ROBUST RECONSTRUCTION OF MOTION VECTORS USING FRAME EXPANSION

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ABSTRACT

Transmitting video streams on channels impaired with transmission errors is a very demanding task, mainly when images are predicted from previous ones. In this case, errors on motion vectors can be very harmful. In order to overcome this problem, this paper presents a modified H263+ scheme without motion vector transmission. This is obtained by reestimating these motion vectors at the receiver, based on a properties of frame expansions. This procedure is obtained at the cost of an increased bit rate, but shows that robust (and efficient) transmission can indeed be obtained in conjunction with image prediction.

1. INTRODUCTION

The reliable transmission of video contents through mobile or mixed internet-mobile channels is a problem of current interest. The main difficulty comes from the fact that when the coding efficiency of source coders increases, so does the sensitivity of the generated bitstream to transmission errors. Video coders which are robust to transmission errors have thus to be developed, particularly when no ARQ is feasible.

Several robustification schemes have been proposed. For example, channel coding can be employed in the context of satellite television or for systems having a broad bandwidth. When the constraints in term of bandwidth are stronger, joint source-channel coding can be an interesting alternative [1]. Results were obtained mainly with multiple descriptions schemes, by using scalable modes offered by MPEG2 [2] or H263 [3]. Soft decoding of VLC codes taking into account the structure of the bitstream generated by H263+ have also been considered [4]. However, these techniques are still very sensitive to motion vectors (MV) transmission errors.

In this paper, joint source-channel coding is performed by introducing redundancy in the video sequence to be encoded using *frame expansion*. This redundancy can be realized by imposing some specific property in each picture of the video sequence. This property has to be partly preserved in each picture of the reconstructed video sequence. MV transmission is then no more necessary, as they can be reestimated choosing the ones which enforce the required property to the reconstructed video sequence (assuming the texture transmission errors to be corrected).

Frames have found many applications in signal processing and communications. The redundant representation of a vector or a signal obtained after expansion on a frame makes it possible, e.g., to improve numerical conditioning at reconstruction [5] or robustness to quantization noise [6]. More recently frames have been applied to multiple description coding of still images, see, e.g. [7]. An analogy between frames and error correcting codes has been introduced by [8]. Error correcting algorithms have been proposed in [9] for frames of \mathbb{R}^k and \mathbb{C}^k and in [10] for frames of $\ell_2(\mathbb{Z})$.

Here, only frames of \mathbb{R}^k are considered, as they are adapted for the expansion of real valued images. The frame expansion must have good interpolation properties in order to preserve satisfying source coder performances. BCH frames, *i.e.*, frames corresponding to BCH codes on the reals [11] satisfy these two constraints. Joint source-channel coding using frame expansion with BCH frames will be presented in Section 2. Section 3 introduces the modified H263+ codec including frame expansion and MV reestimation using the redundancy introduced by the frame expansion. An example is presented in Section 4.

2. JOINT SOURCE-CHANNEL CODING USING THE FRAME EXPANSION

In this paper, some properties of frames of \mathbb{R}^k will be briefly

recalled. For more details, see [5] or [12]. A set $\Phi = \{\varphi_m\}_{m=1}^n \subset \mathbb{R}^k$ of $n \ge k$ vectors of \mathbb{R}^k is a frame of \mathbb{R}^k , if there exits A > 0 and $B < +\infty$ such that

$$A \|\mathbf{x}\|^{2} \leq \sum_{k=1}^{n} |\langle \mathbf{x}, \boldsymbol{\varphi}_{k} \rangle|^{2} \leq B \|\mathbf{x}\|^{2} \text{ for all } \mathbf{x} \in \mathbb{R}^{k}.$$

A frame Φ is *tight* if A = B, it is *uniform* if $\|\varphi_m\| = 1$ for all m = 1, ..., n. Finally, a *Parseval* frame is such that A = B = 1 [13].

The linear operator \mathbf{F} associated with the analysis frame expands vectors of \mathbb{R}^k into vectors of \mathbb{R}^n , as $(\mathbf{Fx})_i = \langle \varphi_i, \mathbf{x} \rangle$.

This work has been partly funded by COSOCATI and VIP RNRT Projects

F is an $n \times k$ matrix whose rows are the transposed vectors of Φ . The expansion on a frame being a redundant representation, the operator associated with the synthesis frame is not unique [14]. An example of a synthesis frame is the pseudo-inverse $\mathbf{F}^p = (\mathbf{F}^H \mathbf{F})^{-1} \mathbf{F}^H$ of **F**.

2.1. Analogy between frames and error correcting codes

In this part, largely inspired from [8], a Parseval frame operator $\mathbf{F}_{(n,k)}$ is considered. It transforms a vector $\mathbf{x}_{(k)} \in \mathbb{R}^k$ into a vector $\mathbf{c}_{(n)} \in \mathbb{R}^n$. The singular value decomposition of $\mathbf{F}_{(n,k)}$ can be written as

$$\mathbf{F}_{(n,k)} = \mathbf{U}_{(n)} \begin{pmatrix} \mathbf{I}_{(k,k)} \\ \mathbf{0}_{(n-k,k)} \end{pmatrix} \mathbf{V}_{(k)}, \qquad (1)$$

where $\mathbf{U}_{(n)}$ and $\mathbf{V}_{(k)}$ are two unitary matrices; $\mathbf{I}_{(k,k)}$ and $\mathbf{0}_{(n-k,k)}$ are respectively the identity and null matrix. This decomposition evidences the redundancy introduced by the expansion. Let $\mathbf{\tilde{x}}_{(k)} = \mathbf{V}_{(k)}\mathbf{x}_{(k)}$ and

$$\widetilde{\mathbf{c}}_{(n)} = \begin{pmatrix} \mathbf{I}_{(k,k)} \\ \mathbf{0}_{(n-k,k)} \end{pmatrix} \widetilde{\mathbf{x}}_{(k)} = \begin{pmatrix} \mathbf{V}_{(k)} \mathbf{x}_{(k)} \\ \mathbf{0}_{(n-k)} \end{pmatrix}$$

Thus $\widetilde{\mathbf{c}}_{(n)}$ is obtained by inserting n - k zeros to $\widetilde{\mathbf{x}}_{(k)}$ and $\mathbf{c}_{(n)} = \mathbf{U}_{(n)}\widetilde{\mathbf{c}}_{(n)}$ belongs to a subspace of dimension k of \mathbb{R}^n . The n - k inserted zeros form a *syndrome*, which can be evaluated using a *parity check* matrix $\mathbf{H}_{(n-k,n)}$ defined as

$$\mathbf{U}_{(n)}^{-1} = \begin{pmatrix} \mathbf{H}_{(k,n)}' \\ \mathbf{H}_{(n-k,n)} \end{pmatrix}$$

When no transmission errors corrupts $\mathbf{c}_{(n)}$, the syndrome calculated from the received vector $\mathbf{r}_{(n)}$

$$\mathbf{s}_{(n-k)} = \mathbf{H}_{(n-k,n)}\mathbf{r}_{(n)} \tag{2}$$

is null. This is no more the case when $\mathbf{c}_{(n)}$ is corrupted by (quantization) noise.

2.2. Frames and BCH codes on the real field

The family of BCH(n, k) codes on reals [11] constitutes a particular class of frame of \mathbb{R}^k . The principle of BCH codes on the reals can be viewed as inserting zeros in the spectrum of the code word on a set \mathcal{A} of n - k frequencies between 0 and n - 1. Let $\delta_{\mathcal{A}}(i)$ be the indicator function of \mathcal{A} , $\delta_{\mathcal{A}}(i) = 1$ if $i \in \mathcal{A}$ and $\delta_{\mathcal{A}}(i) = 0$ else. The BCH encoding transforms an *information word* $\mathbf{x}_{(k)} \in \mathbb{R}^k$ into a *code word* $\mathbf{c}_{(n)} \in \mathbb{R}^n$ as

$$\mathbf{c}_{(n)} = \mathbf{W}_{(n)} \mathbf{P}_{(n,k)} \mathbf{W}_{(k)}^{-1} \mathbf{x}_{(k)} = \mathbf{F}_{(n,k)}^{\text{BCH}} \mathbf{x}_{(k)}, \qquad (3)$$

where $\mathbf{W}_{(k)}$ and $\mathbf{W}_{(n)}$ are the unitary matrices (for example associated with Fourier transform, discrete cosine transform, ...) and $\mathbf{P}_{(n,k)}$ is a matrix of zeros padding with

 $\left(\mathbf{P}_{(n,k)}\right)_{\ell, m} = 1 \text{ if } \delta_{\mathcal{A}}\left(\ell\right) = 0 \text{ and } m = \sum_{j=0}^{\ell} \delta_{\overline{\mathcal{A}}}\left(j\right) - 1$ and $\left(\mathbf{P}_{(n,k)}\right)_{\ell, m} = 0$ in the other cases.

Using Naimark's theorem [15], it is possible to show that the operator $\mathbf{F}_{(n,k)}^{\text{BCH}}$ associated with BCH codes on the reals is a Parseval frame operator. Such frames will be called *BCH frames*.

The syndrome and parity-check matrices of BCH frames are

$$\mathbf{s}_{(n-k)} \left(\mathbf{r}_{(n)} \right) = \mathbf{R}_{(n-k,n)} \left(\mathbf{W}_{(n)} \right)^{-1} \mathbf{r}_{(n)} = \mathbf{H}_{(n-k,n)} \mathbf{r}_{(n)},$$

with $\left(\mathbf{R}_{(n-k,n)} \right)_{\ell,m} = 1$ if $\delta_{\mathcal{A}}(m) = 1$ and $m = \ell + \sum_{i=0}^{m} \delta_{\overline{\mathcal{A}}}(j)$ and $\left(\mathbf{R}_{(n-k,n)} \right)_{\ell,m} = 0$ in the other cases.

Two type of BCH frames will be considered. Frames based on the Fourier transform $\mathbf{F}_{(n,k)}^{\text{BCH-F}}$ are such that $\mathbf{W}_{(k)}$ and $\mathbf{W}_{(n)}$ are Fourier transform matrices of dimension k and n. Obtaining real code words $\mathbf{c}_{(n)}$ from real information words $\mathbf{c}_{(k)}$ requires thus that \mathcal{A} satisfies

if
$$f \in \mathcal{A}$$
 then $n - f \in \mathcal{A}$.

For frames based on the discrete cosine transform $\mathbf{F}_{(n,k)}^{\text{BCH-D}}$, this condition vanishes as $\mathbf{W}_{(k)}$ and $\mathbf{W}_{(n)}$ are real discrete cosine transform matrices of dimension k and n.

2.3. Product expansions

Images are stored as real valued matrices, thus the redundancy has to be introduced by a product frame expansion of blocks $\mathbf{X}_{(k,k)}$ of size $k \times k$ of the images of the initial video sequence to get expanded blocks of size $n \times n$ evaluated as

$$\mathbf{C}_{(n,n)} = \mathbf{F}_{(n,k)}^{\mathrm{BCH}} \mathbf{X}_{(k,k)} \left(\mathbf{F}_{(n,k)}^{\mathrm{BCH}} \right)^{\mathrm{T}}.$$
 (4)

The inverse transformation is obtained by a synthesis on the rows followed by a synthesis on the columns.

$$\widehat{\mathbf{X}}_{(k,k)} = \left(\mathbf{F}_{(n,k)}^{\text{BCH}}\right)^{\text{p}} \mathbf{C}_{(n,n)} \left(\left(\mathbf{F}_{(n,k)}^{\text{BCH}}\right)^{\text{p}}\right)^{\text{T}}, \quad (5)$$

where the pseudo-inverse $\left(\mathbf{F}_{(n,k)}^{\text{BCH}}\right)^p$ of $\mathbf{F}_{(n,k)}^{\text{BCH}}$ is

$$\left(\mathbf{F}_{(n,k)}^{ ext{BCH}}
ight)^{ extsf{p}} = \mathbf{W}_{(k)}^{ ext{BCH}} \left(\mathbf{P}_{(n,k)}^{ ext{BCH}}
ight)^{ ext{T}} \left(\mathbf{W}_{(n)}^{ ext{BCH}}
ight)^{-1}.$$

The syndrome can then be defined as

$$\mathbf{S}_{(n,n)}\left(\mathbf{C}_{(n,n)}\right) = \mathbf{W}_{(n)}^{-1}\mathbf{C}_{(n,n)}\left(\mathbf{W}_{(n)}^{-1}\right)^{\mathrm{T}} - \left(\mathbf{F}_{(n,k)}^{\mathrm{BCH}}\right)^{\mathrm{p}}\mathbf{C}_{(n,n)}\left(\left(\mathbf{F}_{(n,k)}^{\mathrm{BCH}}\right)^{\mathrm{p}}\right)^{\mathrm{T}}$$
(6)

and contains $n \times n - k \times k$ non-zero elements when $C_{(n,n)}$ is corrupted by some transmission errors.

3. MODIFIED H263+ ENCODER SCHEMES

Figure 1 presents a modified H263+ coding scheme incorporating a BCH frame expansion. The BCH and IBCH blocks correspond respectively to the expansion and synthesis on a BCH product frame according to (4) and (5).



Fig. 1. H263+ codec integrating a BCH frame expansion

It is assumed in the remainder of this paper that all transmission errors affecting the texture have been corrected and that no MV have been sent to the modifier H263+ decoder. MV reestimation has then to be realized.

Consider a macroblock $\mathbf{M}_{(n,n)}$. At encoder side, the motion compensation (MC) consists in finding the MV $\hat{\mathbf{m}} = (m_x, m_y)^{\mathrm{T}}$ that minimizes, *e.g.* $J_{\mathrm{MC}}(\mathbf{m})$, which is, the Frobenius norm

$$J_{\mathrm{MC}}\left(\mathbf{m}\right) = \left\|\mathbf{T}_{(n,n)}\left(\mathbf{m}\right)\right\|_{\mathrm{F}} = \sqrt{\sum_{1 \leq i, j \leq n} \left|t_{i,j}\left(\mathbf{m}\right)\right|^{2}} \quad (7)$$

of the texture

$$\mathbf{T}_{(n,n)}\left(\mathbf{m}\right) = \mathbf{M}_{(n,n)} - \mathbf{X}_{(n,n)}\left(\mathbf{m}\right). \tag{8}$$

 $\mathbf{X}_{(n,n)}(\mathbf{m})$ is a block of size $n \times n$ extracted from some search area $\mathbf{N}_{(\ell,\ell)}$ of the previously rebuilt image. The location of $\mathbf{X}_{(n,n)}(\mathbf{m})$ is deduced from that of $\mathbf{M}_{(n,n)}$ by a translation of \mathbf{m} .

Assume that $\mathbf{T}_{(n,n)}(\mathbf{\hat{m}})$ results from the quantization of $\mathbf{T}_{(n,n)}(\mathbf{\hat{m}})$. At decoder side, $\mathbf{\widetilde{T}}_{(n,n)}(\mathbf{\widehat{m}})$ is transmitted and $\mathbf{N}_{(\ell,\ell)}$ is taken from the previously rebuilt image. To reestimate $\mathbf{\widehat{m}}$, the property that the reconstructed macroblock $\mathbf{\widehat{M}}_{(n,n)}$ is an estimate of $\mathbf{M}_{(n,n)}$ which results from the expansion on a BCH frame is put at work. The first idea that

comes to mind is to build an estimate \widehat{m}_{BCH} of \widehat{m} by minimizing the criterion

$$J_{\text{BCH}}\left(\mathbf{m}, \widetilde{\mathbf{T}}_{(n,n)}\left(\widehat{\mathbf{m}}\right)\right) = \\ \left\| \mathbf{S}_{(n,n)}\left(\widetilde{\mathbf{T}}_{(n,n)}\left(\widehat{\mathbf{m}}\right) + \mathbf{X}_{(n,n)}\left(\mathbf{m}\right) \right) \right\|_{\text{F}} (9)$$

However, there is no guarantee that $\hat{\mathbf{m}}$ is the argument of the global minimum of $J_{\text{BCH}}(\mathbf{m})$. Without quantization error, $\hat{\mathbf{m}}$ is *one* of the arguments of the global minimum of $J_{\text{BCH}}(\mathbf{m})$, but it is not necessarily unique. To improve the robustness of the MV reestimation, the classical MC has to be replaced by a *robust MC* which has to ensure at the *encoder side* that the transmitted MV will be reestimated without error.

A list $\mathcal{L} = {\mathbf{m}_1, \mathbf{m}_2, \dots, \mathbf{m}_N}$ of MV is generated by the standard H263+ MC satisfying

$$J_{\mathrm{MC}}(\mathbf{m}_1) \leqslant J_{\mathrm{MC}}(\mathbf{m}_2) \leqslant \cdots \leqslant J_{\mathrm{MC}}(\mathbf{m}_N).$$

 $\mathbf{T}_{(n,n)}(\mathbf{m}_1)$ and \mathbf{m}_1 are then transmitted. The robust MC has to transmit $\mathbf{m}_{\underline{k}}$ and $\mathbf{T}_{(n,n)}(\mathbf{m}_{\underline{k}})$ where

$$\underline{k} = \min \left\{ k \mid \mathbf{m}_{k} = \arg \min_{\mathbf{m}} J_{\text{BCH}} \left(\mathbf{m}, \widetilde{\mathbf{T}}_{(n,n)} \left(\mathbf{m}_{k} \right) \right) \right\},\$$

when such \underline{k} exists. If all texture transmission errors are corrected, the correct reestimation of $\mathbf{m}_{\underline{k}}$ by minimizing (9) at decoder side is ensured at encoder side. When no \underline{k} can be obtained, \mathbf{m}_1 and $\mathbf{T}_{(n,n)}$ (\mathbf{m}_1) may be transmitted, without guarantee that \mathbf{m}_1 can be correctly reestimated.

The IBCH block at decoder side may correct some high amplitude quantization errors as in [16]. It is followed by a BCH block to obtain again images serving as references for MC. The same decoder has to be present at encoder side in order to prevent drift.

Remark 1 This technique could also be put at work for detection and correction of erroneously transmitted MV. At decoder side, without quantization and transmission error, all reconstructed macroblocks should possess a null syndrome matrix (6). However, the texture quantization introduces noise variance the variance of which can be estimated. The syndrome matrix is no more null, and it is possible to calculate the probability distribution of its Frobenius norm of the syndrome under the two hypotheses that there has been no MV transmission error (H_0) and there has been MV transmission errors (H_1). An hypothesis test can then be derived to detect transmission errors corrupting a given macroblock. MV correction is then done by simple reestimation of the errorneous MV.

4. EXAMPLE

The modified H263+ scheme is tested on the first 101 luminance frames of the video sequence *foreman*. The MC is

realized with a 1 pixel accuracy. The first frame is INTRA coded, all the others are INTER coded. Two BCH frames have been put at work : a Fourier-based BCH frame $\mathbf{F}_{(16,15)}^{\text{BCH-F}}$ with $\mathcal{A} = \{8\}$ and a DCT-based BCH frame $\mathbf{F}_{(16,15)}^{\text{BCH-D}}$ with $\mathcal{A} = \{4\}$. For $\mathbf{F}_{(16,15)}^{\text{BCH-D}}$, the set \mathcal{A} has been optimized to obtain the best coding and MV reestimation performances. The quantization parameter QP adjusts the quality of compressed video flow. Before expansion, the original images were supplemented by symmetry so that their dimensions are multiple of 15. Table 1 presents the results of the modified H263+ coder incorporation the robust MC, so that no MV is transmitted.

	QP	16	24
H263+	bit stream size	118 kB	69kB
	PSNR	32.2	29.9
	bit stream size	261 kB	181kB
${f F}_{(16,15)}^{ m BCH-F}$	variation (%)	122	162
$\mathcal{A} = \{8\}$	MV prop. (%)	3.64	5.18
	PSNR	31.6	29
	bit stream size	216 kB	136kB
$\mathbf{F}_{(16,15)}^{\text{BCH-D}}$	variation (%)	84	97
$\mathcal{A} = \{4\}$	MV prop. (%)	3.13	4.72
	PSNR	31.1	28.6

Table 1. Performances of the original H263+ and BCH-frame H263+ video schemes

With both schemes, it a possible to obtain a satisfying quality without transmission of the MV. The increase of the size of the bitstream is due to three factors. (i) For the robust schemes, the size of the pictures to encode has been enlarged before expansion, in order to get dimensions that are multiple of 15. (ii) Redundancy has been introduced due to the expansion. (iii) Finally, the MC is less efficient than in the original H263+ scheme, as only MV that are guaranteed to be reestimated by the minimization of (9) are considered. Textures have thus higher variances and the size of the bitstream increases consequently. Table 1 also shows that for the considered QP, the proportion occupied by the MV is marginal and their removal form the bitstream results only in a partial compensation for the redundancy introduced by the expansion.

5. CONCLUSIONS

In this paper, we showed that using BCH frame expansion of video sequences, it is possible to avoid the transmission of MV generated by video coding schemes such as H263+. The principle is to use the fact that the reconstructed images have to satisfy the constraint introduced by the frame expansion at encoder side.

The performances are reasonable compared to an H263+

scheme where MV are transmitted. Further research is being undertaken in order to combine the results presented with techniques such as soft VLC decoding by taking into account source semantics [4] in order to get a reliable estimate of the textures. This technique could also be combined with video encoders based on 2D+t wavelets expansions, in

6. REFERENCES

order again to avoid transmission of MV.

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