related series $\mathbf{x}_{1,\tau}, \dots \mathbf{x}_{\kappa-1,\tau}$ at a fixed time instant $\tau$. Both short hand notations (the *time*- and *position*-related one) can always be distinguished by the upper subscript. If a *time*-related index $\tau$ or $\Lambda$ appears as upper subscript, the compact notation regards a *time* series. If a *position*-related index $\kappa$ or $M$ is used, this term abbreviates a *position*-related series.

## 2. REVIEW OF THE OPTIMAL VERSIONS TO SBSD

Error concealment by *softbit source decoding* (SBSD) [1–3] utilizes residual redundancy of the source codec parameters $u_{\kappa,\tau}$ as *a priori* knowledge. The residual redundancy may occur as a non-uniform parameter distribution or as (non-)linear mutual dependency of neighboring/consecutive codec parameters. Mutual dependencies of neighboring, $u_{\kappa,\tau}$, $u_{\kappa-1,\tau}$ of a single frame $\underline{\bar{u}}_\tau$ are called *intra-frame* redundancy and these of consecutive realizations, $u_{\kappa,\tau}$, $u_{\kappa,\tau-1}$ of a single codec parameter *inter-frame* redundancy. Applying reasonable assumptions about both terms of redundancy, like a 1st order Markov property, allows to derive efficient rules for computing $P(\mathbf{x}_{\kappa,\tau} \,|\, \mathbf{z}_{\text{SBSD}})$ [1–3]. *A priori* knowledge like $P(\mathbf{x}_{\kappa,\tau})$ (non-uniform distribution), $P(\mathbf{x}_{\kappa,\tau} \,|\, \mathbf{x}_{\kappa,\tau-1})$ (inter-frame redundancy), and $P(\mathbf{x}_{\kappa,\tau} \,|\, \mathbf{x}_{\kappa,\tau-1}, \mathbf{x}_{\kappa-1,\tau})$ (inter- and intra-frame redundancy) can be measured once in advance for a representative signal data base. Combining such histograms with channel-related reliability information provides $P(\mathbf{x}_{\kappa,\tau} \,|\, \mathbf{z}_{\text{SBSD}})$.

Notice, in the subsequent brief review of the optimal versions to SBSD we explain just the concept how to determine the *a posteriori* probabilities $P(\mathbf{x}_{\kappa,\tau} \,|\, \mathbf{z}_{\text{SBSD}})$. The respective detailed rules can be found, e.g., in [1–3]. Some illustrations shall help to clarify notation and to point out the key properties of the different versions.

Using a "jigsaw puzzle" representation, every puzzle element represents either a single bit pattern $\mathbf{x}_{\kappa,\tau}$ or a set of patterns. The irregular shape of the single puzzle elements is constrained to the mutual dependencies between the source codec parameters (see Fig. 2 b.)). The larger the number of interlocking pieces, the higher the accuracy of *a posteriori* probabilities $P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{z}_{\text{SBSD}})$ can be.

### 2.1. Non-Uniform Parameter Distribution – ND Estimator

The ND estimator exploits *a priori* information in terms of a non-uniform parameter distribution (ND). Any additionally given *inter*-frame or *intra*-frame redundancy is ignored. Thus, the determination of *a posteriori* probabilities according to [1–3]

$$P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{z}_{\text{SBSD}}) = C\cdot p(\mathbf{x}_{\kappa,\tau},\mathbf{z}_{\kappa,\tau}) \qquad (2)$$

is restricted to the evaluation of the received word $\mathbf{z}_{\text{SBSD}} = \mathbf{z}_{\kappa,\tau}$ for the desired bit pattern $\mathbf{x}_{\kappa,\tau}$ only. The constant factor $C$ serves for normalization so that the *total probability theorem* $\sum_{\mathbf{x}_{\kappa,\tau}\in\mathbb{X}_\kappa} P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{z}_{\text{SBSD}}) = 1$ is fulfilled. The joint probability density function (PDF) $p(\mathbf{x}_{\kappa,\tau},\mathbf{z}_{\kappa,\tau}) = p(\mathbf{z}_{\kappa,\tau}\,|\,\mathbf{x}_{\kappa,\tau})\cdot P(\mathbf{x}_{\kappa,\tau})$ comprises channel-related knowledge $p(\mathbf{z}_{\kappa,\tau}\,|\,\mathbf{x}_{\kappa,\tau})$ and the non-uniform parameter distribution $P(\mathbf{x}_{\kappa,\tau})$.

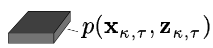


Figure 3: ND estimator exploits $\mathbf{z}_{\text{SBSD}} = \mathbf{z}_{\kappa,\tau}$ only

Figure 3 illustrates the effectively exploited subset $\mathbf{z}_{\text{SBSD}}$ of the *time-position plane* according to Eq. (2). Notice, only one received word $\mathbf{z}_{\text{SBSD}} = \mathbf{z}_{\kappa,\tau}$ is exploited and no puzzle pieces interlock.

### 2.2. Inter-Frame Redundancy – INTER Estimator

The INTER estimator utilizes *a priori* information in terms of a non-uniform parameter distribution and *inter*-frame redundancy (INTER). With respect to a 1st order Markov process and neglecting future received words $\mathbf{z}_{\kappa,\tau+1}^{\Lambda}$, the determination of *a posteriori* probabilities reads [1–3]

$$P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{z}_{\text{SBSD}}) = C\cdot\sum_{\mathbf{x}_{\kappa,\tau-1}} p(\mathbf{x}_{\kappa,\tau},\mathbf{z}_{\kappa,\tau}\,|\,\mathbf{x}_{\kappa,\tau-1})\cdot\alpha_{\tau-1}(\mathbf{x}_{\kappa,\tau-1}). \qquad (3)$$

In Eq. (3), the marginal distribution has to be evaluated by summation over all $\mathbf{x}_{\kappa,\tau-1}\in\mathbb{X}_\kappa$. The parameter *a posteriori* probabilities $P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{z}_{\text{SBSD}})$ exploit the present and all past received words $\mathbf{z}_{\text{SBSD}} = \mathbf{z}_{\kappa,1}^{\tau}$ of the position $\kappa$ under consideration. Received words of adjacent positions $\kappa$ remain unexploited. Past and present received words $\mathbf{z}_{\kappa,1}^{\tau-1}$ resp. $\mathbf{z}_{\kappa,\tau}$ are evaluated separately using (3) in terms of the *forward* recursion $\alpha_{\tau-1}(\mathbf{x}_{\kappa,\tau-1})$ and $p(\mathbf{x}_{\kappa,\tau},\mathbf{z}_{\kappa,\tau}\,|\,\mathbf{x}_{\kappa,\tau-1}) = p(\mathbf{z}_{\kappa,\tau}\,|\,\mathbf{x}_{\kappa,\tau})\cdot P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{x}_{\kappa,\tau-1})$.

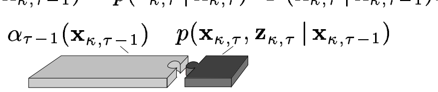


Figure 4: INTER estimator exploits entire history $\mathbf{z}_{\text{SBSD}} = \mathbf{z}_{\kappa,1}^{\tau}$

The "jigsaw puzzle" representation shown in Fig. 4 illustrates that the bit pattern $\mathbf{x}_{\kappa,\tau-1}$ marks the interlocking element of (3).

### 2.3. Inter- and Intra-Frame Redundancy – OPT Estimator

The *optimal* parameter estimator (OPT) jointly exploits *inter*- and *intra*-frame redundancy of the source codec parameters [2]. Assuming a 1st order Markov property for both terms of redundancy, $P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{x}_{\kappa,\tau-1},\mathbf{x}_{\kappa-1,\tau})$, S. Heinen [2] derived a complex *forward-backward* algorithm for the determination of *a posteriori* probabilities $P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{z}_{\text{SBSD}})$. The *forward* $\alpha_\kappa(\mathbf{x}_{\kappa,\tau},\mathbf{x}_{\kappa,\tau-1})$ resp. *backward* recursion $\beta_\kappa(\mathbf{x}_{\kappa,\tau},\mathbf{x}_{\kappa,\tau-1})$ runs over pairs of consecutive bit patterns $\mathbf{x}_{\kappa,\tau}$, $\mathbf{x}_{\kappa,\tau-1}$ in the position direction $\kappa$. This algorithm is able to utilize reliability information about the entire past of received sequences $\mathbf{z}_{\text{SBSD}} = \underline{\mathbf{z}}_1^{\tau}$. The determination rule for the *a posteriori* probabilities is given by

$$P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{z}_{\text{SBSD}}) = C\cdot\sum_{\mathbf{x}_{\kappa,\tau-1}}\Big\{\beta_\kappa(\mathbf{x}_{\kappa,\tau},\mathbf{x}_{\kappa,\tau-1})\cdot \sum_{\mathbf{x}_{\kappa-1,\tau},\mathbf{x}_{\kappa-1,\tau-1}}\Big(p(\mathbf{x}_{\kappa,\tau},\mathbf{z}_{\kappa,\tau}\,|\,\mathbf{x}_{\kappa,\tau-1},\mathbf{x}_{\kappa-1,\tau})\cdot P(\mathbf{x}_{\kappa,\tau-1}\,|\,\mathbf{x}_{\kappa-1,\tau-1},\underline{\mathbf{z}}_1^{\tau-1})\cdot\alpha_{\kappa-1}(\mathbf{x}_{\kappa-1,\tau},\mathbf{x}_{\kappa-1,\tau-1})\Big)\Big\}. \qquad (4)$$

Fig. 5 illustrates the close relations between the four contributions.

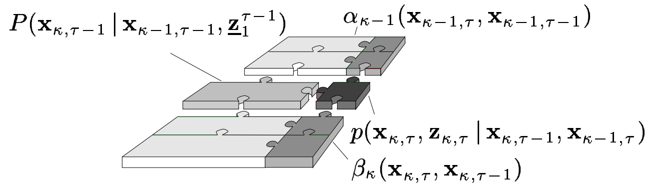


Figure 5: Optimal parameter estimator (OPT)

The center part of Figure 5 describes an extension of Figure 4. Since the desired bit pattern $\mathbf{x}_{\kappa,\tau}$ and the directly preceding pattern $\mathbf{x}_{\kappa,\tau-1}$ depend on adjacent bit patterns $\mathbf{x}_{\kappa-1,\tau}$ or $\mathbf{x}_{\kappa-1,\tau-1}$ respectively as well, the corresponding puzzle pieces have to be refined. The irregular shape expresses both conditional entities.

The puzzle elements denoting the *forward* and *backward* recursion are formed such that they match the reshaped center part. Thus, with respect to the optimal determination rule for the *a posteriori* probabilities $P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{z}_{\text{SBSD}})$, the entire *time-position plane* of received words is exploited. The thorough interlocking of the individual components (see Fig. 5) provides a very robust system.

## 3. EFFICIENT NEAR-OPTIMUM SBSD

The optimal parameter estimator OPT jointly exploits the *inter*- and *intra*-frame redundancy of the source codec parameters. The new efficient, nearly-optimal SBSD version (N-OPT) makes considerable complexity savings possible by separating both terms of residual redundancy. At the same time, the *a posteriori* probabilities determined by N-OPT are still based on the entire history $\underline{\mathbf{z}}_1^{\tau}$ of received sequences and parts of the future $\underline{\mathbf{z}}_{\tau+1}^{\Lambda}$, i.e., $\mathbf{z}_{\text{SBSD}} = \underline{\mathbf{z}}_1^{\Lambda}$. The *a posteriori* probabilities $P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{z}_{\text{SBSD}})$ are determinable by

$$P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{z}_{\text{SBSD}}) = C\cdot\sum_{(\mathbf{x}_{\text{SBSD}}|\mathbf{x}_{\kappa,\tau})} p(\underline{\mathbf{x}}_1^{\Lambda},\underline{\mathbf{z}}_1^{\Lambda}) \quad . \qquad (5)$$

The expression $(\mathbf{x}_{\text{SBSD}}|\mathbf{x}_{\kappa,\tau})$ indicates that the marginal distribution has to be evaluated by summation over all permutations of $\mathbf{x}_{\text{SBSD}} = \underline{\mathbf{x}}_1^{\Lambda}$ with a fixed but arbitrary pattern $\mathbf{x}_{\kappa,\tau}$. That means that with respect to the illustration given in Figure 6 a.) the summation has to be realized successively over all combinations of bit patterns $\mathbf{x}_{k,t}\in\mathbb{X}_k$ with $k=1,\dots M$ and $t=1,\dots\Lambda$ (marked light gray) excluding $\mathbf{x}_{\kappa,\tau}$ itself (marked dark gray). Of course, the formal solution of (5) is by far too complex.

To reduce the computational demands, we assume a 1st order Markov property in time $\tau$ for the sets $\underline{\mathbf{x}}_\tau$. Thus, considering a

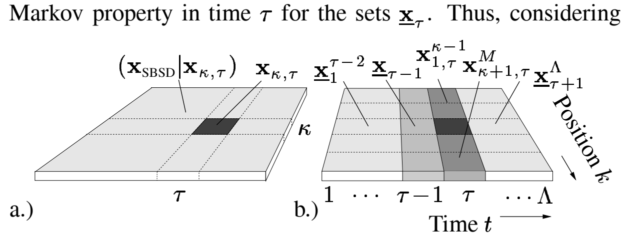


Figure 6: a.) Illustration for the formal solution of Eq. (5)
b.) Partitioning of the formal solution

memoryless transmission channel the joint PDF $p(\underline{\mathbf{x}}_1^{\Lambda},\underline{\mathbf{z}}_1^{\Lambda})$ of (5) can be separated into contributions of future, present, and past sets $\underline{\mathbf{x}}_\tau$. A reordering of the successive summation of (5) provides

$$\begin{aligned} P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{z}_{\text{SBSD}}) = C\cdot \sum_{\mathbf{x}_{1,\tau}^{\kappa-1},\mathbf{x}_{\kappa+1,\tau}^{M}} &\Big\{ \overbrace{\Big(\sum_{\underline{\mathbf{x}}_{\tau+1}^{\Lambda}} p(\underline{\mathbf{x}}_{\tau+1}^{\Lambda},\underline{\mathbf{z}}_{\tau+1}^{\Lambda}\,|\,\underline{\mathbf{x}}_{\tau})\Big)}^{\beta_\tau(\underline{\mathbf{x}}_\tau)} \\ \cdot \sum_{\underline{\mathbf{x}}_{\tau-1}} &\Big[p(\underline{\mathbf{x}}_\tau,\underline{\mathbf{z}}_\tau\,|\,\underline{\mathbf{x}}_{\tau-1})\cdot \underbrace{\Big(\sum_{\underline{\mathbf{x}}_1^{\tau-2}} p(\underline{\mathbf{x}}_1^{\tau-1},\underline{\mathbf{z}}_1^{\tau-1})\Big)}_{\alpha_{\tau-1}(\underline{\mathbf{x}}_{\tau-1})}\Big]\Big\}. \end{aligned} \tag{6}$$

Fig. 6 b.) illustrates the partitioning of the thorough summation over $(\mathbf{x}_{\text{SBSD}}|\mathbf{x}_{\kappa,\tau})$ into the four nested summations of Eq. (6) (each partition has a specific gray scale). In the following the interlacing arrangement of the summations shall be resolved into independent contributions of past, future, and adjacent bit patterns.

Both sums in the parentheses denote *forward* resp. *backward* recursions on sets $\underline{\mathbf{x}}_\tau$, which can efficiently be calculated by

$$\begin{aligned} \alpha_{\tau-1}(\underline{\mathbf{x}}_{\tau-1}) &= \sum_{\underline{\mathbf{x}}_{\tau-2}} p(\underline{\mathbf{x}}_{\tau-1},\underline{\mathbf{z}}_{\tau-1}\,|\,\underline{\mathbf{x}}_{\tau-2})\cdot\alpha_{\tau-2}(\underline{\mathbf{x}}_{\tau-2}) \\ \beta_\tau(\underline{\mathbf{x}}_\tau) &= \sum_{\underline{\mathbf{x}}_{\tau+1}} p(\underline{\mathbf{x}}_{\tau+1},\underline{\mathbf{z}}_{\tau+1}\,|\,\underline{\mathbf{x}}_{\tau})\cdot\beta_{\tau+1}(\underline{\mathbf{x}}_{\tau+1}). \end{aligned} \tag{7}$$

The innovation $p(\underline{\mathbf{x}}_\tau,\underline{\mathbf{z}}_\tau\,|\,\underline{\mathbf{x}}_{\tau-1})$ is the only unknown term in (6), (7) which can also be factorized into preceding, present, and succeeding contributions, but now in position direction $\kappa$,

$$\begin{aligned} p(\underline{\mathbf{x}}_\tau,\underline{\mathbf{z}}_\tau\,|\,\underline{\mathbf{x}}_{\tau-1}) &= p(\mathbf{x}_{\kappa+1,\tau}^{M},\mathbf{z}_{\kappa+1,\tau}^{M}\,|\,\mathbf{x}_{1,\tau}^{\kappa},\mathbf{z}_{1,\tau}^{\kappa},\underline{\mathbf{x}}_{\tau-1}) \\ &\cdot p(\mathbf{x}_{1,\tau}^{\kappa-1},\mathbf{z}_{1,\tau}^{\kappa-1}\,|\,\mathbf{x}_{\kappa,\tau},\mathbf{z}_{\kappa,\tau},\underline{\mathbf{x}}_{\tau-1})\cdot p(\mathbf{x}_{\kappa,\tau},\mathbf{z}_{\kappa,\tau}\,|\,\underline{\mathbf{x}}_{\tau-1}). \end{aligned} \tag{8}$$

Assuming a 1st order Markov property with respect to position $\kappa$, Eq. (8) can be simplified significantly. Moreover, in order to ensure a clear separation of *inter*- and *intra*-frame redundancy, we additionally assume that the first two factors in (8) are independent from the set $\underline{\mathbf{x}}_{\tau-1}$ at the preceding time instant $\tau-1$. Of course, with the latter assumption the optimality of the currently derived algorithm gets lost, because the Markov property in the time domain in the adjacent positions $k=1,\dots M$, $k\neq\kappa$, will not be taken into consideration anymore. Therefore, this SBSD version has been named *near-optimum softbit source decoding* (N-OPT).

Thus, the innovation $p(\underline{\mathbf{x}}_\tau,\underline{\mathbf{z}}_\tau\,|\,\underline{\mathbf{x}}_{\tau-1})$ in (8) can be factorized to

$$\begin{aligned} p(\underline{\mathbf{x}}_\tau,\underline{\mathbf{z}}_\tau\,|\,\underline{\mathbf{x}}_{\tau-1}) = &\overbrace{p(\mathbf{x}_{\kappa,\tau},\mathbf{z}_{\kappa,\tau}\,|\,\mathbf{x}_{\kappa,\tau-1})}^{\textit{inter}\text{-frame dependencies of } \mathbf{x}_{\kappa,\tau}} \\ &\cdot \underbrace{p(\mathbf{x}_{\kappa+1,\tau}^{M},\mathbf{z}_{\kappa+1,\tau}^{M}\,|\,\mathbf{x}_{\kappa,\tau})\cdot p(\mathbf{x}_{1,\tau}^{\kappa-1},\mathbf{z}_{1,\tau}^{\kappa-1}\,|\,\mathbf{x}_{\kappa,\tau})}_{\textit{intra}\text{-frame dependencies of } \mathbf{x}_{\kappa,\tau}}. \end{aligned} \tag{9}$$

Inserting (7), (9) into (6) and rearranging the summations yields

$$\begin{aligned} P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{z}_{\text{SBSD}}) = C\cdot &\sum_{\mathbf{x}_{\kappa,\tau-1}}\Big[p(\mathbf{x}_{\kappa,\tau},\mathbf{z}_{\kappa,\tau}\,|\,\mathbf{x}_{\kappa,\tau-1})\cdot\sum_{\mathbf{x}_{1,\tau-1}^{\kappa-1},\mathbf{x}_{\kappa+1,\tau-1}^{M}}\alpha_{\tau-1}(\underline{\mathbf{x}}_{\tau-1})\Big] \\ \cdot &\sum_{\mathbf{x}_{1,\tau}^{\kappa-1},\mathbf{x}_{\kappa+1,\tau}^{M}}\Big[\beta_\tau(\underline{\mathbf{x}}_\tau)\cdot p(\mathbf{x}_{\kappa+1,\tau}^{M},\mathbf{z}_{\kappa+1,\tau}^{M}\,|\,\mathbf{x}_{\kappa,\tau})\cdot p(\mathbf{x}_{1,\tau}^{\kappa-1},\mathbf{z}_{1,\tau}^{\kappa-1}\,|\,\mathbf{x}_{\kappa,\tau})\Big]. \end{aligned} \tag{10}$$

Some parts of the nested structure of (6) have been resolved. The first line of (10) exhibits information about past sets $\underline{\mathbf{x}}_1^{\tau-1}$ and the pattern $\mathbf{x}_{\kappa,\tau}$ under consideration. The second line comprises knowledge from future sets $\underline{\mathbf{x}}_{\tau+1}^{\Lambda}$ (*backward* recursion $\beta_\tau(\underline{\mathbf{x}}_\tau)$) and adjacent bit patterns $\mathbf{x}_{1,\tau}^{\kappa-1}$, $\mathbf{x}_{\kappa+1,\tau}^{M}$, respectively.

Analogously, the simplified innovation (9) can be introduced to the *forward-backward* recursions (7). Applying the successive summation over $\mathbf{x}_{1,\tau-1}^{\kappa-1}$, $\mathbf{x}_{\kappa+1,\tau-1}^{M}$ of (10) to the *forward* recursion $\alpha_{\tau-1}(\underline{\mathbf{x}}_{\tau-1})$ the reorganized formula reads (note $\tau-1\rightarrow\tilde{\tau}$)

$$\begin{aligned} &\overbrace{\sum_{\mathbf{x}_{1,\tilde{\tau}}^{\kappa-1},\mathbf{x}_{\kappa+1,\tilde{\tau}}^{M}}\alpha_{\tilde{\tau}}(\underline{\mathbf{x}}_{\tilde{\tau}})}^{\alpha_{\tilde{\tau}}^{[\text{TIM}]}(\mathbf{x}_{\kappa,\tilde{\tau}})} = \\ &\sum_{\mathbf{x}_{\kappa+1,\tilde{\tau}}^{M}} p(\mathbf{x}_{\kappa+1,\tilde{\tau}}^{M},\mathbf{z}_{\kappa+1,\tilde{\tau}}^{M}\,|\,\mathbf{x}_{\kappa,\tilde{\tau}})\cdot\sum_{\mathbf{x}_{1,\tilde{\tau}}^{\kappa-1}} p(\mathbf{x}_{1,\tilde{\tau}}^{\kappa-1},\mathbf{z}_{1,\tilde{\tau}}^{\kappa-1}\,|\,\mathbf{x}_{\kappa,\tilde{\tau}}) \\ &\cdot\sum_{\mathbf{x}_{\kappa,\tilde{\tau}-1}}\Big[p(\mathbf{x}_{\kappa,\tilde{\tau}},\mathbf{z}_{\kappa,\tilde{\tau}}\,|\,\mathbf{x}_{\kappa,\tilde{\tau}-1})\cdot\underbrace{\sum_{\mathbf{x}_{1,\tilde{\tau}-1}^{\kappa-1},\mathbf{x}_{\kappa+1,\tilde{\tau}-1}^{M}}\alpha_{\tilde{\tau}-1}(\underline{\mathbf{x}}_{\tilde{\tau}-1})}_{\alpha_{\tilde{\tau}-1}^{[\text{TIM}]}(\mathbf{x}_{\kappa,\tilde{\tau}-1})}\Big]. \end{aligned} \tag{11}$$

As a result, (11) represents a *forward* recursion $\alpha_\tau^{[\text{TIM}]}(\mathbf{x}_{\kappa,\tau})$ for single bit patterns $\mathbf{x}_{\kappa,\tau}$ in the time direction $\tau$. The time relation is indicated by the additional upper subscript TIM (time). The sums in the center line describe the impact of the received words of $\mathbf{z}_{\kappa+1,\tilde{\tau}}^{M}$ and of $\mathbf{z}_{1,\tilde{\tau}}^{\kappa-1}$ on the pattern $\mathbf{x}_{\kappa,\tilde{\tau}}$.

To exploit the *intra*-frame redundancy, the summation over $\mathbf{x}_{\kappa+1,\tilde{\tau}}^{M}$ resp. $\mathbf{x}_{1,\tilde{\tau}}^{\kappa-1}$ combines source statistics as well as channel-related information for the adjacent parameters. Both summations can by itself be measured efficiently using a *forward-backward* algorithm in the position direction (POS, note $\tau-1\rightarrow\tilde{\tau}$)

$$\begin{aligned} \alpha_\kappa^{[\text{POS}]}(\mathbf{x}_{\kappa,\tilde{\tau}}) &= \sum_{\mathbf{x}_{\kappa-1,\tilde{\tau}}} p(\mathbf{x}_{\kappa-1,\tilde{\tau}},\mathbf{z}_{\kappa-1,\tilde{\tau}}\,|\,\mathbf{x}_{\kappa,\tilde{\tau}})\cdot\alpha_{\kappa-1}^{[\text{POS}]}(\mathbf{x}_{\kappa-1,\tilde{\tau}}) \\ \beta_\kappa^{[\text{POS}]}(\mathbf{x}_{\kappa,\tilde{\tau}}) &= \sum_{\mathbf{x}_{\kappa+1,\tilde{\tau}}} p(\mathbf{x}_{\kappa+1,\tilde{\tau}},\mathbf{z}_{\kappa+1,\tilde{\tau}}\,|\,\mathbf{x}_{\kappa,\tilde{\tau}})\cdot\beta_{\kappa+1}^{[\text{POS}]}(\mathbf{x}_{\kappa+1,\tilde{\tau}}). \end{aligned} \tag{12}$$

For initialization serve $\alpha_0^{[\text{POS}]}(\mathbf{x}_{0,\tilde{\tau}})=1$, $\beta_{M+1}^{[\text{POS}]}(\mathbf{x}_{M+1,\tilde{\tau}})=1$.

According to (11), (12), the *backward* recursion of (7) is separable into terms utilizing the *intra*-frame redundancy and terms comprising the non-uniform distribution and *inter*-frame redundancy

$$\begin{aligned} \beta_\tau^{[\text{TIM}]}(\mathbf{x}_{\kappa,\tau}) = &\sum_{\mathbf{x}_{\kappa,\tau+1}}\alpha_\kappa^{[\text{POS}]}(\mathbf{x}_{\kappa,\tau+1})\cdot\beta_\kappa^{[\text{POS}]}(\mathbf{x}_{\kappa,\tau+1}) \\ &\cdot p(\mathbf{x}_{\kappa,\tau+1},\mathbf{z}_{\kappa,\tau+1}\,|\,\mathbf{x}_{\kappa,\tau})\cdot\beta_{\tau+1}^{[\text{TIM}]}(\mathbf{x}_{\kappa,\tau+1}) \end{aligned} \tag{13}$$

with $\beta_\tau^{[\text{TIM}]}(\mathbf{x}_{\kappa,\tau})=\beta_\tau(\underline{\mathbf{x}}_\tau)$. Introducing (11), (12), and (13) to (10) allows to rewrite the determination rule for the *a posteriori* probabilities $P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{z}_{\text{SBSD}})$ of N-OPT as

$$\begin{aligned} P(\mathbf{x}_{\kappa,\tau}\,|\,\mathbf{z}_{\text{SBSD}}) = C\cdot&\beta_\tau^{[\text{TIM}]}(\mathbf{x}_{\kappa,\tau})\cdot\alpha_\kappa^{[\text{POS}]}(\mathbf{x}_{\kappa,\tau})\cdot\beta_\kappa^{[\text{POS}]}(\mathbf{x}_{\kappa,\tau}) \\ \cdot&\sum_{\mathbf{x}_{\kappa,\tau-1}} p(\mathbf{x}_{\kappa,\tau},\mathbf{z}_{\kappa,\tau}\,|\,\mathbf{x}_{\kappa,\tau-1})\cdot\alpha_{\tau-1}^{[\text{TIM}]}(\mathbf{x}_{\kappa,\tau-1}) \end{aligned} \tag{14}$$

with $\mathbf{z}_{\text{SBSD}}=\underline{\mathbf{z}}_1^{\Lambda}$. For initialization serve $\alpha_0^{[\text{TIM}]}(\mathbf{x}_{\kappa,0})=P(\mathbf{x}_{\kappa,0})$ and $\beta_\Lambda^{[\text{TIM}]}(\mathbf{x}_{\kappa,\Lambda})=1$. Figure 7 illustrates Eq. (14).

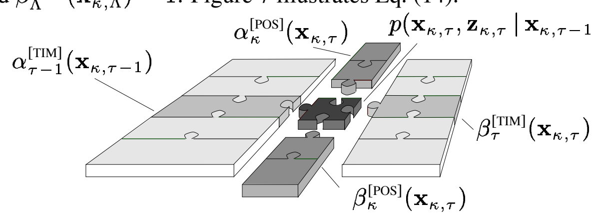


Figure 7: *Near-optimum softbit source decoding* (N-OPT)

In order to determine the *a posteriori* reliability of $\mathbf{x}_{\kappa,\tau}$ (dark gray unit) the position-related recursions $\alpha_\kappa^{[\text{POS}]}(\mathbf{x}_{\kappa,\tau})$, $\beta_\kappa^{[\text{POS}]}(\mathbf{x}_{\kappa,\tau})$ exploit the mutual dependencies to all parameters of the present set at time $\tau$, i.e., $\mathbf{x}_{1,\tau}^{\kappa-1}$ and $\mathbf{x}_{\kappa+1,\tau}^{M}$ (medium gray). Due to the time-related recursions $\alpha_{\tau-1}^{[\mathrm{TIM}]}(\mathbf{x}_{\kappa,\tau-1})$, $\beta_{\tau}^{[\mathrm{TIM}]}(\mathbf{x}_{\kappa,\tau})$ the predecessors and (if available) the successors of these elements (light gray) are evaluated as well. Due to the recursive evaluation all future sets up to the maximum look-ahead $\Lambda$ are utilizable.

The N-OPT version of SBSD exploits the entire received sequence $\mathbf{z}_{\text{SBSD}} = \underline{\mathbf{z}}_1^{\Lambda}$.The interlocking of puzzle pieces is specific because of the separate evaluation of *inter*- and *intra*-frame redundancy. In contrast to the optimal *softbit source decoding* version OPT some neighboring pieces do not interlock at all. In consequence, some robustness will be lost. However, significant savings of computational complexity and data memory become possible.

The number of arithmetical operations required by OPT to quantify the *a posteriori* probabilities $P(\mathbf{x}_{\kappa,\tau} \,|\, \mathbf{z}_{\text{SBSD}})$ is mainly determined by the three nested summations of (4). The data memory demands are mainly characterized by the storage demands for $P(\mathbf{x}_{\kappa,\tau} \,|\, \mathbf{x}_{\kappa,\tau-1}, \mathbf{x}_{\kappa-1,\tau})$. In both cases complexity of the order $O(|\mathbb{X}_\kappa|^3)$ is required. In contrast, as the *inter*- and *intra*-frame redundancy exploited by N-OPT is stored in two separate tables $P(\mathbf{x}_{\kappa,\tau} \,|\, \mathbf{x}_{\kappa,\tau-1})$, $P(\mathbf{x}_{\kappa,\tau} \,|\, \mathbf{x}_{\kappa-1,\tau})$ and as the summations being inherent in (14) are not nested anymore, complexity of the order $O(|\mathbb{X}_\kappa|^2)$ is needed. Thus, the savings have to be considered as significant in particular for codebook sizes $|\mathbb{U}_\kappa| = |\mathbb{X}_\kappa|$ typically used in modern communication systems.

Finally, it has to be mentioned that nearly-optimal versions to SBSD regarding a clear separation of *inter*- and *intra*-frame redundancy have already been studied, e.g., in [4–6]. But, in none of these approaches the entire *time-position plane* of received words is exploited $\mathbf{z}_{\text{SBSD}} = \underline{\mathbf{z}}_1^{\Lambda}$. Therewith, some mutual dependencies remain unutilized. Thus, the new efficient N-OPT version marks a remarkable advancement. As the proposed algorithm is based on two nested *forward-backward* algorithms, one in the time domain and one in position, it is able to provide *a posteriori* probabilities of highest accuracy. At the same time, the computational efficiency (compared to [4–6]) is maintained due to the recursive description.

## 4. SIMULATION RESULTS

For simulations we used the generic source model proposed in [2]. This model allows to adjust correlation properties in time $\tau$ by the *auto correlation factor* $\rho$ and in position $\kappa$ by the *cross correlation factor* $\delta$. In our experiments we use values for $\rho$, $\delta$ which resemble those of realworld applications. For instance, correlation measurements have shown that the differentially encoded *line spectral frequencies* (LSFs) of the GSM-EFR speech codec exhibit a mean *inter*-frame correlation of about $\rho = 0.6$ and *intra*-frame correlation of about $\delta = 0.75$. For the scale factors of audio codec applied to *digital audio broadcasting* (DAB) correlation of about $\rho = 0.95$ and $\delta = 0.8$ was determined [2, 3].

The number of parameters $u_{\kappa,\tau}$ per frame $\underline{u}_\tau$ is set to $M = 10$. The parameters are quantized by a 16-level Lloyd-Max quantizer using 4 bits each. The index assignment is optimized with respect to the MMSE [7]. As transmission channel serves an *additive white Gaussian noise* (AWGN) channel with known $E_s/N_0$.

Fig. 8 depicts the *parameter signal-to-noise ratio* (SNR) between the originally generated parameter $u_{\kappa,\tau}$ and its reconstruction $\hat{u}_{\kappa,\tau}$ as a function of the channel quality $E_s/N_0$.

The bottom curve shows the simulation result for conventional parameter decoding by *hard decision* (HD) and table-lookup in the quantizer codebook. All version to SBSD are able to outperform the reference by several dB in terms of the *parameter* SNR. Note, neither of them makes use of a look-ahead, i.e., $\Lambda = \tau$ (no interpolation). N-OPT reaches the performance bounds of OPT quite closely in a wide range of channel conditions. The maximum loss

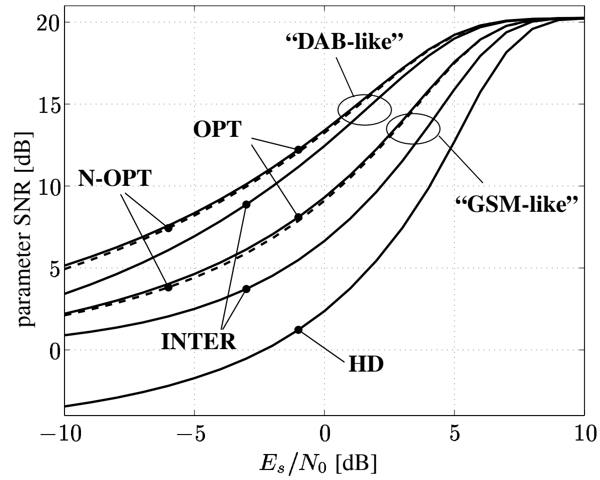


Figure 8: Simulation results

of parameter SNR counts to $\Delta_{\text{SNR}} = 0.24$ dB for the GSM-like settings and to $\Delta_{\text{SNR}} = 0.21$ dB for the DAB-like settings.

If the channel quality decreases the performance loss compared to OPT slightly increases. However, N-OPT is still (much) better than INTER. Although the algorithm of N-OPT comprises remarkable simplifications over OPT and although it resembles the INTER technique with an additional *position*-related recursion, N-OPT yields significant benefits. Compared to INTER, the maximum parameter SNR gain amounts to $\Delta_{\text{SNR}} = 2.49$ dB (GSM-like settings). For the DAB-like settings improvements of about $\Delta_{\text{SNR}} = 1.51$ dB in terms of parameter SNR are obtainable.

## 5. CONCLUSIONS

In this paper, we derived a new efficient, nearly-optimal version to *softbit source decoding*. The new algorithm evaluates the *inter*- and *intra*-frame redundancy of source codec parameters separately using two *forward-backward* recursions (one in the time domain and one in position). The performance loss compared to the joint utilization of both terms of redundancy is negligibly small while the potential for saving computational complexity and data memory is substantial. All SBSD versions are clarified by illustrations.

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